

Trade Policies, Firm Heterogeneity, and Variable Markups

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Abstract

We study unilateral trade liberalization in a model of monopolistic competition with heterogeneous firms, endogenous wages, and non-separable and non-homothetic quadratic preferences that generate variable markups. We show that the optimal rate of the revenue-generating import tariff is strictly positive so that protection is always desirable, whether the liberalizing economy is large or small. Yet, if per-unit trade costs are the only policy instrument available, free trade is optimal. Finally, we show that for both policy instruments, variable markups result in negative pro-competitive effects, reducing gains from trade.

1 Introduction

Recently, there has been a surge in international trade models with imperfect competition and heterogeneous firms. In addition to the expansion in product variety studied by Krugman (1980), these models offer new channels, through which trade affects welfare. A number of papers following on the heels of the seminal work of Melitz (2003) highlight the mechanism of self-selection of more efficient firms into exporting. In the presence of such a mechanism, trade liberalization leads to reallocation of resources towards more efficient firms, improving average productivity in liberalizing countries and potentially raising welfare.¹ Most of these papers rely on the assumptions

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¹One has to be careful while interpreting productivity growth as a welfare-improving change. See, for example, Demidova and Rodríguez-Clare (2009), who show that an increase in average productivity (as a result of an export subsidy) does not necessarily mean an increase in welfare. In fact, welfare falls in their model.

of monopolistic competition and constant elasticity of substitution (CES) preferences as in Dixit and Stiglitz (1977), which imply constant markups charged by firms. While being extremely convenient from the analytical point of view, constant markups are at odds with the empirical evidence. Moreover, models with constant markups ignore so called pro-competitive gains from trade that arise due to the presence of variable markups.² This important channel has become the focus in much of the recent literature, which allows for variable markups by deviating from the assumption of monopolistic competition and/or incorporating non-CES preferences. Some examples of settings with variable markups include, among others, the model of monopolistic competition in Melitz and Ottaviano (2008), the Cournot competition model in Atkeson and Burstein (2008) and Edmond, Midrigan and Xu (2015), and the Bertrand competition setting in Bernard, Eaton, Jensen and Kortum (2003), de Blas and Russ (2015) and Holmes, Hsu and Lee (2014). The pro-competitive mechanism of trade in these models is twofold. First, at the firm level, trade liberalization intensifies foreign competition, reducing market power of local producers and forcing them to decrease their markups. This claim is supported by existing empirical literature, with the notable exception of De Loecker, Goldberg, Khandelwal and Pavcnik (forthcoming). Second, at the industry level, trade liberalization has the ability to affect the markup distribution, reducing its dispersion. As shown, for example, by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), lower markup dispersion, in turn, increases total factor productivity. The reason is that a higher markup dispersion is associated with more extensive distortion that arises due to variability of revenue productivity (the product of physical productivity and a firm's output price) across firms. By reducing this misallocation distortion, trade liberalization can potentially raise welfare.³

On the other hand, Edmond, Midrigan and Xu (2015) and Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2015) point out the possibility of negative pro-competitive effects of trade liberalization. The reason is that, while trade liberalization leads to labor reallocation towards more productive firms, i.e., exporters, these firms could internalize the drop in trade costs and charge higher markups. As a result, whether trade liberalization leads to welfare gains or losses depends on the joint movement of labor reallocation and markup distribution.

²For empirical evidence on variation in markups across firms see Chen, Imbs and Scott (2009), Feenstra and Weinstein (forthcoming), De Loecker and Warzynski (2012), De Loecker, Goldberg, Khandelwal and Pavcnik (forthcoming) and Hottman, Redding and Weinstein (forthcoming). The importance of pro-competitive gains is emphasized, for example, in Edmond, Midrigan and Xu (2015), who show that the size of these gains is especially large in the presence of significant misallocations and weak cross-country comparative advantage in individual sectors.

³See Restuccia and Rogerson (2013) for an excellent survey of the literature on misallocations and productivity.

Given new insights from the recent trade literature on variable markups, what can we say about their policy implications? The main goal of this paper is to partially fill the gap in the literature and provide tractable analytical results for the optimal trade policy in the two-country model of monopolistic competition with firm heterogeneity and variable markups. Our basic environment is a modified version of Melitz and Ottaviano (2008), a well-known extension of Melitz (2003) that incorporates endogenous markups by using the linear demand system with horizontal product differentiation developed by Ottaviano, Tabuchi and Thisse (2002). As pointed out by Nocco, Ottaviano and Salto (2014), who study the closed economy case of this model, endogenous markups create an additional within-sector misallocation: more productive firms do not pass on their entire cost advantage to consumers by absorbing part of it in the markup and end up selling too small quantities compared to the optimal levels. The opposite happens with high cost producers, whose varieties end up being oversupplied. Hence, variable markups in the presence of firm heterogeneity result in misallocation distortion. A similar point is made by Dhingra and Morrow (2012), who study allocation efficiency in the case of separable preferences with a variable elasticity of substitution. The natural question is what the effects of unilateral trade liberalization are in the presence of such distortion.

We modify Melitz and Ottaviano (2008) along two lines. First, we drop the assumption of the linear outside good so that wages in our setting will be determined endogenously. Hence, we allow for the income effect that is muted in Melitz and Ottaviano (2008). Second, in addition to per-unit trade costs considered by Melitz and Ottaviano (2008), we incorporate ad valorem import tariffs that generate revenue. Our key finding is that the choice of the trade instrument used by policymakers, i.e., per-unit trade costs or import tariffs, is crucial for welfare outcomes of unilateral trade liberalization. In particular, with respect to import tariffs, the welfare-maximizing policy from a unilateral perspective of the Home country is a strictly positive import tariff, whereas full trade liberalization is optimal in the case of iceberg-type trade costs. These results hold true whether the Home economy is large relative to the Foreign one or is modeled as a small one similar to Demidova and Rodríguez-Clare (2009, 2013), who study CES preferences.

Comparing our results with Melitz and Ottaviano (2008) immediately shows that the two settings generate the opposite predictions with regard to reductions in trade costs. The reason is that Melitz and Ottaviano (2008) rely on the outside good assumption, i.e., in addition to the Melitz (2003) type sector, they incorporate another one, usually called an “outside good” sector, that produces a freely traded homogenous good under perfect competition. The use of the outside good

assumption is very popular in many extensions of the Melitz (2003) model as well as in other works in the trade literature. One well-known advantage of this assumption is significant simplification, since the outside good pins down wages across all countries exogenously. Its second advantage is the opportunity to study cross-sectoral inefficiencies. Yet, such an assumption comes with a price. Not only does it exclude an income effect, but also it adds an extra distortion to the model, since there is no markup in the outside good sector, whereas in the Melitz (2003) type sector producers charge prices above their marginal costs. As pointed out by Bhagwati (1971), the presence of distortions can result in the breakdown of Pareto-optimality of laissez-faire. Hence, it is not surprising that the use of an outside good in Melitz and Ottaviano (2008) distorts the impact of changes in trade costs, making unilateral trade liberalization welfare-reducing.

Furthermore, to gain better understanding of the differential roles of tariffs and iceberg-type trade costs, let us look at the welfare effect of small changes in these instruments in the same spirit as done in Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2015) (hereafter ACDR). As shown in Appendix A, the welfare change can be described by the following equation:

$$d \ln W = - (1 - \eta) \frac{d \ln \lambda}{\theta} + \left(1 - \frac{2\rho}{1 - \beta + \theta} \right) d \ln \mu, \quad (1)$$

where λ is the share of domestic expenditure on domestic goods, θ is the shape parameter of the Pareto cost distribution, $\eta = \rho \frac{1-\beta}{1-\beta+\theta}$ is a non-negative structural parameter that depends, among other things, on the elasticity of markups with respect to firm productivity, ρ , μ is the ratio of total expenditures and labor income, and $\beta = -1$.

In the absence of import tariffs $\mu = 1$. Hence, the formula above collapses to the same expression as in ACDR, $d \ln W = -(1 - \eta) d \ln \lambda / \theta$, which implies that a small reduction in trade costs, which, in turn, reduces the expenditure share λ on locally produced goods, generates welfare gains. However, the use of import tariffs implies that changes in $\mu > 1$ have to be taken into account while calculating welfare gains/losses. Then protection becomes beneficial. To see this, let us look at two extreme cases. First, consider autarky that can be interpreted as the case of prohibitively high import tariffs. A marginal reduction in these tariffs would reduce λ , but revenues generated by new tariffs would be close to zero, meaning that changes in μ can be ignored. Hence, moving away from autarky is welfare-improving. On the other hand, if we start with the free trade equilibrium, where the market shares of Foreign exporters are high enough, an introduction of a small import tariff would generate tariff revenues, which together with the terms of trade externality can more than compensate for a fall in welfare due to a rise in λ . Thus, deviations from free trade are beneficial

as well. Therefore, when it comes to import tariffs, protection becomes a desirable policy.

Note that the expression above holds for the analysis of the marginal changes in tariffs only and cannot be used for calculations of the optimal tariff rate, which is what the analysis in our paper achieves. Hence, our paper is complimentary to ACDR, who consider a large class of demand functions that generate variable markups in the multiple country setting. The main difference between their work and our setting is that we consider the non-separable quadratic utility function as well as the case of revenue-generating import tariffs. Moreover, given the complexity in the case of large shifts in trade costs, to evaluate welfare gains from trade liberalization, ACDR resort to simulations. The advantage of our paper is that we can study these gains analytically.

Also, our results seem to be in line with Bagwell and Lee (2015), who use the Melitz and Ottaviano (2008) model with the outside good to study the impact of import tariffs and export subsidies in the case of two symmetric countries. Unlike Melitz and Ottaviano (2008), they consider revenue-generating import tariffs and, amongst other results, show that a marginal increase in the tariff imposed by the Home country raises its welfare at the cost of its trading partner. We see this paper as highly complimentary to ours with the main difference, other than asymmetric countries and non-marginal changes in the tariff levels in our model, being that Bagwell and Lee (2015) maintain the assumption of the outside good. It turns out that this difference is important: although it seems that welfare results in their model resemble ours, the mechanisms behind these results are quite different. For instance, their model generates a *Metzler Paradox*: as Home increases its import tariff, its average price increases, while that abroad falls. As shown in Appendix D, this is not the case in our model, where both Home and Foreign average prices rise together, proving again that the popular outside good assumption is not innocuous, and one has to be careful when interpreting the results obtained with this assumption.

Finally, let us discuss the relationship between our work and the trade literature on models with constant markups. Demidova and Rodríguez-Clare (2013) use simple Figure 1 that summarizes the key relationships in the Melitz (2003) model to show that in the absence of the outside good, unilateral trade liberalization by Home is welfare-increasing for both the Home and Foreign economies. In Section 3 of this paper, we show that the analysis of changes in trade costs in our setting with variable markups can be conducted with a help of the similar figure. Next, the comparison of the results for trade costs and import tariffs (see Felbermayr and Jung, 2012, Demidova and Rodríguez-Clare, 2009, 2013, and Felbermayr, Jung and Larch, 2013) shows that, as in our setting with variable markups, the type of trade barriers matters: while full liberalization is opti-

mal in the case of per unit trade costs, in the case of import tariffs protection becomes an optimal policy for the Home government, whether the Home economy is modelled as a small or large one. Moreover, our analysis of formula (1) resembles the one for the Melitz (2003) model with the Pareto productivity distribution with the shape parameter θ , where $d \ln W = -d \ln \lambda / \theta + (1 + 1/\theta) d \ln \mu$ (for the formula for small changes in trade costs and import tariffs see Arkolakis, Costinot and Rodríguez-Clare, 2012, and Felbermayr, Jung and Larch, 2015, respectively).

