Securitization, Ratings, and Credit Supply *

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Abstract

We explore the effect of credit ratings on loan origination and securitization. The model involves two stages: first, banks decide whether to originate a given loan pool or not, and obtain private information about the pools originated. Second, each bank chooses what portion of the pool's cash flow rights to retain and what portion to securitize. All securities are rated and sold to competitive investors. We characterize how credit ratings affect the trade-off between *productive efficiency* (i.e., efficiency of the origination process) and allocative efficiency (i.e., efficiency of the securitization process). In particular, we show that credit ratings increase allocative efficiency by reducing costly retention and increase the supply of credit, but reduce average quality of loans originated and can lead too an oversupply of credit relative to first best. These findings are in contrast to regulators view of credit ratings as a disciplining device. We consider extensions of the model to allow for rating shopping and manipulation. Provided investors are fully rational, shopping/manipulation have effects similar to reducing the informativeness of ratings.

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1 Introduction

Asset-backed securitization is an important driver of credit supply (Loutskina and Strahan (2009) and Shivdasani and Wang (2011)). In recent years, we have witnessed growth in the securitization of many asset classes including mortgages, student loans, commercial loans, auto loans and credit card debt. In particular, the practice financed between 30% and 75% of these consumer lending markets, funding approximately 25% of outstanding US consumer credit (Gorton and Metrick (2012)). The expansion of securitized products has been facilitated by credit rating agencies (CRAs), which have allowed securitizers access to large pools of institutional investors that likely would not have participated in markets for unrated securities (Pagano and Volpin (2010)).

In the aftermath of the recent financial crisis, the role of both asset-backed securitization and rating agencies has been under intense scrutiny.[1] A variety of regulations have been proposed as an attempt to discipline these markets, some of which have already been implemented. For example, the Dodd Frank Act imposed a mandatory "skin in the game" rule on securitizers of assets, and it established disclosure requirements on rating agencies. Clearly, there are important interactions between the information content of ratings and banks' decisions of which loans to originate and securitize. Yet, surprisingly, the academic literature has little to say about these interactions. In this paper, we propose a stylized model to jointly explore the effects of both securitization and ratings on the incentives for loan originators and the implications for the overall supply of credit.

In the model, there is a continuum of banks and a set of competitive investors. Each bank has access to a loan pool, and uses its a screening technology to acquire information about the quality of the pool and then decide whether to fund the loans – the *origination stage*. After origination, banks have an incentive to reallocate the cash flow rights from their loan pools to investors (e.g., due to capital constraints) and do so by selling securities backed by the loan pool in the secondary market – the *securitization stage*. At the latter stage, the bank has private

[1] See Dell'Ariccia et al. (2009), Keys et al. (2010), Jaffee et al. (2009), Mian and Sufi (2009), Agarwal et al. (2012) for how securitization negatively affected lending standards; and Pagano and Volpin (2010) and Benmelech and Dlugosz (2010) for the role, and failures, of CRAs in the securitization process.

information that hinders the efficient allocation of cash flow rights, which in turn distorts its incentives during the origination stage. We interpret the bank's private information about the quality of a given loan pool as information about the exposure of that pool to an economic downturn.

We explore two channels through which information can be conveyed to investors to mitigate these distortions. First, as in Leland and Pyle (1977), the bank can retain some fraction of the loan pool in order to signal its quality to investors. Second, information about the pool of loans underlying each security can be conveyed to investors through a public signal, which we interpret as a *rating*. In order to understand the role of each channel, we begin by analyzing a benchmark with neither as well as benchmarks with ratings and retention separately.

We start by analyzing an originate-to-distribute model in which neither channel is present. That is, banks sells 100% of the loan pool that it originates without obtaining a rating. In this case, the market price for a loan pool in the secondary market is independent of loan quality, which (combined with zero retention) provides no incentive for banks to screen loans during the origination stage. Rather, banks are motivated purely by "volume lending." In the originate-to-distribute equilibrium, a banks originates its loan pools if its expected quality it at least equal to the secondary market price. Since the market price reflects average quality, the average NPV of loan pools originated in the economy is zero! Thus, the originate-to-distribute benchmark involves lending standards that are too low and an oversupply of credit. Moreover, since the secondary price for loans is low, the bank has an incentive to retain good loan pools on its balance sheet.

This motivates our analysis of a second benchmark where the bank optimally chooses how much of the loan pool cash flows to retain. In this second benchmark, the (least-cost) separating equilibrium is the unique outcome of the securitization stage. The bank retains a positive fraction of a good pool and sells 100% of a bad pool. By doing so, investors learn the quality of each loan sold on the secondary market and prices fully reflect all available information. However, because retention is costly, the bank does not realize the full social value, which leads to inefficiently high lending standards and an undersupply of credit.

We then introduce ratings to the model. After the retention decision, but prior to the sale of a security, a rating is publicly announced. We ask how the presence of ratings affect what loans are originated. One natural intuition is that informative ratings will lead to tighter lending standards and increase the quality of loans made. We confirm this intuition is correct in the originate-to-distribute environment (where the retention channel is not present). If banks securitize and sell all of the loans they make regardless of loan quality or rating accuracy, then introducing ratings (or increasing their accuracy) improves the efficiency of loan origination.

The effect of ratings on lending standards and credit supply are more nuanced when the bank optimally chooses its retention level. When ratings are sufficiently informative, the bank relies on them for conveying information to investors and relies less on retention.[2] Since retention is costly and inefficient, ratings improves efficiency in the securitization stage, but because less is being retained, the introduction of ratings actually increases incentives to originate lower quality loans and may induce an *oversupply of credit*. However, the increase in allocative efficiency more than compensates for any decrease in productive efficiency and ratings improve overall ex-ante efficiency. To summarize, the introduction of ratings improves allocative efficiency but may or may not improve productive efficiency.

Our model can be used to evaluate different regulations. An intuitive and often proposed regulation is to require banks to retain a fraction of all loans issued. Proponents argue this will provide incentives for banks to make good loans by ensuring that they have some "skin in the game". Critics argue that such regulation may reduce the availability of financing. This trade-off is nicely captured within our framework. In addition, our model suggests a more subtle consideration in the evaluation of "skin in the game" regulation, which goes as follows. If banks use retention as a way to signal to investors then mandated retention will either reduce the informational content of retention or exacerbate its use as a signal of quality. Our model predicts

[2] This result is consistent with empirical evidence that finds that increased third party certification, such as ratings or number of analysts, increases a firm's debt issuances, and sometimes equity issuances Faulkender and Petersen (2006), Sufi (2007), Derrien and Kecskés (2013).

that the latter case obtains and hence "skin in the game" leads to tighter lending standards and a reduction in credit supply. Productive efficiency may improve but ex-ante efficiency is strictly lower with a "skin in the game" requirement. We also investigate policies related to disclosure requirements, both for securitizers and for CRAs. These policies can be interpreted as attempts to increase rating informativeness. We show that such policies need not improve lending standards and can lead to an oversupply of credit, but do improve ex-ante efficiency.

A caveat to our findings pertaining to ex-ante efficiency is that our model does not include negative externalities associated with making bad loans (e.g., due to an increase in systemic risk). Bad loans have negative NPV, but otherwise impose no social costs to society. Therefore, all of our results pertaining to ex-ante efficiency hold provided any such negative externalities are sufficiently small. Perhaps more importantly, an argument for or against certain policies should hinge on the magnitude of these social costs relative to our measure.

There is an extensive literature that studies the strategic nature of CRAs and their incentives to provide unbiased information.[3] Inspired by the CRA models in Skreta and Veldkamp (2009), Sangiorgi and Spatt (2012), Bolton et al. (2012), and Opp et al. (2013), we consider two extensions of the model: rating shopping and rating manipulation.[4] First, we allow for rating shopping by assuming that the bank can pay for a given rating if it desires, or can choose to stay unrated. Second, we consider the possibility of rating manipulation by allowing the bank to make a side-payment to the CRA to inflate its rating. In both cases, the information content of the rating becomes endogenous. We show that these frictions effectively reduce the information content of ratings and thus, have a similar effect to the comparative static of a reduction in the informativeness of (exogenously generated) ratings.

Several papers have highlighted the trade-off between productive and allocative efficiency studied in this paper. Parlour and Plantin (2008) study the effect of loan sales on banks' origination

[3]Important considerations include Mathis et al. (2009); Bar-Isaac and Shapiro (2013); Fulghieri et al. (2014); Kashyap and Kovrijnykh (2016) who focus on the role of CRA reputation and moral hazard, Boot et al. (2006); Manso (2013) on feedback effects and ratings as coordination devices, and Opp et al. (2013); Josephson and Shapiro (2015) on the implications of rating-contingent regulation.

[4]These extensions are in line with empirical studies on rating shopping and manipulation: Ashcraft et al. (2011), Griffin and Tang (2011), Griffin et al. (2013), Becker and Milbourn (2011), He et al. (2011), Kraft (2015)

and on borrowers' capital structure decisions; while Malherbe (2012) explores the relation between risk-sharing post-origination and market discipline. Chemla and Hennessy (2014) explore a setting in which there is a moral hazard problem followed by a securitization decision. They allow some investors (speculators) to acquire information about the quality of assets. Thus, as in our model, securitizers can choose to signal through retention or can instead rely on the information content of prices (that aggregate speculators' information). In their setting, price informativeness depends on speculators' incentives to acquire information, whereas we take the information content of ratings as exogenously given or as being determined by banks strategic decision (i.e., whether to shop for ratings or try to manipulate them).

There is also a rich literature that focuses on optimal contracting with loan sales and moral hazard (Gorton and Pennacchi (1995), Hartman-Glaser et al. (2012), Vanasco (2016)). In these papers, investors do not have access to information about the assets being traded.

The approach adopted in this paper builds on our previous work. Daley and Green (2014) consider a signaling model in which receivers observe both the sender's costly signal as well as a stochastic grade that is correlated with the sender's type. We enrich this framework by incorporating an ex-ante (investment) stage where assets are originated, as in Vanasco (2016).

The remainder of the paper is organized as follows. In the next section, we introduce the model, our solution concept, and characterize the socially efficient allocations. In Section 3, we present two benchmarks: an originate-to-distribute model, and a model where the bank optimally chooses what to sell in secondary markets in the absence of ratings. We present the complete model in Section 4 where we explore how the interaction of securitization decisions and ratings affects loan origination. In Section 5 we use our model to analyze the effect of several policies on retention levels and credit supply quality and quantity. Finally, in Section 6, we endogenize the information content of ratings by allowing for ratings shopping and manipulation. Section 7 concludes. All proofs are relegated to the Appendix.

2 The Model

The model involves a continuum of loan originators, which we refer to as *banks*, and a set of outside investors. There are two periods and all players are risk neutral. In the first period, each bank makes two decisions: whether or not to originate a given pool of loans (the *Origination Stage*) and, if originated, what fraction of the loan pool to securitize (the *Securitization Stage*)—what is not sold remains on the banks balance sheet. In the second period, the state of the economy, ω, is realized, together with the cash flows from the originated loans. The state of the economy is either strong or weak, $\omega \in \{\mathbf{S}trong, \mathbf{W}eak\}$, and a strong economy occurs with probability π.

Origination Stage. Each bank has access to one potential loan pool, and decides whether to finance it or not. We index loan pools by $q \in [0, \infty)$. A loan pool can be of two types: $t \in \{\mathbf{g}ood,\ \mathbf{b}ad\}$, and the distribution of types is independent of the state. A loan pool requires one unit of capital and it generates a random future cash flow $Y \sim G_{\omega,t}$ that is contingent on type and state. A *good* loan pool is expected to repay $1 + r$ in both states of nature. In contrast, a *bad* loan pool is expected to repay $1 + r$ in a strong economy, but only $\lambda\nu + (1 - \lambda)(1 + r) < 1$ if the economy is weak. One can interpret $\lambda \in (0, 1)$ as the fraction of loans in a bad pool that default in a weak economy and $\nu < 1+r$ as the expected recovery given default. We let v_t denote the expected repayment of a pool of type t and assume that $v_b < 1 < v_g$.

Prior to making its funding decision, each bank observes a private signal about its loan pool opportunity: $p(q) = \mathbb{P}(t(q) = good)$.[5] This can be interpreted as information acquired by the bank during the (unmodeled) loan screening process.[6] Without loss of generality, we can assume that $p : q \mapsto p(q)$ is weakly decreasing. To avoid unnecessary technical complications, we assume the p is strictly decreasing and continuous. Based on its private information, the bank decides

[5]Since each bank is matched with one loan pool, the indexing of loan pools $q \in [0, \infty)$ is also a bank index.

[6]The model implicitly assumes that the bank has a special technology to acquire information and originate loans. Evidence of banks being special lenders can be found in Fama (1985), and of banks having the ability to acquire private information about borrowers in Mikkelson and Partch (1986), Lummer and McConnell (1989), Slovin, Sushka, Polonchek (1993), Plantin (2009), Botsch and Vanasco (2016), among others.

whether to originate the loans in the pool or not.

We make the following assumptions on the distribution of loan opportunities:

Assumption 1. *The following hold:*

(i) $p(0)v_g + (1 - p(0))v_b > 1$

(ii) There exists $\bar{q} \in (0, \infty)$ *such that* $p(\bar{q})v_g + (1 - p(\bar{q}))v_b < A(\bar{q})v_g + (1 - A(\bar{q}))v_b < 1$

where $A(q) \equiv \mathbb{P}(t(s) = g | s \in [0, q]) = \frac{1}{q}\int_0^q p(s)ds$ *is the* average quality *of loan pools in the interval* $[0, q]$

The first condition ensures that there are loan opportunities that are socially efficient for a bank to fund given the information it acquires during screening. The second conditions ensure that there are also loan opportunities that are not socially efficient to fund. Taken together, these two conditions imply an interior socially optimal level of origination; and ensure that, in any equilibrium, there is an interior level of origination in the economy.

After the origination stage, those banks that have originated a loan pool have incentives to raise cash through securitization. This need could arise for a variety of reasons (e.g., credit constraints, binding capital requirements, credit market imperfections combined with profitable investment opportunities). As in DeMarzo and Duffie (1999), we model this incentive in reduced form by assuming that the banks discount second period cash flows by a factor $\delta < 1$, while investors' discount factor is normalized to 1. That is, banks are more impatient than investors. For simplicity, we assume that each bank privately learns the quality of its loan pool $t \in \{g, b\}$ after origination.

Securitization Stage. Each bank can design a security backed by the cash flows of its loan pool. For expositional convenience, we focus on a simple securitization structure where the bank chooses the fraction of the cash flow rights to sell and retains the remaining fraction. Thus, if the bank chooses to sell a fraction $1 - x$ then for any realization of the cash flow y, $(1 - x)y$ and xy are the amounts distributed to investors and to the bank respectively in the second period.

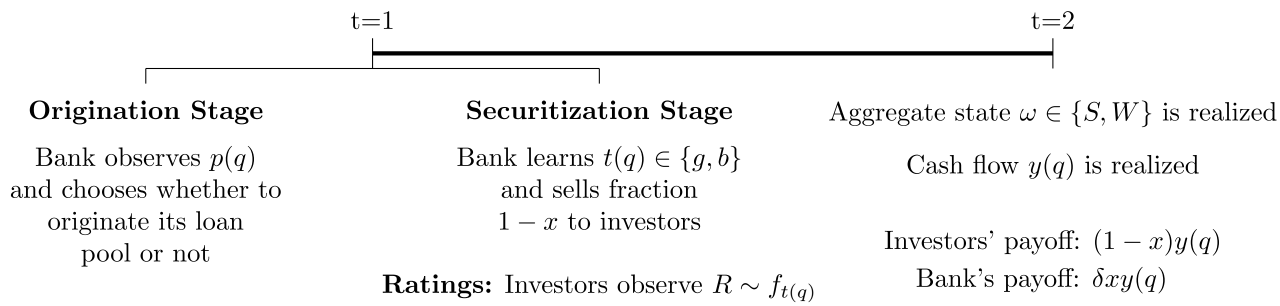

Figure 1: Timeline of the Model for Bank $q \in [0, \infty)$

Choosing a higher x should therefore be interpreted as the bank retaining more, which can serve as a (costly) signal to investors about the quality of the underlying loans.

