# Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries\*

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#### Abstract

Firm dynamics in poor countries show striking differences to those of rich countries. While few firms indeed experience growth as they age, most firms are simply stagnant in that they neither exit nor expand. We interpret this fact as a lack of selection, whereby producers with little growth potential survive because innovative entrepreneurs do not expand enough to force them out of the market. To explain these differences, we develop a theory whereby firms require managerial inputs for production and countries differ in their managerial delegation possibilities. If delegation of managerial tasks to outside managers is difficult in poor countries, entrepreneurs are forced to rely on their own time to supply managerial services. Improvements in the efficiency of delegation will raise the returns to growing large, induce innovative firms to expand, and thereby force stagnant entrepreneurs out of the market. We prove the existence and uniqueness of the dynamic equilibrium and show analytically how the degree of selection depends on some of the key structural parameters. To discipline the quantitative importance of this mechanism, we calibrate our model to micro data from the US and India. Differences in the efficiency of managerial delegation can explain an important fraction of the differences in plants' life-cycles.

**Keywords:** Development, growth, selection, competition, firm dynamics, management, entrepreneurship, creative destruction.

JEL classification: O31, O38, O40

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## 1 Introduction

The process of firm dynamics differs vastly across countries. While firms in rich countries experience rapid growth conditional on survival, firms in poor countries remain small and do not grow as they age. Hsieh and Klenow (2014), for example, show that the average firm in the US has grown by a factor of four by the time it is 30 years old. In contrast, firms in India see very little growth as they age, making 30-year-old firms barely bigger than new entrants. A salient regularity underlying these aggregate numbers is the plethora of tiny producers in developing economies that are entirely stagnant. A case in point is the Indian manufacturing sector, where 77% of the entering firms have at most two workers. More strikingly, this stock of micro-firms is essentially independent of age, i.e., not only are most entering firms in India small, but even for firms at age 30, 73% have never grown beyond the size of two workers. This is very different in developed economies like the US, which are characterized by a pronounced "up-or-out" phenomenon, where firms also enter small but then either exit or expand. This process of selection, whereby stagnant producers are replaced quickly, is all but missing in poor countries like India. In this paper we provide a theory and a quantitative analysis for the reasons behind such lack of selection.

Our theory rests on two premises. First we consider managerial services as being an important input in production but allow countries to differ in the efficiency with which managerial tasks can be delegated to outside managers. That firm owners in developing countries indeed suffer from a lack of managerial delegation was recently argued in a series of papers. Bloom et al. (2013), for example, argue that textile firms in India are severely constrained in their managerial resources, which prevents them from expanding. In particular, they show that the delegation of decision rights hardly extends to managers outside the family and that the number of male family members is the dominant predictor of firm size. Using data across countries, there is also evidence that managerial practices differ across countries (Bloom and Van Reenen, 2007, 2010), that firms in developed countries are both larger and delegate more managerial tasks to outside (non-family) managers (Fukuyama, 1996; La Porta et al., 1997; Bloom et al., 2012) and that both human capital and contractual imperfections are important in explaining the lack of managerial delegation in poor countries (Laeven and Woodruff, 2007; Bloom et al., 2009).

Second, we take seriously the idea that not all firms are destined to grow. While some entrepreneurs have the necessary skills to expand, others might lack the ability to grow their firms beyond a certain size. This idea of heterogeneity in entrepreneurial types and its impact on firm growth has been subject to a growing empirical literature. Hurst and Pugsley (2012) show that there are heterogeneous types of entrepreneurs in the US economy, a majority of whom intentionally choose to remain small. Similar findings are reported in De Mel et al. (2008), Schoar (2010) and Decker et al. (2014), who classify entrepreneurs as transformative versus subsistence entrepreneurs, where the former ones operate with the intention to grow and the latter ones lack the ability to grow

<sup>&</sup>lt;sup>1</sup>Strictly speaking, this evidence stems from the analysis of plant-level (instead of firm-level) data. While we will refer to producers as firms in the theory, we will look at both firm- and plant-level data in our quantitative analysis.

and Shleifer (2008, 2014) provide evidence of a duality between small, informal and large, formal firms and also argue that the "decline of informality is the result of a replacement of inefficient informal firms by efficient formal ones" (La Porta and Shleifer, 2014, p. 121). Hence, while stagnant firms exist in all countries, they are of limited aggregate importance in developed economies as they get replaced quickly. The abundance of small, subsistence producers in poor countries might therefore be a reflection of the fact that other, transformative firms with growth potential are not expanding enough to replace them quickly.