Given qualitative similarity of our policy results to the literature mentioned above, what role then do variable markups play, if any? In the last part of the paper we show that whether we consider a fall in trade costs or in import tariffs, variable markups result in a negative pro-competitive effect due to the misallocation distortion they create. In the former case, a small fall in the trade costs of Foreign exporters causes a reallocation of Home labor to reallocate towards goods that are oversupplied, i.e., those that have a low markup, which, as pointed out by ACDR, worsens the misallocation distortion and leads to smaller welfare gains. In the case of import tariffs, we show that if the level of protection is low enough to begin with, further reductions raise the average markup faced by consumers at Home, which, as discussed by Edmond, Midrigan and Xu (2015), implies smaller welfare gains as well. Thus, variable markups reduce potential gains from trade.

The rest of this paper proceeds as follows. Section 2 describes the underlying environment. Section 3 solves for the optimal values of trade costs and import tariffs. The role of variable markups is studied in Section 4. Section 5 offers concluding remarks.

2 The Model

In this Section we first modify the Melitz and Ottaviano (2008) model by dropping the outside good assumption and derive the equilibrium conditions for the case of two large economies. Further details, derivations, and rationale for the main assumptions in the case of the small Home economy can be found in the online Appendix.

2.1 Demand

There are two countries, Home and Foreign, of size L_i , $i = H, F$. Wage in the Foreign country, w_F , is normalized to unity. By dropping the outside good from the Melitz and Ottaviano (2008) model, we get the following household from country i 's maximization problem with a non-separable quadratic utility function ($\alpha > 0$ can be normalized to 1, but we keep it as is for ease of comparison

with Melitz and Ottaviano, 2008):⁴

$$U_i = \alpha \int_{\Omega_i} q^c(\omega) d\omega - \frac{1}{2}\gamma \int_{\Omega_i} (q^c(\omega))^2 d\omega - \frac{1}{2}\eta \left(\int_{\Omega_i} q^c(\omega) d\omega \right)^2 \quad s.t. \quad \int_{\Omega_i} p(\omega) q^c(\omega) d\omega = w_i,$$

where Ω_i is the set of all available differentiated good varieties in country i , w_i is an income, and $q^c(\omega)$ is the quantity consumed of variety $\omega \in \Omega_i$. $\eta > 0$ is the degree of nonseparability across varieties. The degree of product differentiation across varieties is characterized by positive γ : with lower γ varieties become closer substitutes, and in the limit case of $\gamma = 0$, households care only about the total amount they consume. From the F.O.C., we get

$$\lambda_i p(\omega) = \alpha - \gamma q^c(\omega) - \eta \int_{\Omega_i} q^c(\omega) d\omega, \quad (2)$$

where λ_i is a Lagrangian multiplier. Denote the aggregate quantity of all varieties consumed by an individual from country i , $\int_{\Omega_i} q^c(\omega) d\omega$, by Q_i . Then

$$\lambda_i = \left(\alpha Q_i - \gamma \int_{\Omega_i} (q^c(\omega))^2 d\omega - \eta (Q_i)^2 \right) / w_i. \quad (3)$$

Using the same logic as in Melitz and Ottaviano (2008), it can be shown that the set Ω_i^c of all consumed varieties ($q^c(\omega) > 0$) is the largest subset of Ω_i that satisfies:

$$p(\omega) \leq \frac{1}{\eta M_i + \gamma} \left(\frac{1}{\lambda_i} \gamma \alpha + \eta M_i \bar{p}_i \right) \equiv p_i^{\max},$$

where M_i is the measure Ω_i^c , $\bar{p}_i = (1/M_i) \int_{\Omega_i^c} p(\omega) d\omega$ is the average price, and p_i^{\max} represents the choke price in country i .

2.2 Production and Firm Behavior

Consider a firm from country i that sells its variety ω to L_j consumers in country j . Assume that its marginal cost is $MC_{ij}(\omega)$. Hence, it sells $q_{ij}(\omega) = L_j q_j^c(\omega)$ and maximizes its profit, $\pi_{ij}(\omega) = p_{ij}(\omega) q_{ij}(\omega) - MC_{ij}(\omega) q_{ij}(\omega)$, by choosing the appropriate level of $p_{ij}(\omega)$. From (2),

$$q_{ij}^c(\omega) = \frac{1}{\gamma} (\alpha - \lambda_j p_{ij}(\omega) - \eta Q_j),$$

so that the F.O.C. results in $q_{ij}(\omega) = L_j \lambda_j (p_{ij}(\omega) - MC_{ij}(\omega)) / \gamma$. There is a continuum of domestic firms in country i that derive their unit labor costs from the Pareto cost distribution given by $G_i(c) = (c/c_i^M)^\theta$, $c \in [0, c_i^M]$, $\theta > 1$. In addition, while exporting, each firm faces an iceberg transportation cost τ_{ij} , where $\tau_{ij} > 1$ and $\tau_{ii} = 1$ for $i, j \in \{H, F\}$, as well as an ad-valorem

⁴In Melitz and Ottaviano (2008), q_0^c , a consumption of the outside good, is added to the right-hand side of U_i .

import tariff. To incorporate this into the model, assume that if a firm from country i charges price p_{ij} in market j , then the country j ' government collects tariff revenues of $(t_{ij} - 1) p_{ij}/t_{ij}$ per unit sold, so that this firm receives only p_{ij}/t_{ij} . We assume that only Foreign firms face an import tariff, i.e., the Foreign government is passive, and there are no taxes on local sales. Thus,

$$t_{HH} = t_{FF} = t_{HF} = 1 \quad \text{and} \quad t_{FH} \equiv t \geq 1.$$

Only firms from country i with a positive demand will sell in country j . Define their cost cutoff c_{ij}^* as a solution of $q_{ij}(c_{ij}^*) = 0$ or, given wage w_i , $p_{ij}(c_{ij}^*) = w_i t_{ij} \tau_{ij} c_{ij}^*$, so that in country i only firms with $c \leq c_{ij}^*$ sell in country j . Using (2), we have the following expression for the cost cutoff for local sellers in country i , c_{ii}^* :

$$w_i \lambda_i c_{ii}^* = \alpha - \eta Q_i. \quad (4)$$

Given (4), $\lambda_j p_{ij}(c) = \alpha - \gamma q_{ij}^c(c) - \eta Q_j = w_j \lambda_j c_{jj}^* - \gamma q_{ij}(c) / L_j$ so that

$$\begin{aligned} p_{ij}(c) &= \frac{1}{2} w_i t_{ij} \tau_{ij} (c_{ij}^* + c); & q_{ij}(c) &= \frac{L_j}{2\gamma} w_i t_{ij} \tau_{ij} \lambda_j (c_{ij}^* - c); \\ r_{ij}(c) &= \frac{L_j}{4\gamma} t_{ij} (\tau_{ij} w_i)^2 \lambda_j \left((c_{ij}^*)^2 - c^2 \right); & \pi_{ij}(c) &= \frac{L_j}{4\gamma} t_{ij} (\tau_{ij} w_i)^2 \lambda_j (c_{ij}^* - c)^2; \end{aligned}$$

where the last two equations represent revenues and profits, respectively. Note that as in Melitz and Ottaviano (2008), a higher productivity (a lower cost) firm charges a lower price, makes higher sales, and earns higher profits. Moreover, more productive firms charge higher markups (a markup, $m_{ij}(c) \equiv p_{ij}(c) / MC_{ij}(c)$, falls with c). This gives a rise to the misallocation distortion, since more productive firms end up selling too little, while high cost producers tend to oversupply.

Finally, it can be shown directly that the relationship between the cost cutoffs for local sellers in country i and exporters from abroad is (recall that $w_F = 1$):

$$c_{ij}^* = \frac{w_j c_{jj}^*}{w_i t_{ij} \tau_{ij}} \quad \text{or} \quad c_{HF}^* = \frac{c_{FF}^*}{w_H \tau_{HF}} \quad \text{and} \quad c_{FH}^* = \frac{w_H c_{HH}^*}{t \tau_{FH}}. \quad (5)$$

2.3 Equilibrium Conditions

The free entry condition equalizes the expected profits from entering the market to the entry cost. Given the assumption of the Pareto cost distribution and (5), for firms in country i we have

$$(FE)_i: \quad (c_i^M)^{-\theta} w_i \left[L_i \lambda_i (c_{ii}^*)^{\theta+2} + \lambda_j L_j (w_j/w_i)^{-\theta-2} (t_{ij})^{-\theta-1} (\tau_{ij})^{-\theta} (c_{jj}^*)^{\theta+2} \right] = Const_1, \quad (6)$$

where $Const_1 \equiv 2\gamma f_e (\theta + 1) (\theta + 2)$. Next, let us look at the mass of active firms in country i , M_i . Due to free entry, total after-tariff revenues are equal to the labor payment:⁵

$$R_i = w_i L_i, \quad \text{where} \quad R_i = M_i (\bar{r}_{ii} + G_i (c_{ij}^*) \bar{r}_{ij} / G_i (c_{ii}^*)),$$

and \bar{r}_{ij} is the firm from country i ' expected revenues from sales in country j conditional on getting a cost draw below the corresponding cutoff,

$$\bar{r}_{ij} = \int_0^{c_{ij}^*} r_{ij}(c) dG_i(c) / G_i(c_{ij}^*) = \frac{1}{2\gamma} L_j (w_i)^2 \lambda_j t_{ij} (\tau_{ij} c_{ij}^*)^2 / (\theta + 2), \quad i = H, F. \quad (7)$$

From the (FE) _{i} condition, $G_i(c_{ii}^*) \bar{r}_{ii} + G_i(c_{ij}^*) \bar{r}_{ij} = w_i f_e (\theta + 1)$, so that $M_i = L_i (c_{ii}^* / c_i^M)^\theta / f_e (\theta + 1)$. Then the masses of entrants in country i can be calculated as

$$M_i^e = M_i / G_i(c_{ij}^*) = \frac{L_i}{f_e (\theta + 1)}. \quad (8)$$

Now let us take a closer look at the trade balance condition. It is given by $M_{ij} \bar{r}_{ij} = M_{ji} \bar{r}_{ji}$, where $M_{ij} = (G_i(c_{ij}^*) / G_i(c_{ii}^*)) M_i$ is the mass of exporters from country i to country j . Let

$$w \equiv w_H / w_F,$$

i.e., $w = w_H$, given $w_F = 1$. Then, by using (5) and (8), we get (recall that $t_{HF} = 1$, $t_{FH} = t$)

$$(TB) : \quad \lambda_F (\tau_{FH})^\theta (c_{FF}^*)^{\theta+2} (c_F^M)^\theta = \frac{w^{2\theta+2}}{t^{\theta+1}} \lambda_H (\tau_{HF})^\theta (c_{HH}^*)^{\theta+2} (c_H^M)^\theta. \quad (9)$$

Finally, we need to derive the equation for λ_i . As shown in Appendix B, it can be written as

$$\begin{aligned} (\lambda)_i & : \quad w_i \lambda_i c_{ii}^* = \left(\alpha - \eta \frac{\theta + 2}{\theta + 1} * \frac{1}{c_{ii}^*} \right) / \left(1 + \eta \frac{\theta + 2}{\theta + 1} y_i \right), \quad \text{where} \quad (10) \\ y_H & \equiv \frac{t - 1}{2\gamma (\theta + 2) t^{\theta+1}} \left(\frac{w c_{HH}^*}{\tau_{FH} c_H^M} \right)^\theta M_F^e \quad \text{and} \quad y_F = 0. \end{aligned}$$

Note that in the case of a zero import tariff for Foreign firms ($t_{FH} = 1$), $y_H = 0$. We can use (10) to exclude λ_H and λ_F from the set of equilibrium variables, so that in the equilibrium, we have 3 unknown variables, w , c_{ii}^* , and c_{jj}^* , and 3 conditions.⁶

⁵In addition to the labor payments, the national income at i , I_i , includes tariff revenues T_i : $w_i L_i + T_i = t R_{ji} + R_{ii}$, where R_{ij} are the total revenues of firms from country i earned from sales in country j . Given that $T_i = (t - 1) R_{ji}$ and that balanced trade implies $R_{ji} = R_{ij}$, we get $w_i L_i = R_{ii} + R_{ij}$, i.e., $R_i = w_i L_i$.