The timeline of the model for a given bank is summarized in Figure 1.

Remark 1. *In principle, the bank could design and sell a security that is an arbitrary function of y. We study the problem of optimal security design in the presence of ratings in Daley et al. (2016b). Using the results therein, in Appendix B, we demonstrate that the main insights of this paper remain unchanged when we allow banks to optimally design the securities they issue.*

Ratings. In addition to observing the level of costly retention (x), we consider a second channel through which information may be conveyed to investors, which we refer to as a *rating*. We start by modeling the rating as an exogenous public signal about the quality of the loan pool backing the security. That is, a rating is a publicly observable random variable R with type-dependent density function f_t on $\mathbb{R}$. In Section 6, we endogenize the information content of ratings by incorporating several additional features of the credit-rating industry.

Prior to observing x and r, investors have some prior belief μ_0 about the quality of the loan pool backing each security. Investors then update their beliefs based on both the retention level x and the rating r to some final belief $\mu_f(x, r)$. Investor updating can be decomposed into a first update (based on x) and a second update (based on r). The first update results in an *interim belief*, $\mu(x)$. Along the equilibrium path, the interim belief is pinned down by the retention strategy of banks and Bayes' rule. We denote the retention strategy of banks by $\{\sigma_g, \sigma_b\}$ where

$\sigma_t : [0,1] \to [0,1]$, and $\sigma_t(x)$ denotes the probability that a bank retains a fraction x of a type-t loan. The second update is purely statistical, investors update from their interim belief to a final belief based on the rating as given by

$$\mu_f(x,r) = \frac{\mu(x) f_g(r)}{\mu(x) f_g(r) + (1-\mu(x)) f_b(r)} = \frac{\mu(x)}{\mu(x) + (1-\mu(x))\Gamma(r)}. \tag{1}$$

As can be seen by (1), the informativeness of a rating realization, r, is captured by the likelihood ratio: $\Gamma(r) \equiv \frac{f_b(r)}{f_g(r)}$.[7] Without loss, order the ratings such that Γ is weakly decreasing. A higher rating therefore corresponds to a "better" signal about the quality of the underlying. We assume that the informativeness of ratings is bounded: $\inf_r \Gamma(r) > 0$ and $\sup_r \Gamma(r) < \infty$.

Let $P(x,r)$ denote the price of a security as a function of the retention level chosen by the bank and rating. Since investors are perfectly competitive, the price must be equal to the expected value of the cash flows generated by the security given investors final belief, i.e.,

$$P(x,r) = \mathbb{E}[(1-x)Y|(x,r)] = (1-x)\big(\mu_f(x,r)v_g + (1-\mu_f(x,r))v_b\big). \tag{2}$$

Given a schedule of interim beliefs, the expected payoff of a bank with a t-type pool from choosing a retention level x is

$$u_t(x,\mu(x)) = \mathbb{E}_t[P(x,r)] + \delta x v_t. \tag{3}$$

The retention level x_t^* is optimal for a security backed by a type-t loan pool if and only if it satisfies

$$x_t^* \in \arg\max_x u_t(x,\mu(x)).$$

Given payoffs $u_t^* = u_t(x_t^*, \mu(x_t^*))$ from the securitization stage, banks choose whether to fund their loan pools or not at the origination stage. Naturally, it is optimal for a bank to fund a loan pool if it generates a positive expected profit for the bank. The expected (discounted) profit to

[7] If $f_g(r) = f_b(r) = 0$, we adopt the convention that $\Gamma(r) = 1$.

the q^{th} bank from funding its loan pool is given by:

$$p(q)u_g^* + (1 - p(q))u_b^* - 1. \tag{4}$$

Because good loan pools generate a higher expected cash flow and $p(q)$ is decreasing, the optimal origination policy takes the form of a threshold, q^*, such that the bank originates the loan pool if and only if $q \leq q^*$, where q^* is given by the loan pool with zero NPV to banks: $p(q^*)u_g^* + (1 - p(q^*))u_b^* = 1$.

Aggregate Credit Supply. Given the above described threshold policy of banks, all loan pools with $q \in [0, q^*]$ are originated. As a result, the total amount of credit supplied in this economy is q^*, and the average quality of originated loans is $A(q^*)$.

Allocative and Productive Efficiency. There are two potential sources of inefficiencies in this economy. The first is related to the quality of loans that are originated: efficiency is reduced when negative NPV loans receive funding, as well as when positive NPV loans do not. We refer to this as productive efficiency. The second source is related to the allocation of cash flow rights. Since banks values cash flows in the second period by $\delta < 1$, efficiency is maximized when all of the $t = 2$ cash flows are in the hands of investors, who value these cash flows the most. We refer to this as allocative efficiency. To summarize, *allocative efficiency* requires that banks sell all cash flows to investors at the securitization stage; and *productive efficiency* requires that all positive NPV loan pools are originated. That is, the first-best features:

1. Allocative Efficiency: $x_b^{FB} = x_g^{FB} = 0$.

2. Productive Efficiency: the marginal loan pool q^{FB} is given by $p(q^{FB}) = \frac{1-v_b}{v_g-v_b}$.[8]

[8] Productive efficiency is first-best given all the information available to agents in the economy. If banks could *perfectly observe* the type of loan pool prior to orignation then productive efficiency involves only good loans being originated.

2.1 Equilibrium

We use perfect Bayesian equilibrium (PBE) as our solution concept. Our analysis will first look at each stage in isolation, then characterize equilibrium in the overall game. For this reason, it is convenient to be explicit about the connection between the stages in equilibrium.

Definition 1. *An equilibrium is given by the measure of originated loan pools, q^*, a retention strategy $\{\sigma_b, \sigma_g\}$, a price function P, and beliefs μ_0 such that:*

1. *Given μ_0, for each loan pool originated, the bank and investors strategies at the securitization stage are a PBE of the signaling game that satisfies the D1-refinement.*

2. *Given the profits implied by the signaling game for each type of loan pool, (u_g^*, u_b^*), banks origination decision q^* maximizes bank profits (4).*

3. *Belief Consistency: $\mu_0 = A(q^*)$.*

As stated in Definition 1, we employ D1 (Banks and Sobel, 1987; Cho and Kreps, 1987) to refine off-equilibrium-path beliefs. The D1 refinement requires investors to attribute the offer of an unexpected (i.e. off-equilibrium-path) retention level to the type who is more likely to gain from this offer relative to equilibrium payoffs. See Appendix A.1 for a formal definition.

3 Benchmarks

Our model features two channels through which information can potentially be conveyed to investors: costly retention and informative ratings. In this section, we consider several benchmark cases in which only one or neither of these channels are present.

3.1 Originate-to-Distribute (OTD)

Suppose first that banks sell 100% of the loan pools that they originate and that there are no ratings. In this case, there is no information conveyed to investors, and thus the price for securities backed by g- and b-type pools must be the same. Denote this price by P_d. Since banks are selling the entire loan pool, and prices are not type-contingent, bank profits are equalized across banks with different loan pools; i.e., $u_g = u_b = P_d$. If $P_d > 1$, the optimal origination strategy would be given by threshold $q = +\infty$ and investors would lose money since the value of an average loan pool is be less than one (Assumption 1). If $P_d < 1$, then banks would never originate since all loans have negative NPV for the bank. Hence, the only possible non-trivial outcome involves $P_d = 1$ with origination threshold q_d such that $A(q_d)v_g + (1 - A(q_d))v_b = 1$. That is, the NPV of the *average loan pool* originated in this economy is zero.

Clearly, the OTD benchmark involves allocative efficiency. However, because investors have no information, market prices do not discipline banks' decisions during the origination stage, which leads to an oversupply of credit. Banks with g-type loan pools are subsidizing those with b-type loan pools in the market, and would therefore have an incentive to retain (at least a portion of) their loans provided δ is large enough and/or obtain a third party rating to credibly convey information to investors. In what follows, we analyze these two channels in isolation. In Section 4, we present the complete model where we explore both channels jointly.

OTD with ratings. Now suppose investors observe a rating for each security issued. Since banks do not retain any cash flows, ratings only impact the price each bank receives for the pool sold, which is given by investors' expectations about the quality of the pool conditional on the observed rating, r. The expected payoff for originating a t-type pool given investor's prior belief, μ_0, is:

$$u_t^R = E_R[\mu_f(\mu_0, r)|t](v_g - v_b) + v_b \tag{5}$$

The presence of ratings increases the correlation between expected payoffs and loan pool quality since investor's beliefs (post-rating) are more correlated with loan pool types; that is,

$$E_R[\mu_f(\mu_0, r)|g] > E_R[\mu_f(\mu_0, r)|b] \iff u_g^R > u_b^R \tag{6}$$

This is in contrast to the prediction without ratings, where $u_b^d = u_g^d = 1$. The information content of market prices acts as a disciplining device for banks. As a result, ratings in an OTD environment increase the quality of loans originated and decrease credit supply (fewer negative NPV projects are funded):

$$p(q_d) < p(q^R) = \frac{1 - u_b^R}{u_g^R - u_b^R} \leq \frac{1 - v_b}{v_g - v_b} \tag{7}$$

since $v_b \leq u_b^R < u_g^R \leq v_g$. As ratings become more informative, expected payoffs approach full information payoffs, $u_t^R \to v_t$, and so do credit quality and quantity, $q^R \to q^{FB}$.[9] Thus, in OTD environments where retention levels are fixed, ratings discipline banks and increase productive efficiency.

3.2 Retention without Ratings

Suppose now that the bank can optimally choose which fraction of the pool cash flows to retain, and that this is the only information available to investors (i.e., there are no ratings). In this case, the securitization stage is reminiscent of a standard signaling game in which the least-cost-separating-equilibrium (LCSE) is the unique outcome satisfying standard refinements (e.g., Inuitive Criterion, D1, etc.). In the LCSE, those banks with b-type pools sells 100% of their cash flows (i.e., $x_b = 0$), while those with g-type pools retain fraction $x^{LC} > 0$. Retention is chosen to ensure that banks with bad loan pools do not mimic the retention choice of banks with good loan pools:

$$v_b = (1 - x^{LC})v_g + \delta x^{LC} v_b. \tag{8}$$

The following proposition characterizes the equilibrium allocations of this environment.

Proposition 1 (Retention and No ratings)**.** *[Securitization Stage] Without ratings, the LCSE is*

[9] Where a rating system $\{f_g, f_b\}$ is more informative than rating system $\{f'_g, f'_b\}$ if $E_R[\mu_f(\mu, r)|b] < E'_R[\mu_f(\mu, r)|b]$ and $E_R[\mu_f(\mu, r)|g] > E'_R[\mu_f(\mu, r)|g]$ for all $\mu \in (0, 1)$.

the unique equilibrium of the securitization stage, with implied payoffs:

$$u_b^{LC} = v_b \qquad u_g^{LC} = (1 - x^{LC})v_g + \delta x^{LC} v_g \tag{9}$$

[Origination Stage] Aggregate credit supply is given by q^{LC} *loan pools, where:*

$$p(q^{LC}) = \frac{1 - v_b}{v_g[1 - (1 - \delta)x^{LC}] - v_b} > \frac{1 - v_b}{v_g - v_b}. \tag{10}$$

Thus, $q^{LC} < q^{FB}$*. Credit supply is constrained relative to the first-best.*

In the presence of information frictions, the value to a bank of originating a b-type pool is its full information value, while that of originating a g-type pool is below its full information value due to signaling costs: $(1 - \delta)x^{LC}$. As a result, in equilibrium, there is an under-supply of credit relative to first best. Furthermore, the average quality of originated loan pools is inefficiently high: $A(q^{LC}) > A(q^{FB})$. That is, there are socially positive NPV loans that are not being funded in this economy.

It is important to highlight that in the LCSE, banks perfectly convey the quality of their underlying loans with their retention decisions. Thus, there is a perfect correlation between retention levels, prices, and underlying loan pool quality.

4 The Effect of Ratings

In this section, we analyze the full blown model where banks can optimally choose how much to retain of each loan, and after doing so, investors observe a rating r about the issued security. The goal of this section is to study the interactions between retention decisions and ratings, and their impact on market liquidity, credit supply, and quality of origination. In Section 6, we incorporate some of the intricacies of the credit rating agencies industry to the model, but we believe that it is important to begin by characterizing the role of informative public signals. We will show that informative ratings increase overall efficiency; but that they can do so by increasing allocative efficiency at the expense of decreasing productive efficiency.

4.1 Securitization with Ratings

Recall that for each security that a bank issues (i.e. after the bank chooses its retention level), a rating is publicly observed. Fixing the quality of the loan pool backing the security $t \in \{g, b\}$, a rating is a random variable R, with density function f_t on $\mathbb{R}$, where the likelihood ratio, $\Gamma(r)$, measures the informativeness of a rating realization, r. To understand the banks' decisions during the securitization stage, it is the informativeness of the *rating system* $\{f_b, f_g\}$ that matters.

Definition 2. *Ratings are* Γ*-informative if* $E[\Gamma(r)|b] > \frac{v_g - \delta v_b}{(1-\delta)v_g}$.

$E[\Gamma(r)|b]$ is a measure of the informativeness of the rating system. The more informative the rating system, the higher is $E[\Gamma(r)|b]$. This measure is consistent with the notion of informativeness introduced by Blackwell (1951): if one rating system is Blackwell more informative than another, then $E[\Gamma(r)|b]$ is higher under the more informative system.[10] The right-hand side of the constraint can be written as follows: $\frac{-(v_g - \delta v_b)}{-(1-\delta)v_g}$, which is the ratio of the marginal cost of retaining cash flows for a bank with a b-type pool relative to that of a bank with a g-type pool, in a separating equilibrium. Thus, Γ-informativeness requires that the ratings are informative enough relative to the cost advantage of the g-type to use retention as a signaling device. In what follows, we refer to Γ-informative ratings as *sufficiently informative* ratings.

The following proposition characterizes the equilibrium of the securitization stage in the presence of ratings taking investors prior belief (μ_0) as given.

Proposition 2. *When ratings are* Γ*-informative, the unique equilibrium of the securitization stage features some degree of pooling. In particular, there exists* $\{\mu^*, x^*\}$ *such that:*

$$\{\mu^*, x^*\} = \arg\max_{\mu, x} \quad u_g(\mu, x) \quad s.t. \quad u_b(\mu, x) = v_b \tag{11}$$

and,

[10] Note that $E[\Gamma(r)|g] = 1$ for all rating systems.

- For $\mu_0 < \mu^*$, there is partial pooling at $x^* < x^{LC}$. That is, all banks with g-type pools retain x^*, a fraction $\lambda = \frac{\mu_0(1-\mu^*)}{(1-\mu_0)\mu^*}$ of banks with b-type pools retain x^*, and a fraction $1-\lambda$ retain zero.

- For $\mu_0 > \mu^*$, there is full-pooling at $x = 0$. That is, all banks retain zero, regardless of type.

When ratings are not Γ-informative, the unique equilibrium outcome of the securitization stage is the LCSE of Proposition 1.

Intuitively, with Γ-informative ratings, banks need not signal as vigorously to convey security type. Instead, they rely (to some extent) on the rating to convey information to investors. When investors are sufficiently optimistic ($\mu_0 > \mu^*$), there is full reliance on the rating. Otherwise, when $\mu_0 < \mu^*$, banks rely partially on retention and partially on the rating. That is, banks retain enough of g-backed loans to induce an interim belief of μ^* and rely on the rating beyond that.