To formally study the interaction of these two ingredients and their quantitative importance, we construct a firm-based model of endogenous growth in the spirit of Klette and Kortum (2004). To meaningfully speak about selection, we explicitly assume that firms are heterogenous in their ability to expand. While able transformative entrepreneurs have the necessary skills to grow their business at the expense of other producers, stagnant subsistence entrepreneurs lack such skills and only survive as long as they are not driven out of the market by their more dynamic competitors. To understand why this process of selection is lacking in an economy like India, one therefore has to have a theory for why the returns to expansion for able entrepreneurs might be low.

To this end we introduce an explicit need for managerial delegation. If workers require managerial oversight but the entrepreneur's time to provide such managerial services herself is limited, entrepreneurs need to delegate decision power to outside managers as their firms expand. Without delegation, entrepreneurs of expanding firms have to divide their managerial attention across more and more tasks and hence run into a span of control problems in the spirit of Lucas (1978). The returns to firm growth are therefore crucially dependent on the possibilities of managerial delegation in the economy. If, for instance, managerial human capital is scarce, able entrepreneurs have no incentives to expand as they anticipate not being able to delegate decision power to managers with appropriate skills. Similarly, if a country suffers from contractual frictions due to an underdeveloped legal system, firm owners' desire to expand is limited as outside managers cannot be incentivized appropriately. At the aggregate level, this will reduce the degree of selection, as innovative firms do not expand enough and stagnant entrepreneurs are not replaced sufficiently quickly.

Our model makes tight predictions about how the quality of the delegation environment affects the process of firm dynamics, the resulting equilibrium firm-size distribution and the degree of selection. The model predicts a threshold firm size, below which firms are run only by their owners. As long as firms do not delegate decision rights, firm profits have decreasing returns in firm size as the entrepreneur's time is a fixed factor in production. This implies that growth incentives are declining in size. Once the marginal value of delegation is sufficiently high, firms start to hire outside managers to overcome the decreasing returns. In fact, the model predicts that beyond a certain firm-size threshold, the firm's value function becomes linear, the slope of which is fully determined by the countries' delegation possibilities.

Because the model has an analytic solution, we can derive a set of concise comparative statics results. In particular, increases in the returns to delegation, which could for instance stem from

an increase in managerial human capital or an improvement in the contractual environment, will affect firms asymmetrically. As stagnant, subsistence firms tend to be small and hence do not rely on outside managers, they do not respond to changes in the delegation environment. In contrast, transformative firms, which have high growth potential, will now be able to acquire more managerial services and hence have higher incentives to expand. Improvements in the delegation environment will therefore raise average firm size, induce more selection whereby stagnant firms are wiped out quickly, make the life-cycle of firms steeper and reduce the number of firms running their business without managerial personnel. These predictions are qualitatively consistent with the micro-level evidence for firms in India and other developing economies.

To analyze the importance of this mechanism quantitatively, we then turn to the firm-level data from the manufacturing sector in the US and India. Our theoretical model has the convenient feature that the delegation environment is summarized by a single structural parameter, which we call the delegation efficiency. Because we can calibrate this parameter directly to the data, our quantitative exercise does not have to take any stand on whether the high returns to delegation in the US originate from better managerial human capital or a more developed contractual environment. In contrast, we calibrate the state of the delegation environment directly by using data on managerial hiring in both the US and India. We then use the calibrated model to quantify the extent to which cross-country differences in the delegation environment can explain the variation in the implied processes of firm dynamics. Our analysis suggests that if US firms only had access to the under-developed delegation environment in India, the gap in life-cycle growth between Indian and US firms would have been reduced by 30% to 40% depending on the firm's age. This is due to two effects: Not only do the surviving firms expand at a slower pace with less efficient delegation possibilities, but stagnant firms also exit much less frequently. Both of these margins generate a shallow life-cycle profile. Interestingly, there are important complementarities between the returns to delegation and other differences between the US and India. While a decline in delegation efficiency in the US reduces firm growth substantially, the opposite is less pronounced: If Indian firms were able to hire managers as seamlessly as firms in the US, the implications for the resulting life-cycle are much more modest. The reason is that, according to our estimates, the share of innovative firms in India is low to begin with and that their expansion technology is relatively inefficient. Improvements in the delegation environment therefore have less of an effect in the aggregate.