⁶Note that the (TB) condition below implies that we can solve it for the Home wage w , and then all the other variables in the equilibrium can be calculated as functions of w .

$$\begin{aligned}
(FE)_i &: \frac{\left(\alpha c_{ii}^* - \frac{\theta+2}{\theta+1}\eta\right)}{\left(1 + \eta \frac{\theta+2}{\theta+1} y_i\right)} (c_{ii}^*)^\theta \left[L_i (c_i^M)^{-\theta} + L_j (c_j^M)^{-\theta} (w_i/w_j)^\theta (\tau_{ji})^{-\theta} (t_{ij})^{-\theta-1} \right] = Const_1, \\
(TB) &: t^{\theta+1} L_H (c_H^M)^{-\theta} + L_F (c_F^M \tau_{FH})^{-\theta} w^\theta = w^{2\theta+1} \left(\frac{\tau_{HF} c_H^M}{\tau_{FH} c_F^M} \right)^\theta \left[L_F (c_F^M)^{-\theta} + L_H (c_H^M \tau_{HF})^{-\theta} w^{-\theta} \right].
\end{aligned}$$

2.4 Welfare

As shown in Appendix B, per capita welfare can be written as a function of Q_i :

$$U_i = \frac{\theta + 1}{2(\theta + 2)} Q_i \left[\frac{\alpha(2\theta + 3)}{\theta + 1} - \eta Q_i \right].$$

The behavior of U_i with respect to Q_i depends on whether $Q_i \leq \frac{\alpha(2\theta+3)}{\eta(2\theta+2)}$. The restriction from the model is that $\alpha - \eta Q_i > 0$ (so that prices in equilibrium are non-negative), which means that $Q_i < \alpha/\eta < \frac{\alpha(2\theta+3)}{\eta(2\theta+2)}$. Then rising Q_i increases per capita welfare in country i . Hence, in order to study changes in welfare, it is sufficient to look at the behavior of Q_i .

Note that in the absence of import tariffs ($t = 1$ so that $y_H = 0$), $w_i \lambda_i c_{ii}^* = \alpha - \frac{\eta}{c_{ii}^*} \frac{\theta+2}{\theta+1}$. Then $\eta Q_i = \alpha - w_i \lambda_i c_{ii}^*$ implies that welfare in both economies falls with the cost cutoff for local producers. In the case of $t > 1$, Q_F still depends negatively on c_{FF}^* . However, the relationship between Q_H and c_{HH}^* is no longer that simple due to the changes caused by $t > 1$ in (10) for λ_H ($y_H \neq 0$ anymore). This makes the welfare analysis of import tariffs more complicated.

3 Unilateral Trade Liberalization

There are two types of unilateral trade liberalization (UTL) that are usually studied in the literature, namely, a reduction in the import tariff and/or in per-unit trade costs for Foreign firms, i.e., a fall in t_{FH} and τ_{FH} in our setting, respectively. Let us consider an import tariff first.

3.1 Optimal Import Tariff

We relegate all the proofs to Appendix C (essentially, since $\eta Q_i = \alpha - w_i \lambda_i c_{ii}^*$, the proofs consist of the analysis of $w \lambda_H c_{HH}^*$ as a function of t_{FH}). Here we provide the main results only. First,

Proposition 1 *There exists a unique import tariff that maximizes welfare at Home. It can be found as a solution of the following equation:*

$$t^{opt} = 1 + \frac{1}{\theta} + \frac{1}{\theta} \frac{L_H}{L_F} \left(\frac{c_F^M}{c_H^M \tau_{HF}} \right)^\theta w^{-\theta}, \quad (11)$$

where w is the endogenously determined Home wage.

Not surprisingly, the value of the optimal import tariff in the case of two large economies depends on the terms of trade externality that is captured by the last term: it depends on the relative market size, L_H/L_F , the relative productivity, $(c_F^M/c_H^M)^\theta$, and openness of the Home economy, $(\tau_{HF})^{-\theta}$. Also, this result strongly resembles the one derived by Felbermayr, Jung and Larch (2013) for the case of constant markups, and as in their case, the analysis of the properties of t^{opt} is complicated by the endogenous nature of wage w .⁷

Proposition 1 above allows us to analyze the case of the small Home economy, i.e., the case when $L_H/L_F \rightarrow 0$. As in Demidova and Rodríguez-Clare (2009), this analysis involves 3 assumptions: (i) the Foreign demand for Home varieties depends only on their prices, i.e., aggregate variables in the Foreign demand function are not affected by Home; (ii) the cost distribution of Foreign producers is fixed; and (iii) the mass of available Foreign varieties is fixed. The last two assumptions mean that the mass of Foreign firms and Foreign wages are not affected by changes at Home. As a result, we have (see the online Appendix for more details):

Proposition 2 *In the case of a small Home economy, there exists a unique import tariff that maximizes Home welfare:*

$$t^{opt} = 1 + 1/\theta. \tag{12}$$

The direct comparison of these two Propositions shows that the level of protection is higher for the large Home economy, which is not surprising due to the terms of trade externality. Moreover, in both cases the level of protection depends on the degree of firm heterogeneity: lower θ (higher firm heterogeneity) results in higher need for protection, and when $\theta \rightarrow \infty$, i.e., all firms become identical, protection is no longer needed. Intuitively, if firm dispersion is high, then even without a tariff, only a few low-cost firms survive. This has two effects on the desired policy. On the one hand, low-cost firms charge higher markups, making the markup distortion at Home worse. The markup distortion appears because of the markups charged by local producers, whereas prices of imported goods are equal to opportunity costs. This provides an argument in favor of trade protection, since it reduces the markup distortion by shifting consumption towards domestic goods. On the other hand, since only a few Foreign firms survive, higher tariff results in smaller imported variety, reducing the consumption-surplus distortion that arises because consumers at Home ignore the

⁷See the (TB) condition that implicitly defines w as a function of t .

effect of their spending on the number of imported Foreign varieties, which turns out to be below its optimal level. What higher firm heterogeneity does is that it further increases the former distortion. Hence, higher protection is needed.

3.2 Reduction in Trade Costs

In this Section we ignore import tariffs (by assuming that $t_{ij} = 1$) and focus only on a fall in the iceberg trade cost for the Foreign exporters, τ_{FH} . This is exactly the type of UTL considered in Melitz and Ottaviano (2008). So the natural question is whether the presence of the outside good in their model makes any difference for the results of UTL. The answer is yes: the results in Melitz and Ottaviano (2008) get reversed once the outside good is dropped from the model.

By way of illustration, let us first demonstrate the following property of our setting:

Lemma 1 *Whether Home is a small or large economy, the $(FE)_H$ condition implies a positive relationship between c_{HH}^* and w , while the (TB) condition implies a negative relationship between c_{HH}^* and w .*

Proof. Using the $(FE)_H$ and $(FE)_F$, we can re-write the (TB) condition as

$$w^{\theta+1} \left[L_F (c_F^M)^{-\theta} w^\theta + L_H (c_H^M)^{-\theta} \tau_{HF}^{-\theta} \right] \left(\alpha c_{HH}^* - \frac{\theta+2}{\theta+1} \eta \right) (c_{HH}^*)^\theta \left(\frac{\tau_{HF} c_H^M}{\tau_{FH} c_F^M} \right)^\theta = Const_1 (c_F^M)^\theta.$$

It is straightforward to show that it implies that w and c_{HH}^* are negatively related. Next, by using $(\lambda)_H$ and $(\lambda)_F$ together with the $(FE)_H$ condition, we can re-write the $(FE)_H$ condition as

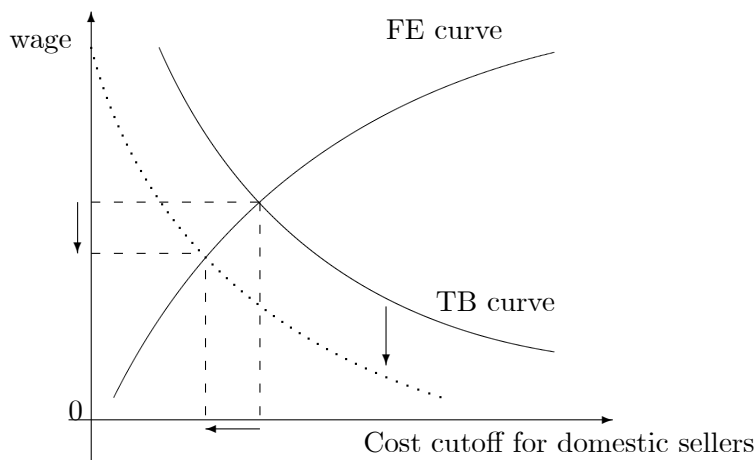
$$(c_F^M)^{-\theta} \left[L_H \left(\alpha c_{HH}^* - \frac{\theta+2}{\theta+1} \eta \right) (c_{HH}^*)^\theta + L_F \frac{Const_1}{\tau_{HF}^\theta w^{\theta+1} L_F (c_F^M)^{-\theta} + w L_H (c_H^M)^{-\theta}} \right] = Const_1,$$

which implies the positive relationship between w and c_{HH}^* . ■

We depict both relationships in Figure 1, where the FE and TB curves represent the $(FE)_H$ and (TB) conditions, respectively.⁸ Intuitively, the FE curve is upward sloping, since high wages deter entry, letting less efficient firms survive. The TB curve is downward sloping, since, to maintain the trade balance, high wages at Home must be compensated by higher efficiency of Home firms. The intersection of two curves gives the unique equilibrium values of w and c_{HH}^* .

⁸Note that our Figure 1 for the case of variable markups resembles the one in Demidova and Rodríguez-Clare (2013) for CES preferences. This is not a coincidence. We relate w and the cost cutoff for domestic sellers, c_{HH}^* , while their graphical analysis relates w and the productivity cutoff for exporters, φ_{HF}^* . In the Melitz (2003) model the productivity cutoff for exporters is negatively related to the one for domestic sellers, which, in turn, is an inverse of the cost cutoff. Hence, we have similar relationships between w and c_{HH}^* in our Figure 1 and w and φ_{HF}^* in Figure 1 in Demidova and Rodríguez-Clare (2013).