An important implication of Proposition 2 is that when ratings are sufficiently informative, they increase allocative efficiency by reducing the inefficient retention of the g-type. Since without ratings the equilibrium is separating and fully reveals the type of loan backing each security, the increase in allocative efficiency occurs at the expense of reducing the total amount of information revealed to investors.

4.2 Origination with Ratings

Having characterized the securitization stage, we now analyze banks' decisions to originate loans. Note that the payoff for a g-type pool is strictly higher than without ratings for all μ_0, since ratings are likely to identify the underlying pool type. On the other hand, the payoff for a b-type pool is weakly higher than without ratings for all μ_0. Furthermore, when ratings are sufficiently informative, the equilibrium payoffs in the securitization stage depend on market beliefs as follows:

$$u_t^*(\mu_0) = u_t(\mu_0, x(\mu_0)) = (1 - x_t(\mu_0))\left[\tilde{\alpha}_t(\mu_0)(v_g - v_b) + v_b\right] + \delta x_t(\mu_0) v_t \tag{12}$$

where $\alpha_t(\mu_0) \equiv E_R[\mu_f(\mu_0, r)|t]$ is the expected posterior given investors' prior beliefs, μ_0, and pool type t, and $x_t(\mu_0)$ is the retention outcome of the securitization stage characterized in Proposition 2. This result is in sharp contrast to the equilibrium when ratings are not sufficiently informative, where the unique equilibrium is the LCSE of Proposition 1, whose type-contingent payoffs are independent of market beliefs, μ_0.

At the origination stage, for given μ_0, the origination threshold decision of banks is determined by the zero NPV condition:

$$p(q^*) = \underbrace{\frac{1 - u_b^*(\mu_0)}{u_g^*(\mu_0) - u_b^*(\mu_0)}}_{\equiv \Psi(\mu_0)} \tag{13}$$

Banks choose to fund any loan pool q with high enough quality; that is, $p(q) \geq p(q^*)$. We refer to q^* as the marginal loan pool originated in the economy, which fully characterizes aggregate supply in the economy. Note that in any equilibrium, $u_g^* > 1$. This implies that the quality of loans the bank is willing to originate is decreasing in the expected payoff for both t-type pools, u_t^*. Consistent with this, as expected payoffs in the securitization stage increase, credit supply increases. The following Corollary characterizes the marginal loan pool, $\Psi(\mu_0)$, as a function of market beliefs μ_0, by using the outcomes of the securitization stage presented in Proposition 2.

Corollary 1. *With Γ-informative ratings, aggregate credit supply depends on investors prior belief, μ_0, as follows:*

$$p(q^*) = \begin{cases} \frac{1-u_b(x^*,\mu^*)}{u_g(x^*,\mu^*)-u_b(x^*,\mu^*)} & \mu_0 < \mu^* \\ \left\{ \frac{1-u_b(x,\mu^*)}{u_g(x,\mu^*)-u_b(x,\mu^*)} \ , \ x \in [0, x^*] \right\} & \mu_0 = \mu^* \\ \frac{1-u_b(0,\mu_0)}{u_g(0,\mu_0)-u_b(0,\mu_0)} & \mu_0 > \mu^* \end{cases} \tag{14}$$

Where $\{x^*, \mu^*\}$ are defined in Proposition 2.

When ratings are Γ-informative, investor's beliefs affect banks' origination decisions. In partic-

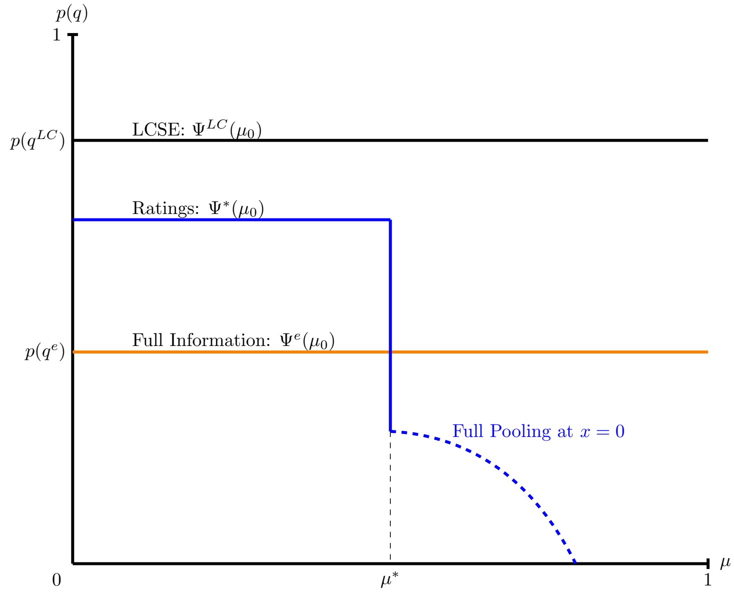

Figure 2: **Loan Origination Decisions.** The figure plots the quality of the marginal originated loan pool, $p(q^*)$, as a function of market beliefs, μ_0.

ular, for low beliefs, $\mu_0 < \mu^*$, the outcome at the securitization stage involves partial pooling at x^*. As a result, credit supply in the economy (characterized by the marginal loan $p(q^*) = \Psi(\mu_0)$) is constant and determined by a full information payoff for b-type pools, v_b, and a payoff of $u_g(\mu^*, x^*)$ for g-type pools. Furthermore, since $u_g(\mu^*, x^*) \in (u_b^{LC}, v_g)$, origination is greater than in the LCSE, but bellow first-best. In the knife-edge case of $\mu_0 = \mu^*$, pooling at any retention level $x \in [0, x^*]$ is an equilibrium, and thus credit supply in the economy decreases in x. This is because for given beliefs μ^*, lower retention increases the payoff to all loan pools types. Finally, when investor's beliefs are high, $\mu_0 > \mu^*$, outcomes of the securitization stage imply full pooling at no retention levels: $x = 0$. In this scenario, credit supply increases in market beliefs μ_0, since higher beliefs increase the payoff to all pool types. These results are summarized in Figure 2, where the quality of the marginal loan originated is plotted as a function of investors' beliefs.

It is useful to compare these results with the origination outcome in the first-best and in the equilibrium with uninformative ratings (i.e. no ratings or ratings that are not Γ-informative). In the first-best, market beliefs play no role since investors can observe the quality of the underlying loan pool. Thus, payoffs are independent of investors' beliefs and so are banks' origination decisions: $\Psi^{FB}(\mu_0) = \frac{1-v_b}{v_g-v_b}$ for all $\mu_0 \in (0,1)$. In a world with uninformative (or no) ratings, banks use retention to convey information to investors. As a result, pool types are fully revealed in the LCSE of the securitization stage, making payoffs and origination decisions independent of beliefs as well: $\Psi^{LC}(\mu_0) = \frac{1-v_b}{\Delta v_g-v_b}$ for all $\mu_0 \in (0,1)$, where $\Delta \equiv 1-(1-\delta x^{LC}) < 1$. From inspecting Figure 2, we see that informative ratings can result in an oversupply (undersupply) of credit relative to first best when beliefs are relatively optimistic (pesimistic). In addition, the amount of credit supplied is always higher than what is obtained with uninformative ratings. It remains to characterize the μ_0 that is consistent with an equilibrium.

The following proposition concludes the characterization of the equilibrium with ratings.

Proposition 3. *There is a unique equilibrium* $\{q^*, \mu_0, x(\mu_0)\}$ *given by:*

$$A(q^*) = \mu_0$$
$$p(q^*) = \Psi(\mu_0)$$

The presence of Γ*-informative ratings leads to an increase in credit supply relative to the LCSE, and can lead to over-investment relative to first-best.*

Figure 3 imposes the belief consistency condition, where μ_0 such that $p(A^{-1}(\mu_0)) = \Psi(\mu_0)$ is an equilibrium of the full game. We see that depending on the distribution of loans, ratings can generate over or underinvestment relative to the first-best. In particular, over-investment is more likely to occur when there are more good quality loan pools in the economy. Graphically, this is reflected in a flatter $p(A^{-1}(\mu_0))$ curve. To see this, consider the scenario in which the first-best level of loan pools, q^{FB}, is originated. The average pool quality originated in this economy determines equilibrium market beliefs, $\mu_{FB} \equiv A(q_{FB})$. For market beliefs μ_{FB}, banks' incentives are such that the marginal loan pool originated in the economy is given by $\Psi(\mu_{FB})$. From inspection of Figure 3, we see that oversupply would occur if $\Psi(\mu_{FB}) < p^{FB}$. That is, oversupply occurs when the market belief implied by the first-best origination level is high-enough that banks have incentives to increase origination relative to the first-best level. When market

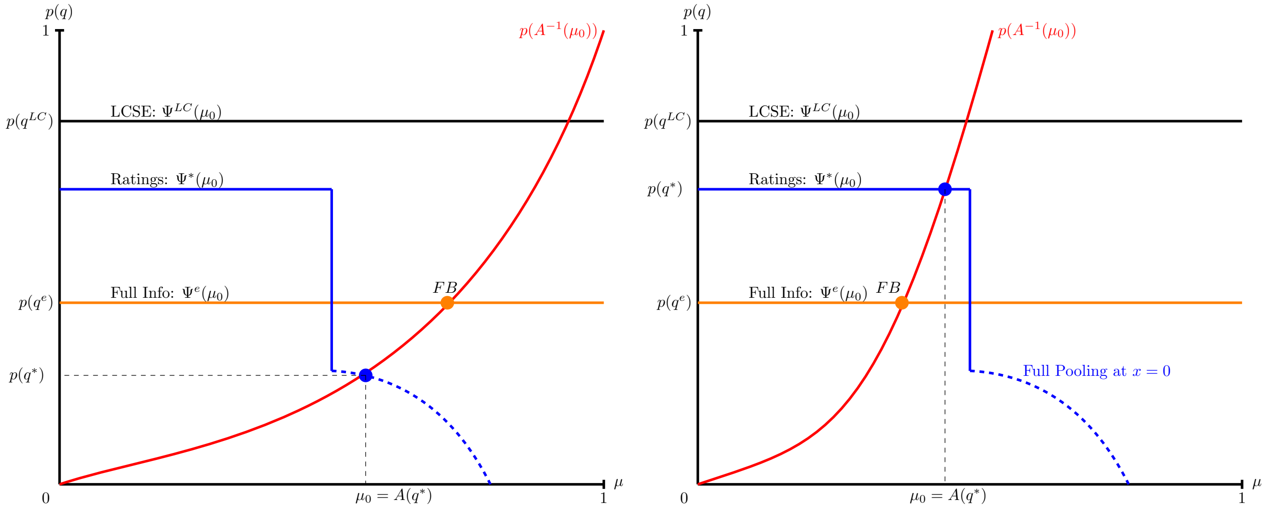

Figure 3: Equilibria with oversupply of credit (left) and undersupply of credit (right).

beliefs are high, the implied subsidy that b-type pools receive in the market is larger, resulting in higher incentives to originate b-type loan pools. In turn, this is more likely to occur when the distribution of loan pools is right-skewed, that is, where there are lots of good-quality loan pools in the economy. When ratings induce over-investment, productive efficiency decreases.

Oversupply: An Example. Let loan pools be distributed according to $F(p) = p^\alpha$ for $\alpha \in \mathbb{R}^+$. Suppose origination is at first-best levels, $p_{FB} = \frac{1-v_b}{v_g - v_b}$, which is independent of the distribution of loan pool qualities. In this case, average credit quality in the economy is

$$A(q_{FB}) = E[p|p \geq p_{FB}] = \left(\frac{\alpha}{\alpha+1}\right) \frac{1 - p_{FB}^{\alpha+1}}{1 - p_{FB}^{\alpha}} \tag{15}$$

Note that $\frac{\partial A(q_{FB})}{\partial \alpha} > 0$ and $\lim_{\alpha \to \infty} A(q_{FB}) = 1$ for all $p_{FB} \in (0,1)$. The condition for over-investment in this economy is that $\Psi(A(q^{FB})) < p_{FB}$. Furthermore, for any q_{FB}, there exists an $\bar{\alpha}$ such that all loan distributions with $\alpha > \bar{\alpha}$ result in over-investment, where $\bar{\alpha}$ solves:

$$\Psi\left(\left(\frac{\bar{\alpha}}{\bar{\alpha}+1}\right) \frac{1 - \left(\frac{1-v_b}{v_g-v_b}\right)^{\bar{\alpha}-1}}{1 - \left(\frac{1-v_b}{v_g-v_b}\right)^{\bar{\alpha}}}\right) = \frac{1-v_b}{v_g - v_b}.$$

Summary of Results. Sufficiently informative ratings improve allocative efficiency by reducing equilibrium retention levels: i.e., ratings always increases market liquidity. As a response, credit supply increases, while average loan quality decreases. Even though overall efficiency is increased, ratings can generate an oversupply of credit relative to first-best. Surprisingly, informative ratings can result in an extension of credit to negative NPV loan pools. In addition, since retention no longer reveals the quality of the underlying pool, and ratings are not perfectly informative, ratings reduce the information available to market investors.

5 Policy Analysis

In this section, we discuss some the main policies implemented in the US and in Europe after the 2008/09 financial crisis to boost firms and households access to credit. To revive the markets where banks off-load some of their risks through the issuance of ABS, reforms where made in the securitization and rating industries. These regulatory responses conceptually fell into four categories: increasing disclosure, requiring risk-retention, reforming rating agencies, and imposing capital requirements.[11] In addition, central banks have intervened in a variety of ways that affect banks liquidity needs, both before and after the crisis. We discuss the effect of these policies through the lenses of our model. First, we analyze the effect of imposing a skin-in-the-game (risk-retention) rule on securitizers of assets. Second, we use our model to enrich our understanding of the effects of improving the disclosure requirements that should accompany a security issuance and we discuss the effect of policies that aim to address the frictions present in the rating industry. Finally, we analyze how changes in bank's liquidity needs impact both securitization and origination decisions.

5.1 Skin-in-the-Game Rules

In October, 2014, the US passed the skin-in-the-game rule that requires sponsors of securitization transactions to retain risk in those transactions, as stated in the risk retention requirements in

[11] See Schwarcz (2015) for an analysis of the regulatory changes in securitization in response to the financial crisis, both in the US and in Europe.

the Dodd-Frank Wall Street Reform and Consumer Protection Act. More specifically, the rule requires sponsors of asset-backed securities to retain not less than five percent of the credit risk of the assets collateralizing the issuance. The rule also sets forth prohibitions on transferring or hedging the credit risk that the sponsor is required to retain. This rule aims to align incentives between the originators of assets and the investors who end up holding these assets. In Europe, the Capital Requirements Regulation (CRR) requires the originator/sponsor/original lender to explicitly disclose that it will retain, on an ongoing basis, a material net economic interest in the securitization for the life of the transaction. As in the US, five per cent has been specified as the minimum net economic interest to be retained.[12]

We study the impact of retention rules on securitization and loan origination decisions in our model. We assume that banks need to retain at least an $(\bar{x} \times 100)\%$ exposure to the loan pool cash flows. As in practice, we assume that retention requirements are not contingent on the choice of security, the rating, nor on other measures of quality of the underlying cash flows.[13] We show that when banks can use cash flow retention to signal the quality of the underlying assets, imposing retention requirements makes it harder for those banks with good quality loans to signal this to investors. This is because retention requirements worsen the outside option of those banks with bad quality pools. As a result, retention weakly increases while credit supply is reduced. The following proposition characterizes the effect of imposing a skin-in-the-game rule in our framework with ratings.

Proposition 4. *Imposing a retention requirement $\bar{x} \in (0, 1]$ to all securitizers increases average credit quality, but it reduces credit supply, increases equilibrium retention levels, and results in a decrease in ex-ante efficiency.*

When banks are already utilizing retention as a signaling device, imposing retention levels

[12] An interesting difference between both regulations is that while in the US the responsibility falls on the originator/sponsor, in Europe it falls on the investor banks (regulated banks), who need to establish mechanisms to monitor the exposure of the originator/sponsor to ensure a lower risk-weight on these assets.