Figure 1: Unilateral Trade Liberalization by Home



Next, it is straightforward to show that a reduction in inward variable trade barriers at Home, τ_{FH} , affects only the TB curve by shifting it down as shown in Figure 1, which immediately proves that both w and c_{HH}^* fall as τ_{FH} falls. Also, from the $(FE)_F$ condition, c_{FF}^* falls with falling w . Finally, recall that in the absence of import tariffs welfare in both economies falls with the cost cutoff for the local producers there. This fact, together with our graphical analysis, leads to the following conclusion:

Proposition 3 *Whether Home is a small or large economy, unilateral trade liberalization (UTL) by Home in the form of falling trade costs raises welfare there. Moreover, in the case of two large economies, welfare of the Foreign country rises as well.*

The comparison of Proposition 3 with the outcome in Melitz and Ottaviano (2008) shows that the outside good assumption incorporated in Melitz and Ottaviano (2008) is crucial for their results regarding unilateral trade liberalization: while UTL by Home is welfare-reducing for the Home government in their model, once the outside good assumption is dropped from the model, full trade liberalization becomes an optimal policy, whether Home is a small or large economy, raising welfare of all trading partners. Intuitively, as argued, for example, by Ossa (2011),⁹ the effect of UTL depends on the interplay between production relocation and terms-of-trade effects. In particular, higher trade barriers for Foreign exporters generate positive profits to local firms at

⁹See also Costinot, Rodríguez-Clare and Werning (2016) for an insightful discussion of the role of the outside good for terms-of-trade manipulation in the generalized version of the Melitz (2003) model.

Home. They can be competed away either through entry (i.e., a production relocation effect) or through an increase in wages (i.e., a terms-of-trade effect). The result of the combination of these two effects depends on the elasticity of the labor supply in the differentiated good sector. In Melitz and Ottaviano (2008), the outside good can release labor at a constant wage, making labor supply perfectly elastic, so that the term-of trade effect is muted. In contrast, in our setting labor supply in the differentiated good sector is perfectly inelastic as it is fixed by country endowment so that the production relocation effect is isolated. Hence, the impact of UTL is the opposite in two models.¹⁰

Moreover, this result is in stark contrast to Propositions 1 and 2: in the case of trade costs, full trade liberalization, and not protection, is a preferable policy for the Home government. Hence, the choice of a trade policy instrument becomes vital for the resulting welfare changes.

4 Role of Variable Markups

Our results for UTL in the setting with variable markups seem to be quite similar to those obtained for the CES models with constant markups by Felbermayr and Jung (2012), Demidova and Rodríguez-Clare (2009, 2013) and Felbermayr, Jung and Larch (2013). Given such similarity, the natural question is what role variable markups play. We answer it in this Section.

To begin, let us discuss a reduction in per unit trade costs. Even though the average local markup as well as the dispersion of local markups remain unchanged as τ_{FH} falls, this does not mean that variable markups play no role as countries liberalize. In particular, as Edmond, Midrigan and Xu (2015) and ACDR point out, tracking down the changes in the local markup distribution alone is not enough. First, Edmond, Midrigan and Xu (2015), who study the revenue-weighted harmonic average of markups, argue that higher markups on imported goods can outweigh the reduction in local markups and make the misallocation distortion worse. In our case, however, a unilateral fall in trade costs of Foreign exporters does not affect the average markup due to the assumption of a Pareto cost distribution. Second, ACDR show that variable markups can create a new source of gains or losses from trade liberalization, depending on whether low cost firms, which charge high markups and under-supply their varieties, end up growing in size or not. In this case the effect of trade liberalization on welfare of country j depends on the sign of the covariance of the markup, $m(\omega, i)$ charged by a firm in country j that produces variety ω for market i and a

¹⁰See Haaland and Venables (2014), who allow the elasticity of labor supply take intermediate values between two polar cases, while analyzing trade policies in the CES model of the small open economy.

change in its labor share that is needed to produce this variety for this market:

$$\text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_j} \right) = \sum_i \int_{\omega \in \Omega_{ji}} \left[m(\omega, i) \frac{dl(\omega, i)}{L_j} \right] d\omega, \quad (13)$$

where $l(\omega, i)$ is the total employment associated with a production of variety ω in country j for sales in country i ,¹¹ and Ω_{ji} is the set of all varieties produced in country j for country i . In other words, trade liberalization has a positive (negative) effect on welfare in country j , if this covariance is positive (negative). The important property of (13) is that it depends not only on the firms' decisions in their local market, but also on their exporting decisions. In our model,

$$\text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_j} \right) = \frac{M_{jj}}{w_j L_j} \int_0^{c_{jj}^*} p(c) d[q(c)] \frac{dG_j(c)}{G_j(c_{jj}^*)} + \frac{M_{ji}}{w_j L_j} \int_0^{c_{ji}^*} p_{ji}(c) d[q_{ji}(c)] \frac{dG_j(c)}{G_j(c_{ji}^*)}.$$

As we show in Appendix E, for a small fall in τ_{FH} , the expression above is negative. Hence, although UTL is beneficial for the Home economy as a whole, its gains are mitigated by the misallocation distortion that gets worse as τ_{FH} starts to fall.¹² In other words, while our UTL results seem to be qualitatively the same as in the case of the Melitz (2003) model with CES preferences, in the case of variable markups welfare gains are smaller.

Another point worth emphasizing here is that unlike movements in trade costs that leave the aggregate markup unchanged, an ad valorem import tariff has the ability to affect it. To avoid analytical difficulties, let us simplify our setting in Section 2 by imposing some symmetry on two countries, i.e., assume that $L_H = L_F$, $c_H^M = c_F^M$, $\tau_{FH} = \tau_{HF} = 1$. Then the average markup becomes

$$\bar{m}_H = \sum_{i=H,F} \frac{M_{iH}}{M_{HH} + M_{FH}} \int_0^{c_{iH}^*} m_{iH}(c) \frac{dG(c)}{G(c_{iH}^*)} = \frac{1}{2} \frac{2\theta - 1}{\theta - 1} \frac{M_{HH} + tM_{FH}}{M_{HH} + M_{FH}}.$$

As we show in Appendix E, when the import tariff t starts to rise from 1, the average markup falls. In other words, when the initial level of protection is already small, trade liberalization by Home in the form of a falling import tariff actually raises the average markup faced by consumers there, which, according to Edmond, Midrigan and Xu (2015), implies a negative pro-competitive effect. To summarize, one has to be careful while trying to use the results derived for trade costs to forecast the effect of changes in import tariffs.

¹¹Note that (13) comes from the fact that for any labor re-allocations, the size of the economy remains the same so that $\sum_i \int_{\omega \in \Omega_{ji}} [dl(\omega, i)/L_j] d\omega = 0$.

¹²Due to complexity of the analysis of large falls in τ_{FH} , we leave it to future work.

5 Conclusion

In this paper we have studied the implications of trade cost reductions and import tariffs in the extension of the Melitz and Ottaviano (2008) model without the outside good. Our conclusions can be broadly summarized as follows. First, we find that in contrast to Melitz and Ottaviano (2008), a reduction in per unit trade costs raises, and does not reduce, welfare of the liberalizing country as well as welfare of its trading partner. Thus, the breakdown of optimality of laissez-faire in Melitz and Ottaviano (2008) can be explained by the distortion created by the presence of the outside good sector. Second, we derive the optimal values of import tariffs for the large and small Home economies and show that as in the models with monopolistic competition and CES preferences, protection is always a desirable policy for the Home government. The main difference between the policy implications from the CES models and our setting is the negative pro-competitive effect caused by variable markups.

Given the results obtained, there are several potential avenues for future research. First, the absence of pro-competitive gains in our model might be partially explained by the assumption made about the cost distribution, which is specified as Pareto. The Pareto distribution assumption can be modified: Feenstra (2014), who studies the model with non-CES preferences, shows that once the support of the distribution becomes bounded, other channels of pro-competitive gains from trade begin working, affecting the average markup as well as the markup dispersion, which otherwise remain unchanged. Alternatively, the Pareto distribution can be replaced by another one. For example, as argued by Fernandes et al. (2015), in the Melitz-Pareto models, conditional on the fixed costs of exporting, falling trade costs affect exports only through the number of exporters, i.e., the extensive margin, while in their empirical analysis, half of the variation in exports occurs on the intensive margin. The authors show that one way to reconcile the theory with the data is to use the log-normal distribution with firm-destination fixed trade costs.

Another limitation of our analysis is that similar to Arkolakis, Costinot and Rodríguez-Clare (2012) and ACDR, it focuses on single-sector economies only. One of the consequences of such an assumption is that entry in each economy is not affected by trade policies.¹³ This might not be the case in a setting with multiple sectors, where the mass of entrants in a particular sector could be affected by the relative expenses on goods produced there. See, for example, Spearot

¹³In the case of CES preferences, Caliendo, Feenstra, Romalis and Taylor (2015) show that entry becomes endogenous if tariff revenues are not rebated to consumers or if one considers cost tariffs instead of revenues ones.

(forthcoming), who studies changes in revenue-generating import tariffs in the Melitz and Ottaviano (2008) model without the outside good assumption, which he further modifies by incorporating multiple countries and multiple industries with heterogeneity in the country-by-industry shape parameters of the Pareto cost distributions.¹⁴ Because of analytical complications, he estimates the model empirically and runs counterfactual experiments. Although his focus is different from ours, he provides some interesting results for the case of unilateral trade liberalization by showing that the US gains both from an increase in all its tariffs by 10% and a removal of the observed tariffs. Moreover, it would be interesting to extend the analysis by incorporating multiple countries as done in Spearot (forthcoming), since in addition to producing potentially interesting third-country effects, multiplicity of trading partners can reduce the negative effect of variable markups by affecting the level of absorption of the drop in trade costs by exporters.

Next, it would be interesting to see whether our trade policy results change, if one considers alternative demand structures that generate variable markups. For instance, Jung, Simonovska and Weinberger (2015) show that the use of the generalized CES utility function allows to match the stylized facts about exporters and their markups qualitatively *and* quantitatively, which other frameworks including ours fail to do.

Finally, we have left Nash trade policies as well as the trade policies, which allow the home government to discriminate between firms from the same country, out of the scope of this paper. In the former case, the question is whether protection remains the optimal policy when all trading countries, and not just the Home economy, have the ability to choose their tariffs.¹⁵ In the latter case, as shown by Costinot, Donaldson, Vogel and Werning (2015) in the setting with a Ricardian economy and perfect competition and Costinot, Rodríguez-Clare and Werning (2016) in the setting with monopolistic competition and CES preferences, even in models with constant (positive or zero markups) the social planner has the incentive to discriminate against firms based on their productivity. In our setting variable markups create a within-sector misallocation distortion, providing the planner with an additional incentive to manipulate terms-of-trade, even at the firm-level. Hence, policies that discriminate against firms become even more relevant. We leave all of these questions to future work.

¹⁴Note that quadratic preferences in Spearot (forthcoming) are different from those in Melitz and Ottaviano (2008) and in our setting, since he excludes the total industry quantity squared from the industry-level sub-utility, making the preferences separable.

¹⁵See Bagwell and Lee (2015) for the discussion of Nash trade policies in the Melitz and Ottaviano (2008) symmetric model with the outside good.

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Appendix A

In this Appendix we derive formula (1) that describes the welfare effect of a small shock in import tariffs or trade costs in the Melitz and Ottaviano (2008) model *without* the outside good. To deal with non-separability of the utility function as well as with the complications caused by accounting for changes in tariff revenues, we modify the approach used in the 2012 version of Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2015) (hereafter ACDR 2012). Before we start, note that to simplify the comparison with ACDR 2012, here we use their notation instead of that used in the main body of the paper.

Consider a world economy comprising $j = 1, \dots, n$ countries, one factor of production, labor, and a continuum of differentiated goods $\omega \in \Omega$. All individuals are endowed with one unit of labor, are perfectly mobile across the production of different goods, and are immobile across countries. L_j denotes the total endowment of labor and w_j denotes the wage in country j .

All consumers have the same preferences. Similar to the case of the separable non-CES and translog utility functions explored in ACDR 2012, if a consumer with income i faces a schedule of prices $\mathbf{p} \equiv \{p_\omega\}_{\omega \in \Omega}$, her Marshallian demand for any differentiated good ω is

$$\ln q_\omega(p, i) = -\beta \ln p_\omega + \gamma \ln i + d(\ln p_\omega - \ln p^*(\mathbf{p}, i)),$$

where $p^*(p, i)$ is symmetric in all prices. Note that in the case of trade costs, income $i = \text{wage } w$, while in the case of import tariffs, when tariff revenues, TR , are distributed equally across L workers, $i = w + (TR/L)$. Let us derive a similar formula for our case of a quadratic, non-separable utility function. From the F.O.C. described in Section 2,

$$\lambda p(\omega) = \alpha - \tilde{\gamma} q^c(\omega) - \tilde{\eta} \int_{\Omega} q^c(\omega) d\omega,$$

where λ is a Lagrangian multiplier, $\lambda = \left[\alpha Q - \tilde{\gamma} \int_{\Omega} (q^c(\omega))^2 d\omega - \tilde{\eta} Q^2 \right] / i$. Using the same logic as in Melitz and Ottaviano (2008), one can show that the set Ω^c of all varieties that are consumed ($q_{\omega}^c > 0$) is the largest subset of Ω that satisfies:

$$p_{\omega} \leq \frac{1}{\eta M + \tilde{\gamma}} \left(\frac{1}{\lambda} \tilde{\gamma} \alpha + \tilde{\eta} M \bar{p} \right) \equiv p^*,$$

where M is the measure of consumed varieties in Ω^c and $\bar{p} = (1/M) \int_{\omega \in \Omega^c} p_{\omega} d\omega$. p^* represents the choke price. By definition, $\lambda p^* = \alpha - \tilde{\eta} \int_{\Omega} q^c(\omega) d\omega$. Let us define the income multiplier μ^I as (we use μ^I instead of μ here to make the comparison with ACDR 2012, who use μ for markups, easy. In the final formula for welfare we will drop superscript “ I ” to relate this formula to the one in Felbermayr, Jung and Larch, 2015):

$$\begin{aligned} \mu^I &\equiv E/wL = \text{income}/\text{wage} = i/w, \quad \text{so that} \\ q^c(\omega) &= \alpha - \tilde{\eta} \int_{\Omega} q^c(\omega) d\omega - \lambda p(\omega) = \frac{p(\omega)}{w} \left(\frac{p^*}{p(\omega)} - 1 \right) \frac{\alpha Q - \tilde{\gamma} \int_{\Omega} (q^c(\omega))^2 d\omega - \tilde{\eta} Q^2}{\tilde{\gamma} \mu^I}. \end{aligned}$$

We can find p^* and λ by solving the following system of equations:

$$\begin{cases} \lambda = \frac{\alpha Q - \tilde{\gamma} \int_{\Omega} (q^c(\omega))^2 d\omega - \tilde{\eta} Q^2}{i} \\ p^* = \frac{1}{\eta M + \tilde{\gamma}} \left(\frac{1}{\lambda} \tilde{\gamma} \alpha + \tilde{\eta} M \bar{p} \right). \end{cases}$$

Hence, we can re-write the Marshallian demand as

$$\ln q^c(\omega) = \ln p(\omega) - \ln w + \ln \left(e^{-(\ln p(\omega) - \ln p^*)} - 1 \right) + \ln \left[\frac{\alpha Q - \tilde{\gamma} \int_{\Omega} (q^c(\omega))^2 d\omega - \tilde{\eta} Q^2}{\tilde{\gamma} \mu^I} \right]. \quad (\text{A.1})$$

The last term in the expression above appears due to non-separability of the utility function and the presence of tariff revenues (it is absent in ACDR 2012). It absorbs the characteristics of the destination market only and is the same for all varieties sold there.

Thus, if a consumer with income $i = \mu^I w$ faces a schedule of prices $\mathbf{p} \equiv \{p_{\omega}\}_{\omega \in \Omega}$, her Marshallian demand for any differentiated good ω takes the form of

$$\ln q_{\omega}(p, i) = -\beta \ln p_{\omega} + \gamma \ln w + d(\ln p_{\omega} - \ln p^*(\mathbf{p}, i)) + \ln \Delta. \quad (14)$$

where $p^*(p, w)$ is symmetric in all prices, $\Delta \equiv \left[\alpha Q - \tilde{\gamma} \int_{\Omega} (q^c(\omega))^2 d\omega - \tilde{\eta} Q^2 \right] / \tilde{\gamma} \mu^I$ is the aggregate term, $\partial \Delta / \partial \ln p_{\omega} = 0$. Finally, $\beta = \gamma = -1$ and $d(x) = \ln(e^{-x} - 1)$. Note that in ACDR 2012 for the non-CES separable utility function $\beta = \gamma = 0$ and for the translog utility function $\beta = \gamma = 1$. Hence, in all cases, theirs and ours, $\beta = \gamma \leq 1$.

“A” Assumptions from ACDR 2012. Now let us verify that the “A” assumptions from ACDR 2012 hold in our case.

- A1. [Existing Demand Systems]. As we have shown already, $\beta = \gamma = -1 \leq 1$.
- A2. [Choke Price] It is straightforward to show that for all $x \geq 0$, $d(x) = \ln(e^{-x} - 1) = -\infty$.
- A3. [Log-concavity] For all $x \leq 0$, $d''(x) \leq 0$. In our case

$$d'(x) = -\frac{e^{-x}}{e^{-x} - 1} = -\frac{1}{e^{-x} - 1} - 1, \quad d''(x) = \frac{-e^{-x}}{(e^{-x} - 1)^2} < 0.$$

A4. [Pareto] Firm environment is exactly the same as in ACDR 2012 including the assumption of the Pareto productivity distribution, where the per unit labor cost of a firm with productivity z is $c = 1/z$. For more details see Section 2.

Firms in Trade Equilibrium. The main difference between ACDR 2012 and our setting is that in addition to trade costs, we allow for import tariffs. In particular, while exporting, each firm faces an iceberg transportation costs τ_{ij} , where $\tau_{ij} > 1$ and $\tau_{ii} = 1$, and if a firm from country i charges price p_{ij} in market j , then the country j ' government collects tariff revenues of $(t_{ij} - 1)p_{ij}/t_{ij}$ per unit sold, so that this firm receives only p_{ij}/t_{ij} , where $t_{ij} \geq 1$.

To deal with the complications caused by tariffs, we are going to deviate from ACDR 2012 by looking at firm characteristics *inclusive of tariffs*. In particular, if $MC_{ij} = \tau_{ij}w_i/z$ is the marginal cost of a firm from country i that serves market j , then $c \equiv t_{ij}MC_{ij} = c_{ij}/z$ is its marginal cost *inclusive of a tariff* and the firm's profit decision is

$$\max_{p_{ij}} (p_{ij}/t_{ij} - MC_{ij}) q_{ij}(p_{ij}) \quad \Longleftrightarrow \quad \max_{p_{ij}} \pi_{ij} = \max_{p_{ij}} (p_{ij} - c) q_{ij}(p_{ij}),$$

where p_{ij} and π_{ij} are the firm's price and profit *inclusive of a tariff*. Hence, we get expressions similar to ACDR 2012 for profits and sales, but now they are *inclusive of tariffs*:

$$\pi(c, p^*, w) = (p - c)q(p, p^*, w) \quad \text{and} \quad x(c, p^*, w) = pq(p, p^*, w).$$

Define the firm-level markups as $m = \ln(p/c)$ and $\nu = \ln(p^*/c)$. Then from F.O.C.,

$$\frac{p - c}{p} = -\frac{1}{\partial \ln q(p, p^*, w) / \partial \ln p},$$

so that we get equation (3) from ACDR 2012:

$$m - \ln \left(\frac{\beta - d'(m - \nu)}{\beta - 1 - d'(m - \nu)} \right) = 0. \tag{A.3}$$

(A.3) is the same as in ACDR 2012, since $\partial \Delta / \partial \ln p_\omega = 0$. Thus, the derivative $\partial \ln q(p, p^*, w) / \partial \ln p$ is also the same as in ACDR 2012.

Remark. The exact form of $d(\cdot)$ and $\beta = \gamma = -1$ allow us to solve equation (A.3) explicitly:

$$m = \ln \left(\frac{d' + 1}{d' + 2} \right) = \ln \left(\frac{1}{2 - e^{-(m-\nu)}} \right), \text{ or } e^m = \frac{p}{c} = \frac{1}{2} (1 + e^\nu) = \frac{1}{2} \left(1 + \frac{p^*}{c} \right),$$

so that $p = (c + p^*)/2$, which is exactly the formula for the price derived in Section 2.

Define $p(c, \nu) = ce^{\mu(\nu)}$, where $\mu(\nu)$ is the optimal markup defined as a solution of (A.3). Then the sales and profits of a firm with marginal cost c are

$$x(c, \nu, w, L) = Lw^\gamma \left(ce^{\mu(\nu)} \right)^{1-\beta} e^{d(\mu(\nu)-\nu)} \Delta, \quad (\text{A.4})$$

$$\pi(c, \nu, w, L) = \left(\frac{e^{\mu(\nu)} - 1}{e^{\mu(\nu)}} \right) x(c, \nu, w, L). \quad (\text{A.5})$$

Note that the only difference between these formulas from those in ACDR 2012 is term Δ , which is specific to the destination market. This allows us to use the same logic as in ACDR 2012 to get *tariff-inclusive* total sales and profits earned by firms from country i in country j :

$$X_{ij} = \chi N_i b_i^\theta (w_i \tau_{ij} t_{ij})^{-\theta} L_j w_j^\gamma (p_j^*)^{1-\beta+\theta} \Delta_j, \quad (\text{A.6})$$

$$\Pi_{ij} = \pi N_i b_i^\theta (w_i \tau_{ij} t_{ij})^{-\theta} L_j w_j^\gamma (p_j^*)^{1-\beta+\theta} \Delta_j. \quad (\text{A.7})$$

In the case of our preferences, $\chi = (2(\theta + 2))^{-1}$ and $\pi = (2(\theta + 2)(\theta + 1))^{-1}$. An important feature of (A.6) and (A.7) is that even though we study non-separable preferences, the term that “absorbs” non-separability, Δ_j , enters both X_{ij} and Π_{ij} multiplicatively so that

$$\Pi_{ij} = \pi X_{ij} / \chi. \quad (\text{A.8})$$

Before making the next step let us point out another difference from ACDR 2012. As explained earlier, we look at the *tariff-inclusive* sales and profits. This means that the total expenditure in country j , $E_j = \Sigma_j X_{ij}$, is no longer equal to the total (after-tariff) revenue earned by firms in this country, $\Sigma_{ij} X_{ij} / t_{ij}$. In turn, we need to make adjustments in the trade balance, free entry, and market clearing conditions. As a result, we have

$$\Sigma_i \Pi_{ji} / t_{ji} = w_j N_j F_j, \quad (\text{A.9})$$

$$\Sigma_i X_{ji} / t_{ji} = w_j L_j. \quad (\text{A.10})$$

Nevertheless, even with the adjustments made in the case of import tariffs, we still get

$$N_j = \pi L_j / \chi F_j. \quad (\text{A.11})$$

This is an important result, since it shows that the mass of entrants in each country is not affected by changes in trade costs and/or import tariffs.

Remark. As shown in Section 2, in our case, $N_j = L_j / [(\theta + 1) F_j]$.