[13] The present regulation does make exceptions for particular asset classes. However, for a given asset class, retention rules are equal for all asset qualities.

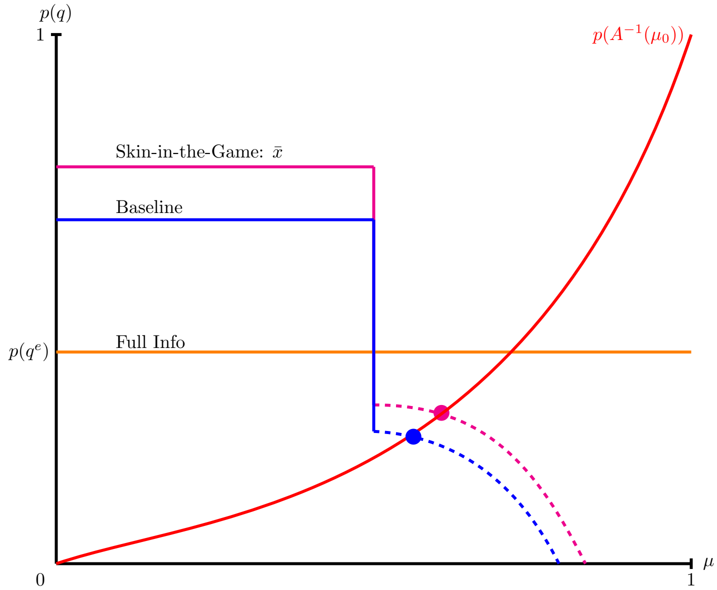

Figure 4: The Effect of a Skin-in-the-Game Rule.

(weakly) decreases ex-ante efficiency. To see this, notice that forced retention levels worsen the outside option of bad banks, i.e. their full information payoff. This, in turn, makes it more costly for good banks to signal their quality, i.e., they have to increase their retention levels to signal successfully. As a result, equilibrium retention levels weakly increase – x^* increases,- and securitization payoffs weakly decrease for all securitizers. The reduction in the expected outcomes from securitization worsens banks incentives to originate loans, decreasing credit supply. Figure 4 exhibits the effect of a skin-in-the-game rule on (i) the marginal loan originated for given market beliefs, $\Psi(\mu)$ and (ii) equilibrium credit supply, given by the intersection of the marginal loan curve with the belief consistency condition curve, $p(A^{-1}(\mu))$. As shown in the Figure, when equilibrium market beliefs are relatively optimistic, retention requirements improve productive efficiency, but they do so at the expense of allocative efficiency.

5.2 Disclosure Requirements and CRA Regulation

Rules have been adopted both in the US and in Europe to improve the disclosure, reporting, and offering process of securitized products. The objective is to increase transparency in markets and to protect investors. The rules require the disclosure of standardized, detailed, loan-level information and of the risk-models used to analyze them. They also increase the time given to investors to process and analyze this information.

In this paper, we study the effect of having a public rating accompanying a security issuance. A public rating in our model is simply noisy public information that is made available to investors at the time a security is issued. In this sense, we believe our model can also be used to think about the effects of improving information disclosure at security issuances. To the extent that it is hard to process and interpret this information, this disclosure of information is noisy, and thus disclosure of information to the market operates as a public signal. The effects of increasing transparency are interpreted as increases in the informativeness of this public signal, i.e. our ratings.

In addition, to increase the reliability of ratings issued by CRAs in the US, the Dodd-Frank Act mandated the creation of the Office of Credit Ratings (OCR) to conduct oversight of the "nationally recognized statistical rating organizations" (NRSROs). The role of the OCR is to monitor and report on the NRSROs internal control structures, rating methodologies and models, conflicts of interest, quality of information disclosure, standards of training, experience, and competence.[14] Europe followed with the creation of the European Securities and Markets Authority (ESMA) in charge of supervising CRAs in the European Union. In Section 6, we show that frictions in the ratings industry (such as rating shopping and manipulation), reduce rating informativeness. This is consistent with the regulation put in place in the US and Europe, where CRA monitoring aims to increase the informative content of ratings.

To understand the effect of both policies, we study the impact on equilibrium outcomes of changes in the informativeness of ratings. To sharpen our predictions, we focus on binary and symmetric rating systems: $\gamma = P(R = 1|g) = P(R = 0|b) > \frac{1}{2}$, where higher γ implies a more informative rating.

Proposition 5. *As γ increases, μ^* and x^* decrease, while credit supply may increase or decrease. Therefore, more informative ratings improve allocative efficiency but may increase or decrease*

[14] For a more detailed description of the OCR mandate, see the *2016 Summary Report of Commission Staff's Examinations of Each Nationally Recognized Statistical Rating Organization* prepared by the SEC.

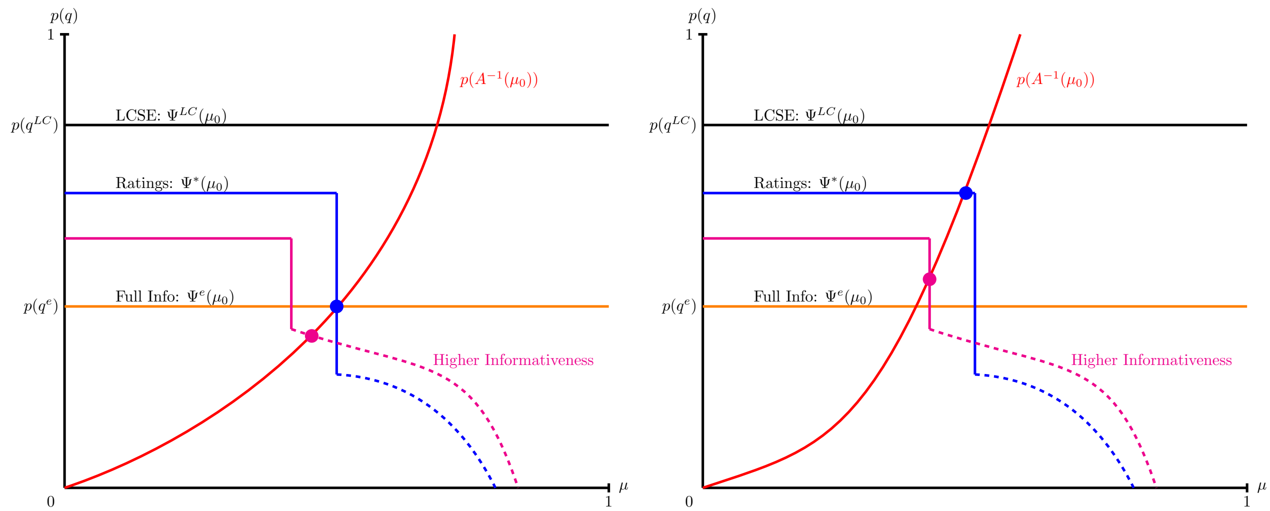

Figure 5: The Effect of an Increase in Rating Informativeness.

productive efficiency.

As rating informativeness increases, the equilibrium becomes more efficient; even though the effect on credit supply and average loan quality is ambiguous. The following corollary characterizes how the response of credit supply to a change in rating informativeness depends on initial equilibrium market beliefs.

Corollary 2. *Let μ_0^{eq} denote the initial equilibrium beliefs. Then two cases can arise:*

- Low initial beliefs, $\mu_0^{eq} \leq \mu^*$: an increase in rating informativeness increases credit supply, reduces average credit quality, and may result in over-investment.

- High initial beliefs, $\mu_0^{eq} > \mu^*$: an increase in rating informativeness reduces credit supply, increases credit quality, and reduces the level of over-investment in the economy.

Improvements in rating informativeness increase allocative efficiency, but do not necessarily improve productive efficiency. In particular, ratings reduce the origination of negative NPV loans only when market beliefs are relatively high. Otherwise, they may incentivize it. We conclude that more informative ratings are beneficial since they increase ex-ante efficiency, but they do not necessarily discipline origination decisions.

Figure 5 shows how the determination of the marginal loan changes with rating informativeness. As stated in Corollary 2, the resulting effect on credit supply depends on initial equilibrium market beliefs, which in turn depend on the shape of the $p(A^{-1}(\mu))$ curve that imposes the belief consistency condition.

5.3 Liquidity Needs

Central banks often undertake policies aimed at easing credit constraints of distressed financial institutions, in attempt to stimulate the economy by inducing banks to lend more.[15] Within our model, such policies can be interpreted as increasing δ. The following proposition summarizes the comparative static effects of such a policy.

Proposition 6. *As δ increases, μ^* and x^* increase, while the effect on the quality and quantity of aggregate credit supply is ambiguous.*

As liquidity needs decrease (higher δ), banks have less incentives to securitize, so they retain a larger fraction of their pools. When equilibrium retention levels are relatively high, a reduction in banks' liquidity needs increases the value of originating g-type loan pools (since retention becomes less costly), and thus credit supply increases. Furthermore, if banks were originating negative NPV loans, a reduction in liquidity needs may improve origination decisions by increasing equilibrium retention levels. These results are summarized in the following corollary.

Corollary 3. *Let μ_0^{eq} and μ_0^* denote the initial equilibrium beliefs and the threshold μ^* before the decrease in liquidity needs (increase in δ), respectively. Three cases can arise:*

(1) $\mu_0^{eq} < \mu_0^*$: lower liquidity needs increase credit supply and worsen credit quality, improving the under-supply of credit problem in the economy.

(2) $\mu_0^{eq} > \mu_0^*$: lower liquidity weakly reduce credit supply, increase credit quality, and can improve the over-supply of credit problem.

[15] For example, in March 2008, the Federal Reserve announced the Term Securities Lending Facility that enabled banks to use MBS as collateral for short-term loans, which naturally reduced their need to sell such securities. Later, during quantitative easing, the Federal Reserve purchased outright billions of dollars in MBS. The European Central Banks adopted similar policy measures during the European Financial Crisis.

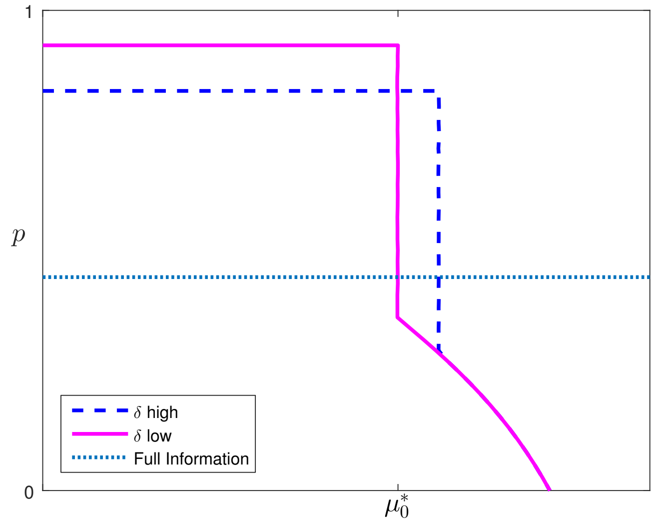

Figure 6: The Effect of an Increase in Liquidity Needs.

(3) $\mu_0^{eq} = \mu_0^*$: the effect depends on the initial equilibrium retention. In particular, lower liquidity needs increase credit supply for sufficiently high retention levels, and decrease it otherwise.

Figure 6 shows how the determination of the marginal loan changes with liquidity needs. As stated in Corollary 3, lower liquidity weakly improve origination standards when equilibrium retention levels are low. As a result, the effect of liquidity policies on credit supply ultimately depends on the distribution of loans, i.e., on the shape of the $p(A^{-1}(\mu))$ curve, that imposes the belief consistency condition.

6 Endogenous Ratings

In this section, we consider two extensions to the model to investigate different features of the CRA industry. First, we incorporate rating shopping by supposing that banks have to pay for a rating if they wish to publish it. Second, we allow for rating manipulation by supposing that banks can inflate their rating by giving a side payment to the CRA. Our main result is that, provided investors are rational, rating shopping and manipulation act mainly to reduce rating informativeness. Hence, the effect of these considerations is largely captured by the comparative static result in Proposition 5. Consistent with this, when investors are not fully rational or naive, the effect of frictions in CRA may be more pervasive. We discuss this possibility in the rating manipulation extension.

6.1 Ratings Shopping

The model outlined in Section 2 is extended as follows. After choosing the fraction of the loan pool to securitize, banks approach the CRA to get a rating for their security. The CRA generates a rating R with type-dependent pdf f_t, as in the baseline model, and reports it to the bank. At this point, banks choose whether to pay the CRA to have R become public or to stay unrated.

We assume that the fee charged by the CRA is proportional to the cash flows being issued, $1-x$, and we normalize it by the difference in expected payoffs from a good vs a bad pool, which is without loss of generality. These normalizations will simplify the characterization of the problem. Given a CRA fee $\phi \geq 0$, a bank with retention x has to pay $\frac{\phi}{v_g - v_b}(1-x)$ to have its rating published. If the bank chooses not to get rated, investors form beliefs $\mu_n(\mu,\phi)$, which in equilibrium will depend on prior beliefs μ and the cost of being rated, ϕ.

Shopping for Ratings. We begin by studying the decision of a bank with a t-type loan pool that has chosen retention $x(t)$ and has been proposed rating r by the CRA. The payoff from hiring the CRA to have rating r published is given by:

$$u_t\left(\mu,r,\phi\right) = \left(1-x\left(t\right)\right)\left\{\left(\mu_f\left(\mu,r\right)-\phi\right)\left(v_g-v_b\right)+v_b\right\} + \delta x\left(t\right)v_t \tag{16}$$

while the payoff from staying unrated is:

$$u_t(\mu,n,\phi) = (1-x(t))\left\{\mu_n(\mu,\phi)(v_g-v_b)+v_b\right\} + \delta x(t)v_t \tag{17}$$

The bank chooses to hire the CRA if:

$$u_t\left(\mu,r,\phi\right) \geq u_t(\mu,n,\phi) \iff \mu_f(\mu,r)-\phi \geq \mu_n\left(\mu,\phi\right) \tag{18}$$

Off-Equilibrium Beliefs. From (18), it follows that the decision to get rated, conditional on the rating r, is not type-dependent. As a result, we characterize equilibria in which investors assign beliefs $\mu_f(\mu, r)$, as defined in (1), for all observed ratings, even if these ratings are not on the equilibrium path. That is, reporting an off-equilibrium path rating does not give more or less information to investors about loan type than the reported rating, r. This is because any observed rating r, even it is off-equilibrium path, must have been drawn from type-contingent pdf, f_t, and thus contains relevant information. It is worth noting that these beliefs are consistent with the D1-refinement.

The following Lemma characterizes the bank's decision to get rated.

Lemma 1. *For any prior belief $\mu \in (0,1)$, banks hire the CRA if the rating is high enough $r \geq \bar{r}(\mu, \phi)$, and remain unrated if the rating is low: $r < \bar{r}(\mu, \phi)$, where threshold rating $\bar{r}$ is such that banks are indifferent between being rated or not:*

$$\mu_f\left(\mu, \bar{r}\right) - \mu_n\left(\mu, \bar{r}\right) = \phi \tag{19}$$

Furthermore, the set of ratings for which banks chooses not to get rated, $\{r : \ r < \bar{r}\left(\mu, \phi\right)\}$, has positive measure iff $\phi > 0$.

From Lemma 1, a bank's decision not to get rated indicates that $r < \bar{r}(\mu, \phi)$. Thus, in any equilibrium, the belief assigned to an unrated issuance is given by

$$\mu_n(\mu, \phi) = \frac{\mu F_g(\bar{r}(\mu, \phi))}{\mu F_g(\bar{r}(\mu, \phi)) + (1-\mu) F_b(\bar{r}(\mu, \phi))}. \tag{20}$$

Lemma 1 yields the first implication of rating shopping. The possibility that banks pay a fee to get a desired rating reduces the information available to investors: the ratings observed by investors are truncated below when $\phi > 0$. As a result, the average rating observed by market investors is "inflated" relative to the baseline model.