Finally, let us derive the gravity equation. Using the expression for the price inclusive of tariffs,

$$p_{ij}(z) = (w_i \tau_{ij} t_{ij} / z) e^{\mu(\ln(p_j^*(p_j, w_j) z / w_i \tau_{ij}))}, \text{ if } z \geq w_i \tau_{ij} t_{ij} / p_j^*(p_j, w_j), \quad (\text{A.12})$$

we can re-write (A.6) as

$$X_{ij} = \frac{N_i b_i^\theta (w_i \tau_{ij} t_{ij})^{-\theta} E_j}{\sum_k N_k b_k^\theta (w_k \tau_{kj} t_{kj})^{-\theta}}, \quad (\text{A.13})$$

where $E_j = \sum_k X_{kj}$. This equation is similar to equation (13) in ACDR 2012, since to derive it, one needs to use *ratios* of X_{ij} , $i = 1, \dots, n$, so that term Δ_j gets cancelled.

Compensating Variation. Consider a small change from $t = \{t_{ij}\}$ to $t' = \{t_{ij} + dt_{ij}\}$. In order to look at compensating variation, we need to study expenditure function of a representative consumer in country j denoted by $e_j \equiv e_j(p_j, u_j)$, for which we have $de_j / dp_{\omega, j} = q(p_{\omega, j}, p_j^*, w_j) \equiv q_{\omega, j}$ for all $\omega \in \Omega$. We consider only infinitesimal changes in tariffs, which allows us to ignore creation of “new” goods and destruction of “old” ones. The reason is that, first, these changes do not affect entry (as shown above, N_j does not depend on trade costs and import tariffs). Second, there are no fixed costs of production in our setting, which means that the price of firms with the productivity close to the cutoff is almost equal to the choke price, i.e., in the absence of fixed costs of production their quantities, revenues, and profits are almost zero. Hence, as in ACDR 2012, the change in expenditures in country j is $de_j = \sum_i \int_{\omega \in \Omega_{ij}} [q_{\omega, j} dp_{\omega, j}] d\omega$. Then, using the same logic as in ACDR 2012, we can rewrite it as:

$$d \ln e_j = \sum_i \int_{\omega \in \Omega_{ij}} [\lambda_{\omega, j} d \ln p_{\omega, j}] d\omega, \quad (\text{A.14})$$

where $\lambda_{\omega, j} \equiv p_{\omega, j} q_{\omega, j} / e_j$. Using (A.12) in (A.4), we get

$$d \ln e_j = \sum_i \int_{z_{ij}^*}^{\infty} \lambda_{ij}(z) [d \ln c_{ij} + dm_{ij}(z)] dG_i(z), \text{ where} \quad (\text{A.15})$$

$$\lambda_{ij} = \frac{X_{ij}}{E_j} = \frac{N_i e^{-(1-\beta)(\ln(z/z_{ij}^*) - \mu(\ln(z/z_{ij}^*)))} e^{d(\mu(\ln(z/z_{ij}^*)) - \ln(z/z_{ij}^*))}}{\sum_k N_k \int_{z_{kj}^*}^{\infty} e^{-(1-\beta)(\ln(z'/z_{kj}^*) - \mu(\ln(z'/z_{kj}^*)))} e^{d(\mu(\ln(z'/z_{kj}^*)) - \ln(z'/z_{kj}^*))}} \quad (\text{A.16})$$

is the expenditure share in country j on goods from country i .

Next, using (A.13) together with the assumption of the Pareto productivity distribution, we get

$$d \ln e_j = \sum_i \lambda_{ij} d \ln c_{ij} - \rho \sum_i \lambda_{ij} d \ln z_{ij}^*,$$

where ρ is the weighted average of the markup elasticities $\mu'(v)$ across all firms. In our case,

$$\rho = \int_0^\infty \mu'(v) \frac{e^{-(1-\beta)(v-\mu(v))} e^{d(\mu(v)-v)} e^{-\theta} dv}{\int_0^\infty e^{-(1-\beta)(v'-\mu(v'))} e^{d(\mu(v')-v')} e^{-\theta v'} dv'} = \frac{\theta + 2}{2(\theta + 1)}.$$

As shown in ACDR 2012, a useful decomposition of the last expression for $d \ln e_j$ is

$$\begin{aligned} d \ln e_j &= \sum_i \lambda_{ij} d \ln c_{ij} + (-\rho) \sum_i \lambda_{ij} d \ln c_{ij} + \rho \ln p_j^* \\ &= (\text{Change in MC}) + (\text{Direct Mark-up effect}) + (\text{GE markup effect}). \end{aligned} \quad (\text{A.17})$$

Given that in our case $\rho > 0$, if trade liberalization reduces the choke price p_j^* , then markups fall and gains from trade liberalization are higher.

Up to this point, most of our derivations strongly resemble those in ACDR 2012. However, the next step introduces one of the crucial differences for our case of import tariffs. In particular, given the use of new term Δ_j and the fact that by X_{ij} we denote the sales *inclusive of tariffs*, the labor market clearing condition becomes

$$\sum_i X_{ij}/t_{ij} = w_j L_j,$$

which is derived by using the (TB) condition, $\sum_{i \neq j} X_{ij}/t_{ij} = \sum_{j \neq i} X_{ji}/t_{ji}$, in (A.10). Then we have

Lemma 2 *For the individual expenditure in country j we have*

$$d \ln e_j = \left(1 - \rho \frac{1 - \beta}{1 - \beta + \theta}\right) \frac{d \ln \lambda_{jj}}{\theta} + d \ln w_j + \frac{2\rho}{1 - \beta + \theta} d \ln \mu_j^I. \quad (\text{A.18})$$

Proof. *We can re-write $\sum_i X_{ij}/t_{ij} = w_j L_j$ as $\sum_i X_{ij}/t_{ij} w_j L_j = 1$ or $\sum_i \psi_{ij} = 1$, where $\psi_{ij} \equiv (X_{ij}/t_{ij})/w_j L_j$ is the after-tariff revenue share of firms from country i in the after-tariff revenues earned by all firms in country j . Note that*

$$\lambda_{ij} = \frac{X_{ij}}{E_j} \quad \text{and} \quad \psi_{ij} = \frac{X_{ij}/t_{ij}}{w_j L_j},$$

so that without tariffs, $\psi_{ij} = \lambda_{ij}$. Using (A.6) and totally differentiating $\sum_i X_{ij}/t_{ij} w_j L_j = 1$, we get

$$\sum_i \psi_{ij} [-\theta d \ln c_{ij} + (1 - \beta + \theta) d \ln p_j^* + (\gamma - 1) w_j - d \ln t_{ij} - d \ln \Delta_j] = 0.$$

Then by using $\sum_i \psi_{ij} = 1$ and the fact that for infinitesimal changes in t , changes in aggregate quantities can be ignored so that $d \ln \Delta_j = -d \ln \mu_j^I$, we get:

$$d \ln p_j^* = \frac{\theta}{1 - \beta + \theta} \sum_i \psi_{ij} d \ln c_{ij} + \frac{1 - \gamma}{1 - \beta + \theta} d \ln w_j + \frac{1}{1 - \beta + \theta} (\sum_i \psi_{ij} d \ln t + d \ln \mu_j^I). \quad (\text{A.19})$$

Next, from $\lambda_{ij}/\lambda_{jj} = (c_{ij}/c_{jj})^{-\theta}$, where $c_{jj} = w_j$, we have

$$d \ln c_{ij} = \frac{1}{\theta} (d \ln \lambda_{jj} - d \ln \lambda_{ij}) + d \ln w_j, \quad \text{and} \quad \sum_i \psi_{ij} d \ln c_{ij} = \frac{d \ln \lambda_{jj}}{\theta} + d \ln w_j - \frac{1}{\theta} \sum_i \psi_{ij} d \ln \lambda_{ij}. \quad (\text{A.20})$$

Then by using the expression above and noting that $\beta = \gamma$, we get

$$\begin{aligned} d \ln p_j^* &= \frac{1}{1 - \beta + \theta} \left(\theta \left[\frac{d \ln \lambda_{jj}}{\theta} + d \ln w_j \right] + (1 - \beta) d \ln w_j + \sum_i \psi_{ij} (d \ln t_{ij} - d \ln \lambda_{ij}) + d \ln \mu_j^I \right) \\ &= d \ln w_j + \frac{\theta}{1 - \beta + \theta} \frac{d \ln \lambda_{jj}}{\theta} + \frac{1}{1 - \beta + \theta} \sum_i \psi_{ij} d \ln (t_{ij} \mu_j^I / \lambda_{ij}). \end{aligned}$$

Moreover, from (A.20)

$$\begin{aligned} \sum_i \lambda_{ij} d \ln c_{ij} &= \frac{d \ln \lambda_{jj}}{\theta} + d \ln w_j - \frac{1}{\theta} \sum_i \lambda_{ij} d \ln \lambda_{ij} = \frac{d \ln \lambda_{jj}}{\theta} + d \ln w_j, \quad \text{and} \\ d \ln e_j &= \left(1 - \rho \frac{1 - \beta}{1 - \beta + \theta} \right) \frac{d \ln \lambda_{jj}}{\theta} + d \ln w_j + \frac{\rho}{1 - \beta + \theta} \sum_i \psi_{ij} d \ln (t_{ij} \mu_j^I / \lambda_{ij}), \quad \text{where} \\ t_{ij} \mu_j^I / \lambda_{ij} &= \frac{t_{ij} E_j}{w_j L_j} * \frac{E_j}{X_{ij}} = \left(\frac{E_j}{w_j L_j} \right)^2 \frac{w_j L_j}{X_{ij} / t_{ij}} = (\mu_j^I)^2 / \psi_{ij}, \quad \text{so that} \\ \sum_i \psi_{ij} d \ln (t_{ij} \mu_j^I / \lambda_{ij}) &= d \ln (\mu_j^I)^2 \sum_i \psi_{ij} - \sum_i \psi_{ij} d \ln \psi_{ij} = 2 d \ln \mu_j^I, \end{aligned}$$

resulting in (A.18), which looks the same as equation (18) in ACDR 2012 except for the last term that appears due to tariff revenues being a part of the national income. ■

Finally, note that the compensating variation in the case of import tariffs implies that

$$\begin{aligned} d \ln W_j &= d \ln \text{income}_j - d \ln e_j = d \ln (\mu_j^I w_j) - d \ln e_j = d \ln \mu_j^I + d \ln w_j - d \ln e_j \\ &= - \left(1 - \rho \frac{1 - \beta}{1 - \beta + \theta} \right) \frac{d \ln \lambda_{jj}}{\theta} + \left(1 - \frac{2\rho}{1 - \beta + \theta} \right) d \ln \mu_j^I, \end{aligned}$$

which leads to formula (1). In our special case of non-separable quadratic preferences, $\rho = \frac{\theta+2}{2(\theta+1)}$

and $\beta = \gamma = -1$ gives $\eta = \rho \frac{1-\beta}{1-\beta+\theta} = \frac{1}{\theta+1}$, so that we can re-write (1) as:

$$d \ln W_j = - (1 - \eta) \frac{d \ln \lambda}{\theta} + \left(1 - \frac{2\rho}{1 - \beta + \theta} \right) d \ln \mu = \frac{\theta}{1 + \theta} \left(- \frac{d \ln \lambda}{\theta} + d \ln \mu \right). \quad (\text{A.21})$$

Appendix B: Equilibrium Conditions

Derivations for Lagrangian Multipliers. From (4), $\lambda_i p q^c = (\alpha - \eta Q_i - \gamma q^c) q^c = (\lambda_i w_i c_{ii}^* - \gamma q^c) q^c$.