The following lemma shows that the informativeness of ratings does not only depend on the cost ϕ, but also on investor's prior beliefs, μ. Furthermore, ratings are less informative when investors are more confident about the underlying loan type.

Lemma 2. *For $\phi > 0$, $\bar{r}(\mu, \phi)$ is non-monotonic in μ. In particular, there exists $\mu_m(\phi) \in (0, 1)$ such that $\bar{r}(\mu, \phi)$ strictly decreases in μ for all $\mu < \mu_m(\phi)$ and strictly increases in μ for all $\mu > \mu_m(\phi)$.*

Retention Decisions. Given the previous characterization of banks' decision to get rated, we proceed to the retention decision of a bank with a t-type pool. The expected payoff of retaining a fraction x of a t-type loan pool is:

$$u_t(\mu, x, \phi) = (1 - x)(\tilde{\alpha}_t(\mu, \phi)(v_g - v_b) + v_b - \phi) + \delta x v_t \tag{21}$$

where $\tilde{\alpha}_t(\mu, \phi) \equiv E_R[\max\{\mu_f(\mu, \bar{r}); \mu_f(\mu, r)\}|t]$ is the new expected posterior, which incorporates the truncated nature of ratings. The expression is obtained after incorporating the indifference condition between being rated or not in (19).

From Lemma 1, we know that if ratings are free, $\phi = 0$, all ratings are published. Thus, $\tilde{\alpha}_t(\mu, 0) = E_R[\mu_f(\mu, r)|t] = \alpha_t(\mu)$ from Section 4. In this case, the two problems are identical. The following lemma shows that as the CRA fee increases, the expected payoff for a g-type pool approaches that of a b-type.

Lemma 3. *For all $\mu \in (0, 1)$, $\tilde{\alpha}_g(\mu, \phi) - \tilde{\alpha}_b(\mu, \phi) \geq 0$ and decreasing in ϕ.*

If the presence of rating shopping decreases the informativeness of ratings to the point that they are no longer Γ-informative, then the equilibrium outcome is as the one described in Proposition 1. This would occur, for example, when ϕ is large enough. However, it is more interesting to analyze the scenario in which ratings continue to be Γ-informative and used in equilibrium. In Appendix A.2, we discuss the conditions under which an equilibrium with rating shopping exists and is unique. Most importantly, we show that any equilibria must have the same properties of the equilibrium of the baseline model (described in Propositions 2 and 3).

With rating shopping, the informativeness of ratings and the payoffs at the securitization stage depend on the CRA fee, ϕ. This can be seen by comparing the expected payoff from a t-type pool (21) in this model with that from the baseline model with no shopping (3). There are two differences: first, now banks have to pay ϕ if they get rated, and, more importantly, the expected market belief for a t-type pool with rating shopping, $\tilde{\alpha}_t(\cdot, \phi)$, differs from that in the baseline model $\alpha_t(\cdot)$ since now the distribution of ratings is truncated bellow. That is, with rating shopping, ratings are costlier and less informative. The effect of reducing the informativeness of ratings was described in Section 5.2.

6.2 Manipulable Ratings

In this extension, we suppose that banks can manipulate ratings upward by paying m to the CRA. This can be interpreted as a side payment a bank gives to the CRA to obscure the value of the underlying loan pool. To keep this extension simple, we assume that when a bank with a b-type loan pool pays the CRA, it obtains a rating from the distribution corresponding to a g-type loan pool with probability σ_m. There is no rating shopping in this model, so after the payment m is given to the CRA, the model is as the one described in Section 4. The main difference is that now the CRA may report a manipulated rating.

Naive Investors. Assume investors are not aware of the ability of the bank to manipulate ratings. Recall that (x^*, μ^*) is the pooling action/belief absent manipulation, as stated in Proposition 2. If investors are not aware of manipulation, the expected gain from paying to manipulate for banks with b-type pools is given by:

$$\Delta = \sigma_m(\alpha_g(\mu^*) - \alpha_b(\mu^*))(1 - x^*)(v_g - v_b) \tag{22}$$

Thus, manipulation takes place if $m < \Delta$. As a result, the payoff associated with originating a b-type loan pool, $u_b(\mu_0)$, increases, while that of originating a g-type loan remains unchanged, $u_g(\mu_0)$. When investors are not aware of manipulation, they transfer value to the bank selling

b-backed security. As a result, banks increases their willingness to originate loans. With manipulation and naive investors, credit supply increases, efficiency decreases, and investors make losses.

Sophisticated Investors. Now we suppose that investors are aware of banks' ability to manipulate their ratings. Then the belief updating of investors is adjusted as follows:

$$\mu_f(\mu, r) = \frac{\mu f_g(r)}{(\mu + \sigma_m(1-\mu))f_g(r) + (1-\sigma_m)(1-\mu)f_b(r)} \tag{23}$$

It is straightforward from equation (23) that for σ_m large enough, ratings are not sufficiently informative to support the pooling outcome, and the g-type plays LCSE, making rating manipulation not worth it for the b-type. In the extreme case of $\sigma_m = 1$, we can see that investors do not update after observing a rating; that is, ratings are fully uninformative, and we recover the results from Section 3.2. In these cases, an equilibrium may fail to exist.

However, to the extent that σ_m is small enough so that ratings continue being Γ-informative at the x^{LC}, manipulation would take place to the extent that the cost m is lower than the benefit to the b-type. Thus, when investors are aware of manipulation, the latter acts as a reduction in rating informativeness, with the consequences for securitization and credit supply discussed in Section 5.2.

7 Conclusion

We have studied the effect of ratings on loan origination and securitization decisions. Sufficiently informative ratings increase market liquidity and improve allocative efficiency; but may worsen incentives to originate positive NPV loans. In particular, we show that if banks cannot adjust their securitization decisions in response to the presence or informativeness of ratings, the latter discipline markets and improve loan origination, increasing productive efficiency. However, when banks can adjust their securitization/retention decisions in response to ratings, their disciplining role is weakened. Furthermore, we show that informative ratings can induce an oversupply of credit relative to first-best, by improving incentives to originate negative NPV loans. Our results are robust to extensions that incorporate some features of the rating industry, such as shopping and manipulation; where these market frictions act mainly by reducing rating informativeness. We then use our model to discuss the implications of policies, such as the "skin-in-the-game" rule for securitizaters or the information disclosure requirements for CRAs.

We highlight the interactions between securitization decisions and ratings, and their importance in understanding incentives to originate loans. Even though not considered in this paper, we believe that incorporating more strategic aspects of the ratings industry, such as competition, is the next step to continue furthering our understanding of the role of CRAs in financial markets.

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A Appendix

A.1 Preliminaries and Definitions

Fact 1. *For any* $t \in \{b, g\}$,

1. $\alpha_t(\mu)$ *is strictly increasing in* μ *for any* $x \in (0, 1]$.

2. $\alpha_g(\mu) - \alpha_b(\mu)$ *is concave and achieves a unique maximum at* $\hat{\mu} \in (0, 1)$.

3. $\frac{\partial}{\partial \mu}\left(\frac{\alpha'_g(\mu)}{\alpha'_b(\mu)}\right) < 0$ *for all* $\mu \in (0, 1)$.

4. $E[(1 - x)Y|g] > E[(1 - x)Y|b]$ *for any* $x \in (0, 1]$

5. $u_t(x, \mu)$ *is strictly increasing in* μ *for any* $x \in (0, 1]$.

6. $u_b(x, \mu)$ *is strictly decreasing in* x *for any* $\mu \in [0, 1]$.

Fact 2. *In any PBE,* $u_t \in [v_b, v_g)$ *for any* $t \in \{b, g\}$.

The D1 Refinement

Fix $k \in [v_g, v_g)$ and $x \in [0, 1]$, and consider the equation $u_t(\mu, x) = k$. By Fact 1(5), there is at most one solution for μ. If it exists, denote it by $b_t(x, k)$—that is, $u_t(x, b_t(F, k)) = k$. Next, let $B_t(x, k) \equiv \{\mu : u_t(x, \mu) > k\}$. From Fact 1(1), the connection between b_t and B_t is immediate: if $b_t(x, k)$ exists, then $B_t(x, k) = (b_t(x, k), 1]$. If $b_t(x, k)$ fails to exist, then either $B_t(x, k) = [0, 1]$ or $B_t(x, k) = \emptyset$.

In our model, the D1 refinement can be stated as follows. Fix an equilibrium endowing expected payoffs $\{u_b, u_g\}$. Consider a retention choice x that is not in the support of either type's strategy. If $B_L(x, u_b) \subset B_H(x, u_g)$, then D1 requires that $\mu(x) = 1$ (where $\subset$ denotes strict inclusion). If $B_H(x, u_g) \subset B_L(x, u_b)$, then D1 requires that $\mu(x) = 0$.

A.2 Proofs of Section 3.2

Proof of Proposition 1. To check that this is a PBE, we need to check that neither type wishes to deviate at any stage. [Securitization Stage] First, note that banks with b-type pools do not profit from deviating to mimic the retention of those banks with g-type pools since x_g is chosen so that the incentive compatibility (IC) for b-type pools binds. Second, a binding IC for the b-type implies a slack IC for the g-type since $v_g > v_b$ and thus $v_b < x_g v_g + \delta(1 - x_g)v_g$. It is easy to check that the following off-equilibrium beliefs: $\mu(x) = 0$ for all $x \neq x_g$, support this equilibrium. Single-crossing ensures that the LCSE is the unique equilibrium that satisfies D1 (see DeMarzo (2005)).

[Origination Stage] From the previous results, the payoffs associated with originating a g- and a b-type loan pool, respectively, are:

$$u_g^{LC} = (1 - x_g)v_g + \delta x_g v_g \tag{24}$$

$$u_b^{LC} = v_b \tag{25}$$

where $u_b^{LC} < 1 < u_g^{LC}$. Most importantly, these payoffs are independent of the actual origination threshold decision of the banks. Since $p(q)$ is a decreasing function, all loan pools with $q < q^{LC}$ give the bank a positive NPV, and thus would be originated. Credit supply resulting from the aggregation of all pools originated is given by q^{LC}, the loan pool that generates zero NPV to banks. This origination threshold maximizes banks' ex-ante value given securitization payoffs, and it is not observed by investors so deviations cannot impact market beliefs directly. As a result, there are no profitable deviations at the origination stage either.

□

A.3 Proofs of Section 4

Lemma 4. *The solution to the following $\mathcal{M}(k)$ problem:*

$$\max_{\mu,x} \; u_g(\mu, x) \quad s.t. \quad u_b(\mu, x) = k \tag{26}$$

denoted by $\{\mu(k), x(k)\}$ is unique and characterized by the problem's first-order conditions. Furthermore, the solution is interior with $\mu^* < 1$ if ratings are Γ-informative: $\frac{\alpha_b'(1)}{\alpha_g'(1)} > \frac{v_g - \delta v_b}{(1-\delta)v_g}$. In addition, $\mu(k)$ is independent of k and $x(k)$ is decreasing in k.

Proof. We have that:

$$u_t(x, \mu) = (1 - x)(\alpha_t(\mu)(v_g - v_b) + v_b) + \delta x v_t \tag{27}$$

where $\alpha_t(\mu) \equiv E_R[\mu_f(\mu, r)|t]$. Let $\alpha(\mu) \equiv \alpha_g(\mu) - \alpha_b(\mu)$ be the difference between expected posteriors for prior beliefs μ. It will be useful to re-state the problem as follows:

$$\max_{\mu,x} \; u_g(\mu, x) - k \quad s.t. \quad u_b(\mu, x) = k \tag{28}$$

By plugging in the corresponding expressions and the binding constraint, we obtain:

$$\max_{\mu,x} \; (1 - x)(\alpha(\mu) - \delta) + \delta(v_g - v_b) \tag{29}$$

$$s.t. \quad (1 - x)(\alpha_b(\mu)(v_g - v_b) + v_b) + \delta x v_b = k \tag{30}$$

After some algebra, the solution implied by the first-order conditions $\{\mu(k), x(k)\}$ solves:

$$\frac{v_g - v_b}{v_b} = \frac{1 - \delta}{(\alpha(\mu) - \delta) \frac{\alpha_b'(\mu)}{\alpha'(\mu)} - \alpha_b(\mu)} \tag{31}$$

$$k = (1 - x)(\alpha_t(\mu)(v_g - v_b) + v_b) + \delta x v_b \tag{32}$$

First, note that $\mu(k)$ is independent of k. Second, note that $x(k)$ is given by the constraint. To see that $\mu(k)$ is unique, note that the LHS of equation (31) is constant, while the RHS is monotonic in μ, since $\frac{\partial RHS(\mu)}{\partial\mu}$ is given by:

$$
\begin{aligned}
&-\frac{1-\delta}{\left(\left(\alpha\left(\mu\right)-\delta\right)\frac{\alpha_b'(\mu)}{\alpha'(\mu)}-\alpha_b\left(\mu\right)\right)^2}\left(\alpha'\left(\mu\right)\frac{\alpha_b'\left(\mu\right)}{\alpha'\left(\mu\right)}+\left(\alpha\left(\mu\right)-\delta\right)\frac{\partial}{\partial\mu}\left(\frac{\alpha_b'\left(\mu\right)}{\alpha'\left(\mu\right)}\right)-\alpha_b'\left(\mu\right)\right)\\
&=-\frac{1-\delta}{\left(\left(\alpha\left(\mu\right)-\delta\right)\frac{\alpha_b'(\mu)}{\alpha'(\mu)}-\alpha_b\left(\mu\right)\right)^2}\frac{\alpha\left(\mu\right)-\delta}{\alpha\left(\mu\right)^2}\left(\alpha_b''\left(\mu\right)\alpha'\left(\mu\right)-\alpha''\left(\mu\right)\alpha_b'\left(\mu\right)\right)\\
&=\frac{1-\delta}{\left(\left(\alpha\left(\mu\right)-\delta\right)\frac{\alpha_b'(\mu)}{\alpha'(\mu)}-\alpha_b\left(\mu\right)\right)^2}\frac{\alpha\left(\mu\right)-\delta}{\alpha\left(\mu\right)^2}\frac{\partial}{\partial\mu}\left(\frac{\alpha_g'\left(\mu\right)}{\alpha_b'\left(\mu\right)}\right)
\end{aligned}
$$

From Lemma A.1. of Daley and Green (2014), we know that $\frac{\partial}{\partial\mu}\left(\frac{\alpha_g'(\mu)}{\alpha_b'(\mu)}\right)<0$ for all $\mu\in(0,1)$. Then, if $\alpha\left(\mu\right)<\delta$ for all $\mu\in(0,1)$, the RHS is increasing in μ and there is a unique solution $\mu(k)$. Otherwise, there exists μ such that $\alpha(\mu)>\delta$. In this scenario, by inspection of the objective function (29) it follows that the solution requires $\alpha(\mu)>\delta$, in which case the RHS is decreasing in the relevant range, and a unique solution in that range exists as well.

Thus, we conclude that a unique pair $\{\mu^*(k),x^*(k)\}$ is a candidate solution. Furthermore, $\mu^*(k)$ is constant and given by (31), and $x^*(k)$ is strictly decreasing in k and given by the constraint (32).

The solution to the above problem is interior with $\mu^*<1$ (and thus $x<x^{LC}$) if and only if condition (31) holds for an interior μ. Note that the RHS of the condition is negative at $\mu=0$ ($\alpha(0)=0$,$\alpha_g'(0)>0,\alpha_b'(0)>0$) and increasing. It follows that an intersection between the RHS and the LHS for $\mu<1$ exists iff:

$$
\frac{v_g}{v_b}-1<\frac{1-\delta}{\left(\alpha\left(1\right)-\delta\right)\frac{\alpha_b'(1)}{\alpha'(1)}-\alpha_b\left(\right)}
$$

$$
\frac{v_g}{v_b}-1<\frac{1-\delta}{\left(-\delta\right)\frac{\alpha_b'(1)}{\alpha'(1)}-1}\iff\frac{\alpha_b'(1)}{\alpha_g'(1)}>\frac{v_g-\delta v_b}{(1-\delta)v_g}
$$

Which is our Γ-informative condition. This condition is a statement about the slope of the indifference curves at the LCSE outcome $\{\mu = 1, x^{LC}\}$, and it states that an interior solution exists iff:

$$\left(\frac{\partial u_g(\mu,x)}{\partial x} \,/\, \frac{\partial u_g(\mu,x)}{\partial \mu}\right)|_{\mu=1,x^{LC}} > \left(\frac{\partial u_b(\mu,x)}{\partial x} \,/\, \frac{\partial u_b(\mu,x)}{\partial \mu}\right)|_{\mu=1,x^{LC}}$$

That is, if the slope of the indifference curve at the LCSE outcome is steeper for the g-type than it is for the b-type, breaking the single-crossing condition necessary for separation.

It remains to verify the second order conditions of this problem. We verify that the determinant of the Bordered Hessian is negative at our interior candidate:

$$BH = \begin{bmatrix} 0 & \frac{\partial u_b(\mu,x)}{\partial x} & \frac{\partial u_b(\mu,x)}{\partial \mu} \\ \frac{\partial u_b(\mu,x)}{\partial x} & L_{xx} & L_{x\mu} \\ \frac{\partial u_b(\mu,x)}{\partial \mu} & L_{\mu x} & L_{\mu\mu} \end{bmatrix}$$

where $L(x,\mu) = u_g(\mu,x) - \lambda\left(u_b(\mu,x) - k\right)$ and λ is the Lagrange multiplier.

$$L_{xx} = 0$$

$$L_{\mu\mu} = \left(\alpha_g''(\mu^*) - \lambda^* \alpha_b''(\mu^*)\right)(1-x^*)(v_g - v_b)$$

$$L_{x\mu} = L_{\mu x} = -(v_g - v_b)(\alpha_g'(\mu^*) - \lambda^* \alpha_b'(\mu^*)) = 0$$

A sufficient condition for our solution to be a local maximum is that the bordered Hessian is negative definite. That is, $|BH_1| < 0$ and $|BH_2| > 0$. It is easy to see that $|BH_1| = -(\frac{\partial u_b(\mu,x)}{\partial x})^2 < 0$ and that $|BH_2| = -\left(\frac{\partial u_b(\mu,x)}{\partial x}\right)^2 L_{\mu\mu} > 0$. Thus, SOC are satisfied since $L_{\mu\mu} < 0$ from $\frac{\partial}{\partial \mu}\left(\frac{\alpha_g'(\mu)}{\alpha_b'(\mu)}\right)|_{\mu=\mu^*} < 0$. □

Definition 3. *We define $b_t(x,v)$ as the belief necessary to provide the t-type utility v if retention is x; that is, $u_t(x, b_t(x,v)) = v$, and by $B_t(x,v) = (b_t(x,v), 1]$ the set of beliefs for which the t-type obtains strictly higher utility than v when retention is x.*

Proof of Proposition 2. Let $x(k), \mu(k)$ be given by the solution to the constrained maximization problem $\mathcal{M}(k)$ in Lemma 4. From that same Lemma, we know that the solution exists and is unique, that $\mu^*(k)$ is constant and that $x^*(k)$ is continuous and decreasing in $k \in [v_g, v_g]$. We define $\mu^* = \mu(v_b)$ and $x^* = x(v_b)$. First, note that $\mu^* < 1$ iff $E[\Gamma(r)|b] - E[\Gamma(r)|g] > \frac{\delta}{1-\delta}\frac{v_g - v_b}{v_g}$ since $\alpha_b'(1) = E[\Gamma(r)|b]$ and $\alpha_g'(1) = E[\Gamma(r)|g] = 1$. That is, the equilibrium is not separating if ratings are Γ-informative.

The next step is to show that the equilibrium proposed in Proposition 2 is a PBE that satisfies D1. Before doing so, we introduce the following definitions. Let S_t be the support of the t-type's strategy. In the proposed unique equilibrium, the good type plays a pure strategy, denoted it x_g, so $S_g = \{x_g\}$, while the bad type could mix, and thus $S_b \subseteq \{0, x_g\}$. For completeness, we must specify the off-path beliefs: $\mu(x) = 0$ for all $x \neq x_g$.

Verifying that the proposed profile is a PBE is straightforward. To see that it satisfies D1, fix a μ_0 and consider the Proposition's unique equilibrium candidate. Denote the good type's equilibrium payoff u_g^e and the bad type's equilibrium payoff k, so $x_g = x^*(k)$. Let x be an arbitrary retention level in $[0,1]$ such that $x \neq x^*(k)$. First, if $B_b(x,k) = [0,1]$, then the low type could deviate to x and obtain a payoff strictly greater than k, regardless of $\mu(x)$, breaking the PBE. Hence, either $b_b(x,k) \in (0,1]$ exits or $u_b(x,1) < k$. If $b_b(x,k)$ exits, then since $\{x^*(k), \mu^*(k)\}$ is the unique solution to (26), $u_g(x, b_b(x,k)) < u_g(x^*(k), \mu^*(k)) = u_h^e$. By Fact 1(5) then, $b_g(x, u_h^e) > b_b(x,k)$ implying $B_g(x, u_h^e) \subseteq B_b(x,k)$. So, $\mu(x) = 0$ is consistent with D1. If instead $u_b(x,1) < k$ (so $B_b(x,k) = \emptyset$), then there exists a unique $\epsilon > 0$ such that $u_b(x+\epsilon, 1) = k$. Since $\{x^*(k), \mu^*(k)\}$ solves 26, $u_g(x^*(k), \mu^*(k)) \geq u_g(x+\epsilon, 1) > u_g(x,1)$. Hence, $B_g(x, u_h^e) = \emptyset$ as well, and D1 places no restriction on $\mu(x)$.

We now establish uniqueness. Fix an equilibrium with $u_g = u_g^e$ and $u_b = k$. Since the bank with a bad loan type has the option to choose the same retention as for a good type loan, $u_b(x, \mu(x)) \leq k$ for all $x \in S_g$. Fix now $x \in S_g$ and suppose that $u_b(x, \mu(x)) < k$. Then $x \notin S_b$, so $\mu(x) = 1 = b_g(x, u_g^e)$ and $B_b(x,k) = \emptyset$. Further, it must be that $x \neq 0$ since $u_b(0,1) = v_g > k$. Then for $\epsilon > 0$ small enough $b_g(x+\epsilon, u_g^e) \in (0,1)$ and $B_b(x+\epsilon, k) = \emptyset$.

Therefore, $x + \epsilon \notin S_b$ and $\mu(x+\epsilon) = 1$ (by belief consistency if $x + \epsilon \in S_g$, by D1 if not). Since $u_g(x+\epsilon, 1) > u_g(x, 1) = u_g^e$, the high type would gain by deviating to $x + \epsilon$, breaking the equilibrium. Therefore, $u_b(x, \mu(x)) = k$, or equivalently $\mu(x) = b_b(x, k)$, for all $x \in S_g$.

Suppose now there exists $x \in S_g$ such that $x \neq x^*(k)$. Then

$$u_g(x, \mu(x)) = u_g(x, b_b(x, k)) < u_g(x^*(k), \mu^*(k)) = u_g(x^*(k), b_b(x^*(k), k)),$$

and thus $b_g(x^*(k), u_h^e) < \mu^*(k) = b_b(x^*(k), k)$. D1 then implies that $\mu(x^*(k)) = 1$, meaning that deviating to $x^*(k)$ is profitable for the high type and breaking the equilibrium. Hence, if the b-type's equilibrium payoff is k, then $S_g = \{x^*(k)\}$ and $\mu(x^*(k)) = \mu^*(k)$.

Further, if the b-type selects $x \notin S_g \cup \{0\}$, then $\mu(x) = 0$, and $u_b(x, 0) < u_b(0, 0) \leq u_b(0, \mu(0))$ for any value of $\mu(0)$. She could therefore profitably deviate to $x = 0$. Hence, $S_b \subseteq S_g \cup \{0\}$.

The final step is to characterize which values of $u_b = k$ are consistent with equilibrium, which depends on the prior, μ_0. Recall that μ^* does not vary with k. First, let $\mu_0 < \mu^*$, and let $u_b = k$. Therefore, $S_g = \{x^*(k)\}$ and $\mu(x^*(k)) = \mu^* > \mu_0$. For this belief to be consistent with seller strategies, $S_b \neq \{x^*(k)\}$. Hence, $S_b = \{x^*(k), 0\}$ and $k = v_b$. The precise mixing probabilities given in the proposition are required for the Bayesian consistency: $\mu(x^*(v_b)) = \mu^*(v_b)$. Second, let $\mu_0 \geq \mu^*$. Then $S_g = S_b = \{x^*(k)\}$ is consistent with $\mu^*(x^*(k)) = \mu^*(k) = \mu^*$, as stated in the Proposition.

□

Proof of Proposition 3. An equilibrium exists if there exists μ_0 such that $p(A^{-1}(\mu_0)) = \Psi(\mu_0)$, and it is unique if the two curves only intersect one. First, note that $p(A^{-1}(\cdot))$ is strictly increasing and that by Assumption 1, there exists μ_0 and μ_1 with $\mu_0 < \mu_1$ such that $p(A^{-1}(\mu_0)) < p^{FB}$ and $p(A^{-1}(\mu_1)) > p^{FB}$. In addition, we know that $\lim_{\mu \to 0} \Psi(\mu) > p^{FB}$ and that $\lim_{\mu \to 1} \Psi(\mu) < p^{FB}$ (see Corollary 1). Therefore, since both functions are continuous, an equilibrium exists. To see that it is unique, it suffices to show that $\Psi(\mu) = \frac{1 - u_b(\mu)}{u_g(\mu) - u_b(\mu)}$ is decreasing in $\mu \in (0, 1)$. Let $G = \{\mu : 0 \leq \Psi(\mu) \leq 1\}$. Let $\mu^* \in [0, 1]$ be an equilibrium of the full game, then $\mu^* \in G$.

In addition, for any $\mu \in G$, it must be that $u_g(\mu) \geq 1 \geq u_b(\mu)$. Then, since $u'_g(\cdot) \geq 0$ and $u'_b(\cdot) \geq 0$, it follows that $\Psi'(\mu) \leq 0$ for all $\mu \in G$. □

Proof of Proposition 4. The skin-in-the-game rule requires all securitizers to retain at least a fraction $\bar{x}$. As a result, $\{x^*, \mu^*\}$ that characterize the marginal loan determination as stated in Proposition 2 are now given by the solution to:

$$\max_{\mu,x}(1-x)(\alpha_g(\mu)(v_g - v_b) + v_b) + \delta x v_g \tag{33}$$

$$s.t.(1-x)(\alpha_g(\mu)(v_g - v_b) + v_b) + \delta x v_g = (1 - \bar{x} + \delta\bar{x})\, v_b$$

where the only adjustment has been a change in the outside option (full information payoff) of the banks with b-type pools in the constraint. The proof of Proposition 2 can be used to see that the that the solution to the re-stated problem is a D1 PBE of the securitization stage with skin-in-the-game rule.

First, note that for the constraint to hold $x \geq \bar{x}$, and thus $x^* \geq \bar{x}$. For the skin-in-the-game rule to hold, retention will be $x \in [\bar{x}, x^*]$, depending on bank type and on market beliefs. This rule affects the marginal loan determination by affecting the low-type's outside option. Second, $\{x^*, \mu^*\}$ are given by the problem's FOC. In particular, μ^* continues to be determined by FOC (31), while x^* is determined by the new constraint in (33). Thus, μ^* is unaffected by the rule. It follows that increases a skin-in-the-game rule weakly increase retention levels as follows. For $\mu > \mu^*$, equilibrium retention increases from 0 to $\bar{x}$. For $\mu = \mu^*$, retentions in the range $x \in [x^*, \bar{x}]$ can be D1-equilibria. Finally, for $\mu < \mu^*$, there is partial pooling at the new (higher) x^*, where banks with g-type pools retain x^* and those with b-type pools mix between $\{x^*, \bar{x}\}$ with probability $\lambda = \frac{\mu_0(1-\mu^*)}{\mu^*(1-\mu_0)}$ and $(1-\lambda)$ respectively. Since the payoff associated to both type of pools has decreased with the skin-in-the-game rule, it follows that credit supply weakly decreases, and so does ex-ante efficiency.

□

Proof of Proposition 5. The equilibrium at the securitization stage is fully characterized by

$\{\mu^*, x^*\}$ as stated in Proposition 2. Furthermore, as shown in the proof of this proposition, $\{\mu^*, x^*\}$ are given by the first-order conditions of the $\mathcal{M}(v_b)$, which can be re-written as follows:

$$\frac{(1-\delta)\, v_b}{v_g - v_b} = (\alpha(\mu^*) - \delta)\, \frac{\alpha_b'(\mu^*)}{\alpha'(\mu^*)} - \alpha_b(\mu^*) \tag{34}$$

$$x^* = \frac{\alpha_b(\mu^*)(v_g - v_b)}{\alpha_b(\mu^*)(v_g - v_b) + (1-\delta) v_b} \tag{35}$$

We proceed to characterize how this solution changes with γ. We have that:

$$\alpha_g(\mu) = \frac{\mu\gamma^2}{\mu\gamma + (1-\mu)(1-\gamma)} + \frac{\mu(1-\gamma)^2}{\mu(1-\gamma) + (1-\mu)\gamma} \tag{36}$$

$$\alpha_b(\mu) = \frac{\mu\gamma(1-\gamma)}{\mu\gamma + (1-\mu)(1-\gamma)} + \frac{\mu\gamma(1-\gamma)}{\mu(1-\gamma) + (1-\mu)\gamma} \tag{37}$$

$$\alpha(\mu) = \frac{(2\gamma-1)^2 \mu(1-\mu)}{(\mu\gamma + (1-\mu)(1-\gamma))(\mu(1-\gamma) + (1-\mu)\gamma)} \tag{38}$$

After some algebra, from the RHS of condition (34) we have that:

$$\frac{\partial RHS}{\partial \gamma}|_{\mu=\mu^*} = \frac{\delta}{(2\gamma-1)^3(1-2\mu^*)}$$

$$\frac{\partial RHS}{\partial \mu}|_{\mu=\mu^*} = -\frac{\alpha(\mu^*) - \delta}{\alpha(\mu^*)^2} \frac{\partial}{\partial \mu}\left(\frac{\alpha_g'(\mu^*)}{\alpha_b'(\mu^*)}\right)$$

If $\delta > \alpha(\mu)$ for all $\mu \in [0,1]$, then from FOC the solution requires $\alpha'(\mu^*) < 0 \iff \mu^* > \arg\max_\mu \alpha(\mu) = \frac{1}{2}$ (see Fact 2). As a result, $\frac{\partial RHS}{\partial \gamma} < 0$ and $\frac{\partial RHS}{\partial \mu^*} < 0$. Otherwise, $\max_\mu \alpha(\mu) > \delta$ and, as stated in the proof Proposition 2, the solution has to satisfy $\alpha(\mu^*) > \delta$, which from FOC requires $\alpha'(\mu^*) > 0$. Therefore, $\mu^* < \frac{1}{2}$, and thus $\frac{\partial RHS}{\partial \gamma} > 0$ with $\frac{\partial RHS}{\partial \mu^*} > 0$. If follows that as γ increases, μ^* has to decrease.