By integrating both parts over all varieties sold in country i , we get

$$\lambda_i \int_{\Omega_i} p(\omega) q^c(\omega) d\omega = i_i \lambda_i = \lambda_i w_i c_{ii}^* Q_i - \gamma \int_{\Omega_i} (q^c(\omega))^2 d\omega, \quad \text{so that}$$

$$\lambda_i = \frac{\gamma \int_{\Omega_i} (q^c(\omega))^2 d\omega}{w_i c_{ii}^* Q_i - i_i}, \quad (\text{B.1})$$

where from (5),

$$Q_i = \frac{1}{2\gamma} \frac{1}{\theta + 1} \lambda_i (c_{ii}^*)^{\theta+1} \left[M_i^e w_i (c_i^M)^{-\theta} + M_j^e (c_j^M)^{-\theta} (t_{ji} \tau_{ji})^{-\theta} (w_i)^{\theta+1} \right],$$

$$\int_{\Omega_i} (q^c(\omega))^2 d\omega = \frac{1}{2\gamma^2} \frac{w_i (\lambda_i)^2 (c_{ii}^*)^{\theta+2}}{(\theta + 1)(\theta + 2)} \left[M_i^e w_i (c_i^M)^{-\theta} + M_j^e (c_j^M)^{-\theta} (t_{ji} \tau_{ji})^{-\theta} (w_i)^{\theta+1} \right] = \frac{1}{\gamma} w \lambda_i c_{ii}^* \frac{1}{\theta + 2} Q_i.$$

Thus, (B.1) can be re-written as $\lambda_i (w_i c_{ii}^* Q_i - i_i) = w_i \lambda_i c_{ii}^* Q_i / (\theta + 2)$, so that

$$Q_i = \frac{\theta + 2}{\theta + 1} \frac{i_i}{w_i c_{ii}^*}, \quad \text{and} \quad (\text{B.2})$$

$$i_i \lambda_i = \alpha Q_i - \gamma \int_{\Omega_i} (q^c(\omega))^2 d\omega - \eta (Q_i)^2 = \alpha \frac{\theta + 2}{\theta + 1} \frac{i_i}{w_i c_{ii}^*} - i_i \lambda_i \frac{1}{\theta + 1} - \eta \left(\frac{\theta + 2}{\theta + 1} \frac{i_i}{w_i c_{ii}^*} \right)^2,$$

which we can solve for $i_i \lambda_i$:

$$w_i c_{ii} \lambda_i = \alpha - \frac{\theta + 2}{\theta + 1} \eta \frac{i_i}{w_i c_{ii}^*}, \quad \text{where} \quad (\text{B.3})$$

$$\frac{i}{w_i} = 1 + \frac{(t_{ji} - 1)}{w_i L_i} M_{ji} \bar{r}_{ji} = 1 + t_{ji} M_e^* \frac{(t_{ji} - 1)}{2\gamma (\theta + 2) t_{ji}^{\theta+2}} \left(\frac{w_i c_{ii}^*}{c_i^M \tau_{ji}} \right)^\theta c_{ii}^* (w_i \lambda_i c_{ii}^*) \equiv 1 + y_i c_{ii}^* (\alpha - \eta Q_i),$$

with $y_i \equiv \frac{t_{ji} - 1}{2\gamma(\theta + 2)(t_{ji})^{\theta+1}} \left(\frac{w_i c_{ii}^*}{\tau_{ji} c_i^M} \right)^\theta M_j^e$. Then by using (B.2) in the equation above, we get:

$$\begin{aligned} \frac{i}{w_i} &= (1 + \alpha c_{ii}^* y_i) / \left(1 + \eta \frac{\theta + 2}{\theta + 1} y_i \right), \quad \text{so that} \\ w_i c_{ii} \lambda_i &= \alpha - \frac{\theta + 2}{\theta + 1} \eta \frac{i_i}{w_i c_{ii}^*} = \left(\alpha - \eta \frac{\theta + 2}{\theta + 1} * \frac{1}{c_{ii}^*} \right) / \left(1 + \eta \frac{\theta + 2}{\theta + 1} y_i \right). \end{aligned}$$

Welfare. Since $\gamma \int_{\Omega_i} (q^c(\omega))^2 d\omega = w \lambda_i c_{ii}^* \frac{1}{\theta + 2} Q_i = (\alpha - \eta Q_i) \frac{1}{\theta + 2} Q_i$, per capita welfare is

$$U_i = \alpha Q_i - \frac{\gamma}{2} \int_{\Omega_i} (q^c(\omega))^2 d\omega - \frac{\eta}{2} (Q_i)^2 = \frac{\theta + 1}{2(\theta + 2)} Q \left[\frac{\alpha(2\theta + 3)}{\theta + 1} - \eta Q \right].$$

Appendix C: Optimal Import Tariff

As discussed in Section 2.4, we need to study the behavior of $z \equiv w \lambda_H c_{HH}^*$. Using the new notation and replacing $\lambda_F (c_{FF}^*)^{\theta+2}$ from the (TB) condition, we get:

$$(FE)_H : z (c_{HH}^*)^{\theta+1} \left[L_H + L_F \left(\frac{c_H^M}{c_H^M \tau_{FH}} \right)^\theta \frac{w^\theta}{t^{\theta+1}} \right] = \text{Const}_1 (c_H^M)^\theta, \quad (\text{C.1})$$

$$(FE)_F : z (c_{HH}^*)^{\theta+1} \left[L_F \left(\frac{c_H^M \tau_{HF}}{c_F^M \tau_{FH}} \right)^\theta \frac{w^{2\theta+1}}{t^{\theta+1}} + L_H (\tau_{FH})^{-\theta} \frac{w^{\theta+1}}{t^{\theta+1}} \right] = \text{Const}_1 (c_F^M)^\theta, \quad (\text{C.2})$$

$$(z) : z + \frac{\eta}{c_{HH}^*} \frac{\theta + 2}{\theta + 1} [1 + zy_H c_{HH}^*] = \alpha. \quad (\text{C.3})$$

Combining (C.1) and (C.2) gives

$$L_H t^{\theta+1} + L_F \left(\frac{c_H^M}{c_H^M \tau_{FH}} \right)^\theta w^\theta = L_F \left(\frac{c_H^M \tau_{HF}}{c_F^M \tau_{FH}} * \frac{c_H^M}{c_H^M} \right)^\theta w^{2\theta+1} + L \left(\frac{c_H^M}{c_F^M \tau_{FH}} \right)^\theta w^{\theta+1}. \quad (\text{C.4})$$

From the implicit function theorem applied to (C.4) and (C.2), we get:

$$t \frac{w'}{w} = \left[1 + \frac{\theta L_F \left(\frac{c_H^M \tau_{HF}}{c_F^M \tau_{FH}} * \frac{c_H^M}{c_H^M} \right)^\theta w^{2\theta+1} + L_F \left(\frac{c_H^M}{c_F^M \tau_{FH}} \right)^\theta w^\theta}{(\theta + 1) L t^{\theta+1}} \right]^{-1}, \quad \text{and} \quad (\text{C.5})$$

$$\frac{(c_{HH}^*)'}{c_{HH}^*} = \frac{1}{t} - \frac{z'}{(\theta + 1) z} - \frac{w'}{(\theta + 1) w} \frac{(2\theta + 1) L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^{2\theta+1} + (\theta + 1) L_H w^{\theta+1}}{L^* (c_H^M \tau_{HF}/c_F^M)^\theta w^{2\theta+1} + L_H w^{\theta+1}} \quad (\text{C.6})$$

Finally, from $(FE)_F$, $zy_H c_{HH}^* = (t - 1) L_F / \left[w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right) \right]$, so that

$$(z) : z + \frac{\eta}{c_{HH}^*} \frac{\theta + 2}{\theta + 1} \left[1 + (t - 1) L_F \left[w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right) \right]^{-1} \right] = \alpha.$$

After applying the implicit function theorem to the equation above, we get

$$z' - \frac{\eta}{c_{HH}^*} \frac{\theta + 2}{\theta + 1} \left[\frac{(c_{HH}^*)'}{c_{HH}^*} \left(1 + \frac{(t - 1) L_F}{w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right)} \right) - \frac{L_F}{w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right)} + \frac{L_F}{w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right)^2} \frac{w'}{w} \left(L_H + (\theta + 1) L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right) \right] = 0.$$

Using the expression for $(c_{HH}^*)'/c_{HH}^*$, we can re-write it as

$$z' = \frac{\eta}{t c_{HH}^*} \frac{\theta + 2}{\theta + 1} \left(1 + \frac{1}{(\theta + 1) z} \eta \frac{\theta + 2}{\theta + 1} \frac{1}{c_{HH}^*} \left(1 + \frac{(t - 1) L_F}{w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right)} \right) \right)^{-1} * \left[\left(1 + \frac{(t - 1) L_F}{w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right)} \right) * \left(1 - \frac{t w'}{(\theta + 1) w} \frac{(\theta + 1) L_H w^{\theta+1} + (2\theta + 1) L_F \left(\frac{c_H^M \tau_{HF}}{c_F^M} \right)^\theta w^{2\theta+1}}{L_H w^{\theta+1} + L_F \left(\frac{c_H^M \tau_{HF}}{c_F^M} \right)^\theta w^{2\theta+1}} \right) - \frac{t L_F}{w \left(L_H + L_F \left(\frac{c_H^M \tau_{HF}}{c_F^M} \right)^\theta w^\theta \right)} + \frac{(t - 1) L_F}{w \left(L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta \right)} \frac{t w' L_H + (\theta + 1) L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta}{L_H + L_F (c_H^M \tau_{HF}/c_F^M)^\theta w^\theta} \right].$$

The first multipliers are positive so that to know the sign of z' , we need to look at the last term in squared brackets that can be re-written as

$$\begin{aligned} \text{sign}(z') &= \text{sign} \left[\frac{L_F}{w \left(L_H + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta \right)} \left(\frac{(t-1)tw'}{(\theta+1)w} \theta^2 \left(1 - \frac{L_H}{L_H + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta} \right) - 1 \right) \right. \\ &\quad \left. + 1 - \frac{tw'}{(\theta+1)w} \frac{(\theta+1)L_H w^{\theta+1} + (2\theta+1)L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^{2\theta+1}}{L_H w^{\theta+1} + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^{2\theta+1}} \right]. \end{aligned}$$

Note that the $(FE)_H$ condition can be written as $\tilde{r}_{HH} + \tilde{r}_{HF} = w f_e(\theta+1)$, so that from (C.1),

$$\begin{aligned} \tilde{r}_{HF} / \tilde{r}_{HH} &= L_F (c_H^M / \tau_{FH} c_F^M)^\theta w^\theta / L_H t^{\theta+1}, \quad \text{and from (C.5)} \\ \left[\frac{tw'}{(\theta+1)w} \right]^{-1} &= (\theta+1) + \left(1 + \theta (c_H^M \tau_{HF} / c_F^M)^\theta w^{\theta+1} \right) \frac{\tilde{r}_{HF}}{\tilde{r}_{HH}}. \end{aligned} \quad (\text{C.7})$$

Moreover, from (C.4),

$$\frac{L_F}{w \left(L_H + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta \right)} = \frac{L_F (c_H^M / \tau_{FH} c_F^M)^\theta w^\theta}{t^{\theta+1} L_H + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta} = \frac{\tilde{r}_{HF}}{\tilde{r}_{HH} + \tilde{r}_{HF}}.$$