We have established that μ^* decreases in γ. It remains to characterize how x^* changes in γ. Let RHS_c denote the left-hand side of the constraint (35). Then, we have that:

$$\frac{dx^*}{d\gamma} = \frac{\partial RHS_c}{\partial \mu^*}\frac{d\mu^*}{d\gamma} + \frac{\partial RHS_c}{\partial \gamma} < 0 \tag{39}$$

Where the results follows from (i) $\frac{\partial \alpha_b(\mu)}{\partial \gamma} < 0$ for all $\mu \in (0,1)$; (ii) $\alpha_b'(\cdot) > 0$ for all $\gamma \in (\frac{1}{2}, 1)$; and (iii) RHS_c being increasing in $\alpha_b(\mu)$ for all $\mu \in (0,1)$ since $v_b > 0$, which are all easy to check. □

Proof of Proposition 6. We know that $\{x^*, \mu^*\}$ are given by the solution to the $\mathcal{M}(v_b)$ problem in (26) and determine the shape of the marginal loan locus $\Psi(\mu)$. The following first-order condition determines μ^*:

$$\frac{v_g - v_b}{v_b} = \frac{1-\delta}{\frac{(\alpha(\mu^*)-\delta)}{\alpha'(\mu^*)}\alpha_b'(\mu^*) - \alpha_b(\mu^*)} \tag{40}$$

Let $RHS(\mu^*, \delta)$ denote the right-hand side of equation (40). Since the left-hand side is independent of δ and $\{\mu^*, x^*\}$ we have that $\frac{\partial RHS}{\partial \mu^*} d\mu^* + \frac{\partial RHS}{\partial \delta} d\delta = 0$ where:

$$\frac{\partial RHS}{\partial \mu^*} = -\frac{1-\delta}{[...]^2}(\alpha(\mu^*) - \delta)\frac{\partial}{\partial \mu}\left(\frac{\alpha_b'(\mu)}{\alpha_g'(\mu)}\right)$$

since $\frac{\partial}{\partial \mu}\left(\frac{\alpha_b'(\mu)}{\alpha_g'(\mu)}\right) >$) the derivative takes the sign of $-(\alpha(\mu^*) - \delta)$. We also have that:

$$\frac{\partial RHS}{\partial \delta} = \frac{1}{[...]^2}\frac{1}{\alpha'(\mu^*)}\left[(1-\alpha_g(\mu^*))\alpha_b'(\mu^*) + \alpha_g'(\mu)\alpha_b(\mu^*)\right]$$

which takes the sign of $\alpha'(\mu^*)$. Since the left-hand-side of the constraint is positive, so has to be the the RHS, which requires $\alpha'(\mu^*) \times (\alpha(\mu^*) - \delta) \geq 0$. Thus, we have that $\frac{\partial \mu^*}{\partial \delta} \geq 0$. To see the effect on retention, we do a total differentiation of the constraint:

$$[-\alpha_b(\mu^*)(v_g - v_b) - (1-\delta)v_b]\, dx + \left[(1-x)\alpha_b'(\mu^*)(v_g - v_b)\frac{d\mu^*}{d\delta} + x v_b\right] d\delta = 0$$

$$\frac{dx^*}{d\delta} = \frac{\left[(1-x)\alpha_b'(\mu^*)(v_g - v_b)\frac{d\mu^*}{d\delta} + x v_b\right]}{[\alpha_b(\mu^*)(v_g - v_b) + (1-\delta)v_b]} > 0$$

Finally, to study the effect of a change in δ on credit supply, we study the effect on $\Psi(\mu)$. We know that μ^* has increased, and that since δ does not affect payoffs when retention is zero, the Ψ function weakly increases for $\mu > \mu_0^*$. It remains to characterize the change in $\Psi(\mu_0)$ for $\mu_0 < \mu_0^*$, which is fully determined by the effect on $u_g(\mu^*, x^*)$:

$$u_g^* \equiv u_g(\mu^*, x^*) = \max_{\mu,x} (1-x)\left[\alpha_g(\mu)(v_g - v_b) + v_b\right] + \delta x v_g$$
$$s.t.\ (1-x)\left[\alpha_b(\mu)(v_g - v_b) + v_b\right] + \delta x v_b = v_b$$

We have that

$$\frac{\partial u_g^*}{\partial \delta} = v_g - \frac{\alpha_g'(\mu^*)}{\alpha_b'(\mu^*)} v_b \tag{41}$$

since $\frac{\alpha_g'(\mu^*)}{\alpha_b'(\mu^*)}$ is the Lagrange multiplier of the constraint in this problem at $\{\mu^*, x^*\}$.

Therefore, to complete the proof it suffices to show that $\frac{\alpha_g'(\mu^*)}{\alpha_b'(\mu^*)} < \frac{v_g}{v_b}$. To see this, first rewrite the FOC of $\mathcal{M}(v_b)$ as

$$\frac{\alpha_g'(\mu^*)}{\alpha_b'(\mu^*)} = \frac{\alpha_g(\mu^*)v_g + (1-\alpha_g(\mu^*))v_b - \delta v_g}{\alpha_b(\mu^*)v_g + (1-\alpha_b(\mu^*))v_b - \delta v_b} \tag{42}$$

and observe that the numerator on the RHS is increasing in α_g, while the denominator is increasing in α_b, therefore

$$\frac{\alpha_g(\mu^*)v_g + (1-\alpha_g(\mu^*))v_b - \delta v_g}{\alpha_b(\mu^*)v_g + (1-\alpha_b(\mu^*))v_b - \delta v_b} < \frac{\alpha_g(1)v_g + (1-\alpha_g(1))v_b - \delta v_g}{\alpha_b(0)v_g + (1-\alpha_b(0))v_b - \delta v_b} = \frac{v_g}{v_b}.$$

□

A.4 Proofs of Section 6

Proof of Lemma 1. Let $\mathcal{R}$ be the set of ratings that the bank chooses to report in equilibrium, where $\mu_n(\mu, \phi)$ is the belief assigned to a bank not being rated. Define $\bar{r}$ be such that

$$\mu_f(\mu, \bar{r}) - \mu_n(\mu, \phi) = \phi,$$

where $\bar{r}$ exists since $\mu_f(\mu, r) = \frac{\mu}{\mu+(1-\mu)\Gamma(r)}$ is increasing and continuous in r for $\mu \in (0,1)$, and in any equilibrium it must be that $\mu_n(\mu, \phi) \in [\inf_r \mu_f(\mu, r), \sup_r \mu_f(\mu, r)]$. Suppose $r < \bar{r}$ and $r \in \mathcal{R}$. Since $\mu_f(\mu, r)$ is increasing in μ, $u_t(\mu, x, r) < u_t^n(\mu, n, r)$. Contradiction, since it violates bank's optimality. Now suppose there exists $r > \bar{r}$ such that $r \notin \mathcal{R}$. Then, if a bank receives rating r and chooses to pay and report it, investors assign beliefs $\mu_f(\mu, r)$ and the bank profits from this deviation. Contradiction. Thus, it must be that $\mathcal{R} = \{r : r \geq \bar{r}\}$.

Then, in any equilibrium, not being rated indicates that $r < \bar{r}(\mu, \phi)$. As a result:

$$\mu_n(\mu, \phi) = \frac{\mu}{\mu + (1-\mu)\frac{F_b(\bar{r}(\mu,\phi))}{F_g(\bar{r}(\mu,\phi)))}}.$$

It remains to show that the set $\mathcal{R}$ differs from the full set of ratings iff $\phi > 0$. ($\Leftarrow$) Let the set $\{r : \ r < \bar{r}\}$ has zero measure. Then, $\bar{r} = \inf r$ and $\mu_f(\mu, \bar{r}) - \mu_n(\mu, \bar{r}) = 0$ since by L'Hopital,

$$\lim_{r \to \inf r} \frac{F_b(r)}{F_g(r)} = \frac{f_b(r)}{f_g(r)}.$$

This is only consistent with with $\phi = 0$. ($\Rightarrow$) Now assume that $\phi = 0$. Then, $\bar{r}$ is given by: $\mu_f(\mu, \bar{r}) - \mu_n(\mu, \bar{r}) = 0$, which implies $\bar{r} = \inf r$ since $\mu_f(\mu, r) - \mu_n(\mu, r) > 0$ for $\mu \in (0,1)$ and all $r > \inf r$. Thus, $\{r : \ r < \bar{r}(\mu, \phi)\}$ has zero measure. □

Proof of Lemma 2. From equation 19, we have that:

$$\frac{\partial \bar{r}(\mu, \phi)}{\partial \mu} = -\frac{\frac{\partial\left(\mu_f(\mu,\bar{r}) - \mu_n(\mu,\bar{r})\right)}{\partial \mu}}{\frac{\partial\left(\mu_f(\mu,\bar{r}) - \mu_n(\mu,\bar{r})\right)}{\partial \bar{r}}}|_{\bar{r}=r(\mu,\phi)}$$

[Denominator] Since (i) $\lim_{\bar{r}\to\inf r} \mu_f(\mu, \bar{r}) - \mu_n(\mu, \bar{r}) = 0$, (ii) $\lim_{\bar{r}\to\inf r} \frac{\partial\left(\mu_f(\mu,\bar{r}) - \mu_n(\mu,\bar{r})\right)}{\partial \bar{r}} > 0$, and (iii)$\mu_f(\mu, \bar{r}) - \mu_n(\mu, \bar{r})$ is continuous in $\bar{r}$, it must be that $\frac{\partial\left(\mu_f(\mu,\bar{r}) - \mu_n(\mu,\bar{r})\right)}{\partial \bar{r}}|_{\bar{r}(\mu,\phi)} > 0$. [Numerator] After some algebra, the numerator can be expressed as follows:

$$\frac{\left(\frac{F_b(\bar{r})}{F_g(\bar{r})}-\frac{f_b(\bar{r})}{f_g(\bar{r})}\right)}{\left(\mu+(1-\mu)\frac{f_b(\bar{r})}{f_g(\bar{r})}\right)^2\left(\mu+(1-\mu)\frac{F_b(\bar{r})}{F_g(\bar{r})}\right)^2}\left[(1-\mu)^2\frac{F_b(\bar{r})}{F_g(\bar{r})}\frac{f_b(\bar{r})}{f_g(\bar{r})}-\mu^2\right]$$

The first term is positive for all $\mu \in (0,1)$ and for all $\phi > 0$. The second term, however, is negative for μ close enough to one, and positive for μ close enough to zero; in addition, it is easy to check that it is continuously decreasing in μ. Thus, there exists $\mu_m(\phi)$ such that $\frac{\partial\left(\mu_f(\mu,\bar{r})-\mu_n(\mu,\bar{r})\right)}{\partial\mu}|_{\mu=\mu_m(\phi)} = 0$, negative for $\mu < \mu_m(\phi)$, and positive otherwise. Furthermore, $\mu_m(\phi)$ is given by:

$$\frac{\mu_m(\phi)}{1-\mu_m(\phi)} = \left[\frac{F_b(\bar{r}(\mu,\phi))}{F_g(\bar{r}(\mu,\phi))}\frac{f_b(\bar{r}(\mu,\phi))}{f_g(\bar{r}(\mu,\phi))}\right]^{\frac{1}{2}}.$$

□

Proof of Lemma 3. We have that:

$$\frac{\partial\alpha_t(\mu,\phi)}{\partial\phi} = F_t(\bar{r}(\mu,\phi))\frac{\partial\mu_f(\mu,\bar{r})}{\partial\bar{r}}\frac{\partial\bar{r}(\mu,\phi)}{\partial\phi}|_{r=r(\mu,\phi)} \geq 0$$

and thus:

$$\frac{\partial\alpha(\mu,\phi)}{\partial\phi} = \left[F_g(\bar{r}(\mu,\phi)) - F_b(\bar{r}(\mu,\phi))\right]\left(\frac{\partial\mu_f(\mu,\bar{r})}{\partial\bar{r}}\frac{\partial\bar{r}(\mu,\phi)}{\partial\phi}\right) \leq 0$$

since it is easy to check that $\frac{\partial\mu_f(\mu,\bar{r})}{\partial\bar{r}} > 0$ and that $\frac{\partial\bar{r}(\mu,\phi)}{\partial\phi} > 0$. □

The following proposition characterizes the allocations at the securitization stage in the model with rating shopping. As we show bellow, the condition for an equilibrium in the securitizatin stage to exist and be unique is that $\frac{\partial}{\partial\mu}\left(\frac{\tilde{\alpha}'_g(\mu,\phi)}{\tilde{\alpha}'_b(\mu,\phi)}\right) < 0$, for all $\mu \in (0,1)$.

Proposition 7. *If* $\frac{\partial}{\partial\mu}\left(\frac{\tilde{\alpha}'_g(\mu,\phi)}{\tilde{\alpha}'_b(\mu,\phi)}\right) < 0$*, for all* $\mu \in (0,1)$*, and ratings are* Γ*-informative:* $\frac{\tilde{\alpha}'_b(1,\phi)}{\tilde{\alpha}'_g(1,\phi)} > \frac{v_g-\delta v_b-\phi}{(1-\delta)v_g-\phi}$*, the unique equilibrium at the securitization stage features some degree of pooling. In particular, there exists* $\{\mu^*, x^*\}$ *with* $u_b(\mu^*, x^*) = v_b$ *such that:*

- *For* $\mu_0 < \mu^*$*, there is partial pooling at* $x(\mu_0) = x^* < x^{LCSE}$

- *For $\mu_0 > \mu^*$, there is full-pooling at $x(\mu_0) = 0$*

If ratings are not Γ-informative, the unique equilibrium is the LCSE of Proposition 1.

Proof. Let $\{\mu(k), x(k)\}$ be given by the solution of the adjusted $\mathcal{M}(k)$ problem:

$$\begin{aligned} \max_{\mu,x} \quad & u_g(\mu, x, \phi) \\ s.t. \quad & u_b(\mu, x, \phi) = k \end{aligned} \tag{43}$$

where $\phi = 0$ indexes the star problem for the baseline ratings model of Section 4, and $\phi > 0$ the extension ratings shopping model of Section 6. When $\phi > 0$, the problem is:

$$\begin{aligned} \max_{\mu,x} \quad & (1-x)\left[(\tilde{\alpha}_g(\mu,\phi) - \phi)(v_g - v_b) + v_b\right] + \delta x v_g \\ s.t. \quad & (1-x)\left[(\tilde{\alpha}_b(\mu,\phi) - \phi)(v_g - v_b) + v_b\right] + \delta x v_b = k \end{aligned} \tag{44}$$

In an abuse of notation, let $\tilde{\alpha}'(\mu,\phi)$ denote $\frac{\partial \tilde{\alpha}(\mu,\phi)}{\partial \mu}$. From the first-order conditions of the $\mathcal{M}(k)$ problem, we obtain:

$$\frac{\tilde{\alpha}'_g(\mu^*,\phi)}{\tilde{\alpha}'_b(\mu^*,\phi)} = \frac{(\tilde{\alpha}_g(\mu^*,\phi) - \delta)(v_g - v_b) + (1-\delta)v_b - \phi}{\tilde{\alpha}_b(\mu^*,\phi)(v_g - v_b) + (1-\delta)v_b - \phi} \tag{45}$$

$$x^* = \frac{(\tilde{\alpha}_b(\mu^*,\phi) - \phi)(v_g - v_b) + (1-\delta)v_b - (k - \delta v_b)}{(\tilde{\alpha}_b(\mu^*,\phi) - \phi)(v_g - v_b) + (1-\delta)v_b} \tag{46}$$

Let $\tilde{\alpha}(\mu,\phi) \equiv \tilde{\alpha}_g(\mu,\phi) - \tilde{\alpha}_b(\mu,\phi)$. After some algebra, condition (45) becomes:

$$(\tilde{\alpha}(\mu^*,\phi) - \delta)\frac{\tilde{\alpha}'_b(\mu^*,\phi)}{\tilde{\alpha}'(\mu^*,\phi)} - \tilde{\alpha}_b(\mu^*,\phi) = \frac{(1-\delta)v_b - \phi}{v_g - v_b} \tag{47}$$

The RHS is constant and independent of μ. As for the LHS, note that:

$$\frac{\partial LHS(\mu,\phi)}{\partial \mu} = -\frac{\tilde{\alpha}(\mu,\phi) - \delta}{\tilde{\alpha}(\mu,\phi)^2}\frac{\partial}{\partial \mu}\left(\frac{\tilde{\alpha}'_g(\mu,\phi)}{\tilde{\alpha}'_b(\mu,\phi)}\right)$$

As shown in Proposition 2, if $\frac{\partial}{\partial \mu}\left(\frac{\tilde{\alpha}'_g(\mu,\phi)}{\tilde{\alpha}'_b(\mu,\phi)}\right) < 0$, then there is a unique μ^* that solves the maximization problem.

In addition, the solution to the above problem is interior with $\mu^* < 1$ (and thus $x < x^{LC}$) if and only if condition (47) hold for an interior μ. Since the LHS of the condition is positive at $\mu = 0$ and decreasing, and the RHS of the condition is negative at $\mu = 0$ and increasing, it follows that an intersection between the RHS and the LHS does exists iff there exists $\bar{\mu}$ such that:

$$\begin{aligned}\frac{\tilde{\alpha}'_g(1,\phi)}{\tilde{\alpha}'_b(1,\phi)} &< \frac{(\tilde{\alpha}_g(1,\phi)-\delta)(v_g-v_b)+(1-\delta)v_b-\phi}{\tilde{\alpha}_b(1,\phi)(v_g-v_b)+(1-\delta)v_b-\phi} \\ &< \frac{(1-\delta)(v_g-v_b)+(1-\delta)v_b-\phi}{v_g-v_b+(1-\delta)v_b-\phi} = \frac{(1-\delta)v_g-\phi}{v_g-\delta v_b-\phi}\end{aligned}$$

This is an adjusted Γ-informativeness condition, which is now a function of ϕ, since the informativeness of ratings is a function of the cost of being rated.

It remains to verify the second order conditions of this problem. We verify that the determinant of the Bordered Hessian is negative at our interior critical point, where λ denotes the multiplier on the constraint of the problem:

$$BH = \begin{bmatrix} 0 & \frac{\partial u_b(\mu,x,\phi)}{\partial x} & \frac{\partial u_b(\mu,x,\phi)}{\partial \mu} \\ \frac{\partial u_b(\mu,x,\phi)}{\partial x} & L_{xx} & L_{x\mu} \\ \frac{\partial u_b(\mu,x,\phi)}{\partial \mu} & L_{\mu x} & L_{\mu\mu} \end{bmatrix}$$

where $L(x,\mu,\phi) = u_g(\mu,x,\phi) - \lambda\left(u_b(\mu,x,\phi)-k\right)$ and λ is the Lagrange multiplier.

$$\begin{aligned} L_{xx} &= 0 \\ L_{\mu\mu} &= \left(\tilde{\alpha}''_g(\mu^*,\phi) - \lambda^*\tilde{\alpha}''_b(\mu^*,\phi)\right)(1-x^*)(v_g-v_b) \\ L_{x\mu} &= L_{\mu x} = -(v_g-v_b)(\tilde{\alpha}_g(\mu^*,\phi)-\lambda^*\tilde{\alpha}_b(\mu^*,\phi)) = 0 \end{aligned}$$

A sufficient condition for our solution to be a local maximum is that the bordered Hessian is negative definite. That is, $|BH_1| < 0$ and $|BH_2| > 0$. It is easy to see that $|BH_1| = -(\frac{\partial u_b(\mu,x,\phi)}{\partial x})^2 < 0$ and that $|BH_2| = -\left(\frac{\partial u_b(\mu,x,\phi)}{\partial x}\right)^2 L_{\mu\mu} > 0$. Thus, SOC are satisfied if $L_{\mu\mu} < 0$, that is, if $\frac{\partial}{\partial\mu}\left(\frac{\tilde{\alpha}'_g(\mu,\phi)}{\tilde{\alpha}'_b(\mu,\phi)}\right)|_{\mu=\mu^*} < 0$.

Finally, the proof that this characterizes the solution of unique D1 PBE of the securitization stage is as the one in Proposition 2. □

Let $u_t(\mu,\phi)$ denote the outcomes of the securitization stage, as described in Proposition 7. Then, the decision to originate a given loan pool is as in the baseline model, and determined by $\Psi(\mu,\phi) = \frac{1-u_b(\mu,\phi)}{u_g(\mu,\phi)-u_b(\mu,\phi)}$. An equilibrium $\{x^*, \mu_0, x(\mu_0)\}$ of the model with rating shopping exists if there exists x^* such that $A(x^*) = \mu_0$ and $p(x^*) = \Psi(\mu_0)$.

B Security Design

Thus far, we have studied how much banks will retain taking the "class" of securities, $F = (1-x)Y$, as given. In this section, we demonstrate that the main results of the paper remain unchanged when banks can choose the design of the security. Our demonstration relies heavily on Daley et al. (2016b) (henceforth, DGV16) in which, we study the optimal security design in the presence of public information (e.g., ratings).

B.1 Summary

In DGV16, we characterize the equilibrium of the securitization stage where the securitizer can choose any *security*, $F = \psi(Y)$, to offer for sale. Specifically, for any realization of the cash flow y, $\psi(y)$ is the amount paid to the purchaser of the security and $y - \psi(y)$ is the amount retained by the securitizer, where $0 \leq \psi(y) \leq y$ for all y. As is standard in the security design literature, we focus on securities for which both the amount paid and the amount retained must be nondecreasing in y. We retain the assumption that each pool of loans is either good or bad (i.e., $t \in \{b, g\}$). Further, a type-t pool delivers a cash flow distributed according to the cdf and pdf denoted by Π_t and π_t respectively on a common support $[0, \bar{y}]$, where $\frac{\pi_g(y)}{\pi_b(y)}$ is weakly increasing (i.e., MLRP holds). We refer to this setting as the *Security Design Game.*

We show that the form of the security that emerges in the equilibrium of the Security Design Game depends on the measure of rating informativeness (denoted RI) and the cost of retention, δ.[16] In particular, if $RI < \delta$, then the securitizer issues debt and retains a levered-equity claim, while if $RI > \delta$, then the securitizer issues a levered-equity claim and retains debt (see DGV16, Theorem 1).[17] We also show that a result analogous to Proposition 2 holds in the Security Design Game. That is, when ratings are informative enough the unique equilibrium involves some degree of pooling (either partial or full) whereas when ratings are not sufficiently informative the unique equilibrium is separating (see DGV16, Theorem 2).

Below, we characterize the equilibrium payoffs of the security design game as a function of the prior belief, μ_0, about the type of the pool. Denote these payoffs by $u_\theta^{SDG}(\mu_0)$. Importantly, these payoff functions share similar characteristics to the ones derived earlier, where the bank is restricted to issuing (and retaining) equity.

Fact 3. *There exists a unique equilibrium of the security design game. Moreover, the following statements are generically true.*

(i) $u_\theta^{SDG}(\mu_0)$ *is continuous and weakly increasing in* μ_0.

(ii) There exists a $\mu_1 \in (0, 1]$ *such that for all* $\mu_0 \leq \mu_1$, $u_g^{SDG}(\mu_0) \in (u_g^*(\mu_0), v_g)$ *and* $u_b^{SDG}(\mu_0) = v_b$. .

Proof. See DGV16. □

B.2 Security Design: Lending Standards and Credit Supply

Let us now turn to the implications for lending standards and the supply of credit when the securitization stage is replaced by the security design game. Analogous to (13), the origination

[16] The measure is defined as $RI \equiv \max_\mu \alpha_g(\mu) - \alpha_b(\mu)$.
[17] If $RI = \delta$, the security designed in equilibrium is not pinned down.

threshold is given by

$$p(q^{SDG}) = \frac{1 - u_b^{SDG}(\mu_0)}{u_g^{SDG}(\mu_0) - u_b^{SDG}(\mu_0)} \equiv \Psi^{SDG}(\mu_0). \tag{48}$$

From Fact 3, we know that $\Psi^{SDG}(\mu_0)$ is decreasing and continuous in the relevant range. Hence, there is a unique solution to (48) and a unique level of credit supply that is consistent with an equilibrium. We also know that $\Psi^{SDG}(\mu_0) \in \left(\frac{1-v_b}{v_g-v_b}, \Psi^*(\mu_0)\right)$ for $\mu_0 \leq \mu^*$. In this region, a bank with a bad pool of loans gets the full-information value, while the bank with a good pool of loans does strictly better by being able to choose the security design, which eases banks lending standards.

Provided the rating is Γ-informative, one can also show that Ψ^{SDG} lies weakly above Ψ^* for all priors above a threshold (and strictly above for at least some priors).[18] In the region where the inequality is strict, banks use retention to signal quality when they can design the security, but rely purely on ratings when they are restricted to equity. These properties are summarized in Figure 7 and formally stated in Lemma 5.

Lemma 5. *If the rating is Γ-informative, then the following statement is generically true.*[19]

(i) $\Psi^{SDG}(\mu_0) \in \left(\frac{1-v_b}{v_g-v_b}, \Psi^*(\mu_0)\right)$ *for all* $\mu_0 < \mu^*$.

(ii) $\Psi^{SDG}(\mu_0) \geq \Psi^*(\mu_0)$ *for all* $\mu_0 \geq \mu^*$, *where the inequality holds strictly for at least some* μ_0.

If the rating is not Γ-informative then $\Psi^{SDG} < \Psi^*$ *for all* μ_0.

The next proposition summarizes the implications of security design on the supply of credit and productive efficiency.

Proposition 8. *If each bank can optimally design the security that they issue (i.e., if the securitization stage is replaced with the security design game) then:*

[18] Under some conditions (e.g., if the rating is not β-informative at $\bar{y}$), Ψ^{SDG} lies strictly above Ψ^* for *all* priors above a threshold.

[19] In the non-generic case that $\max_\mu \alpha(\mu) = \delta$, the security design game has the exact same payoffs as the securitization stage and hence $\Psi^{SDG} = \Psi$.

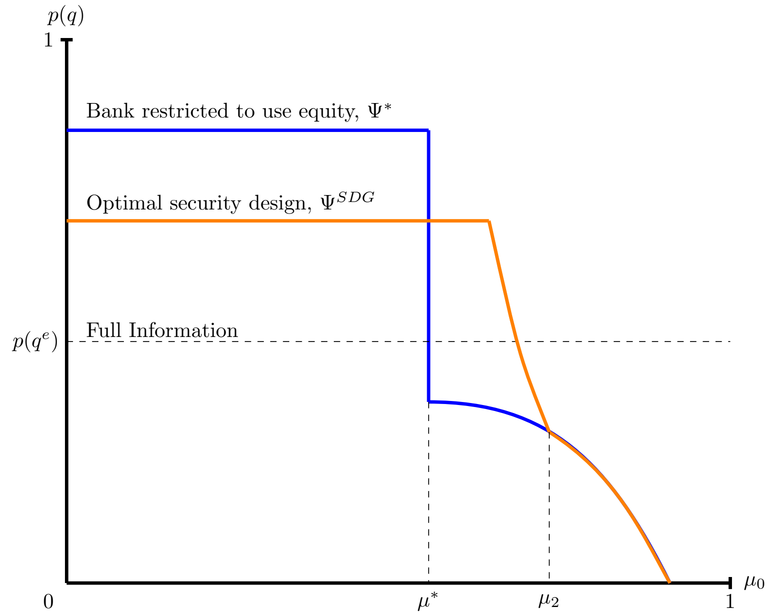

Figure 7: Security Design and Lending Standards.

- *If* $p(A^{-1}(\mu^*)) > \Psi^{SDG}(\mu^*)$*: credit supply increases, which improves productive efficiency, though an undersupply of credit persists.*
- *If* $p(A^{-1}(\mu^*)) < \Psi^{SDG}(\mu^*)$*: credit supply decreases, which may improve or reduce productive efficiency.*
- *If* $p(A^{-1}(\tilde{\mu})) < \Psi^{SDG}(\tilde{\mu})$*, where* $\tilde{\mu}$ *is defined implicitly by* $\Psi^{SDG}(\tilde{\mu}) = p(q^e)$*: credit supply decreases, which improves productive efficiency, though oversupply of credit persists.*

In essence, Proposition 8 says that the ability to design the security improves productive efficiency when the distribution of pools is sufficiently strong or weak. For intermediate distributions, optimal security design results in tighter lending standards, which may improve or hinder productive efficiency depending on the distribution of loan pools.

Proof of Lemma 5. The proof of the case in which the rating is not Γ-informative is trivial. Regardless of the prior, in any equilibrium of the security design game, the b-payoff is weakly greater than v_b and the g-payoff is strictly greater than u_g^{LC}. Noting the Ψ is decreasing in both u_b and u_g (whenever it is non-negative) yields the result.

When the rating is Γ-informative. The statement in (i) follows immediately from Fact 3. To prove (ii), it will be useful to break the proof into three cases.

Case 1: Rating is not β-informative at $\bar{y}$. In this case, the equilibrium of the security design game does not converge to full-pooling with zero retention as $\mu_0 \to 1$. In particular, g retains a non-trivial levered equity claim and the low-type either pools are full separates with zero retention. In either case, $u_t^{SDG}(\mu_0) < u_t^*(\mu_0)$ for all $\mu_0 \geq \mu^*$, which implies the result.

Case 2: Rating is β-informative at $\bar{y}$*, but not α-informative.* In this case, there is full-pooling with zero retention for μ_0 large enough. Let μ_2 denote the smallest prior belief at which the zero-retention, full-pooling outcome obtains in the SDG. It suffices to show that $\mu_2 > \mu^*$. The FOC characterizing μ_2 is

$$g(\mu_2) = \frac{\pi_g(\bar{y})}{\pi_b(\bar{y})} - 1, \tag{49}$$

where $g(\mu) \equiv \frac{1-\delta}{(\alpha(\mu)-\delta)\frac{\alpha_b'(\mu)}{\alpha'(\mu)} - \alpha_b(\mu)}$. The FOC characterizing μ^* (see (40)) is $g(\mu^*) = \frac{v_g}{v_b} - 1$. Note that $g(\mu) \leq 0$ for all $\mu < \hat{\mu} \equiv \arg\max_\mu \alpha(\mu)$, whereas the RHS of the FOC is strictly positive in both cases. Hence, it must be that both μ_2 and μ^* are above $\hat{\mu}$. Further, g is positive, strictly increasing, continuous, and tends to $+\infty$ on the interval $(\hat{\mu}, \bar{m})$ and g is negative above $\bar{m}$, where $\bar{m}$ is such that the denominator of g is zero. Finally, by MLRP $\frac{\pi_g(\bar{y})}{\pi_b(\bar{y})} > \frac{v_g}{v_b}$. Therefore $g(\mu_2) > g(\mu^*)$ and thus $\mu_2 > \mu^*$.

Case 3: Rating is α-informative. The proof for this case is similar to case 2. Again, there is full-pooling with zero retention for μ_0 large enough in the SDG. Let μ_2 denote the smallest prior belief at which the zero-retention, full-pooling outcome obtains in the SDG. We will show that $\mu_2 > \mu^*$. The FOC characterizing μ_2 in this case is

$$g(\mu_2) = \frac{1 - \Pi_g(\underline{y})}{1 - \Pi_b(\underline{y})} - 1 = 0 \tag{50}$$

where $g(\mu)$ is defined in Case 2. When the rating is α-informative, g is strictly decreasing over the relevant domain and equal to zero only at $\hat{\mu}$.[20] Therefore, $\mu_2 = \hat{\mu}$, whereas $g(\mu^*) = \frac{v_g}{v_b} - 1 >$

[20]The relevant domain includes all $\mu \in (\underline{\mu}, \bar{\mu})$, where any solution to either FOC must lie.

$0 \implies \mu^* < \hat{\mu}$, which implies the desired result. □

Proof of Proposition 8. Follows immediately from Lemma 5. □