Finally,

$$\frac{(\theta+1)L_H w^{\theta+1} + (2\theta+1)L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^{2\theta+1}}{L_H w^{\theta+1} + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^{2\theta+1}} = (\theta+1) + \theta w^{\theta+1} (c_H^M \tau_{HF} / c_F^M)^\theta \frac{\tilde{r}_{HF}}{\tilde{r}_{HH} + \tilde{r}_{HF}}.$$

Using all the relationships together, we get

$$\begin{aligned} \text{sign}(z') &= \text{sign} \left[\frac{\tilde{r}_{HF}}{\tilde{r}_{HH} + \tilde{r}_{HF}} \frac{tw'}{(\theta+1)w} (t-1)\theta^2 \left(1 - \frac{L_H}{L_H + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta} \right) + \frac{\tilde{r}_{HF}}{\tilde{r}_{HH} + \tilde{r}_{HF}} \right. \\ &\quad \left. - \frac{tw'}{(\theta+1)w} \left((\theta+1) + \theta w^{\theta+1} (c_H^M \tau_{HF} / c_F^M)^\theta \frac{\tilde{r}_{HF}}{\tilde{r}_{HH} + \tilde{r}_{HF}} \right) \right] \\ &= \text{sign} \left[\frac{\tilde{r}_{HF}}{\tilde{r}_{HH} + \tilde{r}_{HF}} \frac{tw'}{(\theta+1)w} \left((t-1)\theta^2 \frac{L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta}{L_H + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta} - (\theta+1) \frac{\tilde{r}_{HH}}{\tilde{r}_{HF}} + \right. \right. \\ &\quad \left. \left. + 1 + \theta (c_H^M \tau_{HF} / c_F^M)^\theta w^{\theta+1} - (\theta+1) \left(\frac{\tilde{r}_{HH}}{\tilde{r}_{HF}} + 1 \right) - \theta w^{\theta+1} (c_H^M \tau_{HF} / c_F^M)^\theta \right) \right]. \end{aligned}$$

By noting that $w' > 0$ and rearranging the term in brackets, we have

$$\text{sign}(z') = \text{sign} \left[\left((t-1)\theta^2 \frac{L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta}{L_H + L_F (c_H^M \tau_{HF} / c_F^M)^\theta w^\theta} - \theta \right) \right].$$

Thus, $dz/dt \gtrless 0$ for $t \gtrless t^{opt} \equiv 1 + \frac{1}{\theta} + \frac{1}{\theta} \frac{L_H}{L_F} \left(\frac{c_F^M}{c_H^M \tau_{HF}} \right)^\theta w^{-\theta}$. Given (C.5), w rises with t , meaning that t^{opt} is unique. Finally, recall that as z falls, welfare rises. Hence, we proved Proposition 1.

Appendix D: No Metzler Paradox

Given the Pareto cost distribution assumption, the average price in country i can be written as $\bar{p}_i = (2\theta + 1) w_i c_{ii}^* / 2(\theta + 1)$. First, let us look at $w_H c_{HH}^* = w c_{HH}^*$. From (C.5) and (C.6),

$$(w c_{HH}^*)' = w c_{HH}^* \left[-\frac{z'}{(\theta + 1)z} + \frac{1}{t} \left(1 - \frac{\theta L_F (\tau_{HF} c_H^M / c_F^M)^\theta w^{2\theta+1}}{(\theta + 1) (L_F (\tau_{HF} c_H^M / c_F^M)^\theta w^{2\theta+1} + L_H w^{\theta+1})} t \frac{w'}{w} \right) \right],$$

where for $t \in [1, t^{opt}]$, $z'/z < 0$, $0 < tw'/w < 1$, and the multiplier in front of tw'_H/w is positive and less than 1. Hence, we get

$$\frac{d\bar{p}_H}{dt} = \frac{2\theta + 1}{2(\theta + 1)} \frac{d(w c_{HH}^*)}{dt} > 0,$$

i.e., the average price at Home rises with t . Next, let us look at c_{FF}^* . From the (TB) condition,

$$\lambda_F (c_{FF}^*)^{\theta+2} = (c_{FF}^*)^\theta \left(\alpha_{FF}^* - \frac{\theta + 2}{\theta + 1} \eta \right) = \frac{w^{2\theta+1}}{t^{\theta+1}} w \lambda_H (c_{HH}^*)^{\theta+2} \left(\frac{\tau_{HF} c_H^M}{\tau_{FH} c_F^M} \right)^\theta,$$

where the left-hand side is monotonically increasing in c_{FF}^* . From the $(FE)_F$ condition,

$$\frac{w^{2\theta+1}}{t^{\theta+1}} w \lambda_H (c_{HH}^*)^{\theta+2} \left(\frac{\tau_{HF} c_H^M}{\tau_{FH} c_F^M} \right)^\theta \left[L_F + L_F \left(\frac{\tau_{HF} c_H^M}{\tau_{FH} c_F^M} \right)^{-\theta} (\tau_{FH})^{-\theta} \frac{1}{w^\theta} \right] = Const_1 (c_F^M)^\theta.$$

Given that w rises with t , the term in the squared brackets above falls, so that the multiplier in front of these brackets has to rise, meaning that c_D^* rises too. Thus, we have

$$\frac{d\bar{p}_F}{dt} = \frac{2\theta + 1}{2(\theta + 1)} \frac{d(c_{FF}^*)}{dt} > 0,$$

so that the average prices everywhere rise with an increase in t , implying no Metzler paradox.

Appendix E: Role of Variable Markups

A rise in misallocation distortion in the case of falling trade costs. Here we show that the Home country's covariance introduced in Section 4 is negative for a marginal fall in τ_{FH} . First, note that $dq_{Hi}(c) = d(L_i w_H \lambda_i (c_{Hi}^* - c) / 2\gamma) = L_i [d(w_H \lambda_i c_{Hi}) - cd(w_H \lambda_i)] / 2\gamma$, $i = H, F$, so that using the Pareto cost distribution assumption, we get (here we use $d\tau_{HF} = 0$),

$$\begin{aligned} & \frac{M_{Hi}}{w_H L_H} \int_0^{c_{Hi}^*} p_{Hi}(c) d[q_{Hi}(c)] \frac{dG_H(c)}{G_H(c_{Hi}^*)} = \frac{1}{4\gamma w_H L_H} \frac{M_{Hi} L_F}{(\theta + 1)(\theta + 2)} \\ & * \left[(2\theta + 1)(\theta + 2) (\tau_{Hi})^2 c_{Hi} d(w_H \lambda_i c_{Hi}) - (2\theta^2 + 3\theta) (\tau_{Hi} c_{Hi}^*)^2 d(w_H \lambda_i) \right]. \end{aligned}$$

Hence, the expression for the covariance of markups and labor share changes is

$$\begin{aligned} \text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_H} \right) &= \sum_i \frac{M_{Hi}}{w_H L_H} \int_0^{c_{Hi}^*} p_{Hi}(c) d[q_{Hi}(c)] \frac{dG_H(c)}{G_H(c_{Hi}^*)} \\ &= \frac{(2\theta + 1)}{4\gamma w_H L_H (\theta + 1)} \left[M_{HH} L_H c_{HH}^* d(w_H \lambda_H c_{HH}^*) + M_{HF} L_F (\tau_{HF})^2 c_{HF}^* d(w_H \lambda_F c_{HF}^*) \right] \\ &\quad - \frac{(2\theta^2 + 3\theta)}{4\gamma w_H L_H (\theta + 1) (\theta + 2)} \left[M_{HH} L_H (c_{HH}^*)^2 d(w_H \lambda_H) + M_{HF} L_F (\tau_{HF} c_{HF}^*)^2 d(w_H \lambda_F) \right]. \end{aligned}$$

The total labor size at Home does not change. Hence, $\sum_i \int_{\omega \in \Omega_{Hi}} [dl(\omega, i) / L_H] d\omega = 0$, or

$$\begin{aligned} M_{HH} \int_0^{c_{HH}^*} d[cq_{HH}(c)] \frac{dG_H(c)}{G_H(c_{HH}^*)} + M_{HF} \int_0^{c_{HF}^*} d[\tau_{HF} c q_{HF}(c)] \frac{dG_H(c)}{G_H(c_{HF}^*)} &= 0, \quad \text{or} \\ \frac{\theta}{2\gamma (\theta + 1) (\theta + 2)} \sum_i M_{Hi} L_i \left[(\theta + 2) (\tau_{Hi})^2 c_{Hi}^* d(w_H \lambda_i c_{Hi}^*) - (\theta + 1) (\tau_{Hi} c_{Hi}^*)^2 d(w_H \lambda_i) \right] &= 0. \end{aligned}$$

Multiplying the expression above by $2\gamma(2\theta + 3)$ and rearranging the terms result in

$$\begin{aligned} &\frac{2\theta + 3}{\theta + 2} \left[M_{HH} L_H c_{HH}^* d(w_H \lambda_H) + M_{HF} L_F (\tau_{HF} c_{HF}^*)^2 d(w_H \lambda_F) \right] \\ &= \frac{2\theta + 3}{\theta + 1} \left[M_{HH} L_H c_{HH}^* d(w_H \lambda_H c_{HH}^*) + M_{HF} L_F (\tau_{HF})^2 c_{HF}^* d(w_H \lambda_F c_{HF}^*) \right]. \end{aligned}$$

We can use this equation in the expression for the covariance to get:

$$\text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_H} \right) = \frac{M_{HH} L_H c_{HH}^* d(w_H \lambda_H c_{HH}^*) + M_{HF} L_F (\tau_{HF})^2 c_{HF}^* d(w_H \lambda_F c_{HF}^*)}{4\gamma w_H L_H (\theta + 1)^2}.$$

Finally, note that from (5) and (11), we get

$$\begin{aligned} d(w_H \lambda_i c_{Hi}^*) &= \frac{1}{\tau_{Hi}} \frac{\theta + 2}{\theta + 1} \eta \left(dc_{ii}^* / (c_{ii}^*)^2 \right), \quad \text{and} \\ \text{cov} \left(m(\omega, i), \frac{dl(\omega, i)}{L_H} \right) &= \frac{(\theta + 2) \eta}{4\gamma w_H L_H (\theta + 1)^3} \left[M_{HH} L_H \frac{dc_{HH}^*}{c_{HH}^*} + M_{HF} L_F \frac{dc_{FF}^*}{w_H c_{FF}^*} \right]. \end{aligned}$$

From Proposition 1, c_{HH}^* and c_{FF}^* fall as τ_{FH} falls, so the covariance is negative for $d\tau_{HF} < 0$.

A fall in the average markup in the case of a rising import tariff. By definition,

$$\bar{m}_H = \frac{1}{2} \frac{2\theta - 1}{\theta - 1} \frac{M + tM_x^*}{M + M_x^*} = \frac{1}{2} \frac{2\theta - 1}{\theta - 1} \frac{t^{\theta+1} + w^\theta}{t^{\theta+1} + tw^\theta}, \quad \text{so that}$$

$$\frac{d\bar{m}_H}{dt} = \frac{2\theta - 1}{2(\theta - 1)(t^{\theta+1} + tw^\theta)^2} \left[(1 - t) \theta w^{\theta-1} t^{\theta+1} \frac{dw}{dt} - w^{2\theta} - \theta t^\theta w^\theta \left(1 + \frac{1}{\theta} - t \right) \right].$$

When t starts to rise from 1 (consider t between 1 and $1 + 1/\theta$), w rises too (see (C.5)), so that for small increases in t we get $d\bar{m}_H/dt < 0$.