We then consider various extensions to our analysis. First, we try to dig deeper into the fundamental determinants of the delegation environment. Using data on managerial employment shares, human capital, the quality of legal institutions, and the degree of financial development for almost 50 countries of the world, we further decompose the life-cycle differences that are due to the variation in delegation environment between the US and India into three components. We find that while 54% of the differences can be explained by the difference in human capital, 41% is accounted for by better legal institutions in the US. The observed variation in the state of financial development is responsible for the remaining 5%. Second, we also conduct a number of robustness

and counterfactual exercises to show in what sense mechanisms other than the delegation margin highlighted in our theory would have failed in explaining the observed life-cycle dynamics.

Related Literature On the theoretical side, this paper provides a new theory of firm dynamics in developing countries.<sup>2</sup> While many recent papers have aimed to measure and explain the static differences in allocative efficiency across firms,<sup>3</sup> there has been little theoretical work explaining why firm dynamics differ so much across countries. A notable exception is the work by Cole et al. (2012), who argue that cross-country differences in the financial system affect the type of technologies that can be implemented. Like them, we let the productivity process take center stage. However, we turn to the recent generation of micro-founded models of growth, following Klette and Kortum (2004), who have been shown to provide a tractable and empirically successful theory of firm dynamics (Lentz and Mortensen (2005, 2008), Akcigit and Kerr (2010), Atkeson and Burstein (2014), Garcia-Macia et al. (2015)). In the model, firm dynamics are determined through creative destruction à la Aghion and Howitt (1992), whereby successful firms expand through replacing other producers.<sup>4</sup> Importantly, we explicitly allow for heterogeneity in firms' growth potential, which is not only required to empirically match the micro data but also at the heart of our selection mechanism (Acemoglu et al. (2013); Lentz and Mortensen (2015)).<sup>5</sup>

Our theory is consistent with the dual economy view of economic development. As stressed in La Porta and Shleifer (2014), the vast majority of small, informal firms in poor countries do not seem to transition to formality, but get replaced as the economy develops. That managerial delegation might be a key aspect to this process goes back to the early work of Penrose (1959), who argues not only that managerial resources are essential for firms to expand but that this scarcity of managerial inputs prevents the weeding out of small firms as "the bigger firms have not got around to mopping them up" (Penrose, 1959, p. 221). The importance of managerial and entrepreneurial human capital for economic development is also stressed in the empirical work of Gennaioli et al. (2013).

We focus on inefficiencies in the interaction between managers and owners of firms to explain the differences in firms' demand for expansion. Caselli and Gennaioli (2013) also stress the negative consequences of inefficient management, but focus on static misallocation on the market for control, whereby (untalented) firm owners might not be able to sell their firms to (talented) outsiders. We, in contrast, argue that managerial frictions within the firm reduce growth incentives and hence prevent competition from taking place sufficiently quickly on product markets. Powell (2012), Bertrand and

<sup>&</sup>lt;sup>2</sup>An overview of some regularities of the firm size distributions in India, Indonesia and Mexico is contained in Hsieh and Olken (2014).

<sup>&</sup>lt;sup>3</sup>The seminal papers for the recent literature on misallocation are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). As far as theories are concerned, there is now a sizable literature on credit market frictions (Buera et al. (2011); Moll (2010); Midrigan and Xu (2010)), size-dependent policies (Guner et al. (2008)), monopolistic market power (Peters (2013)) and adjustment costs (Collard-Wexler et al. (2011)). A synthesis of the literature is also contained in Hopenhayn (2012) and Jones (2013).

<sup>&</sup>lt;sup>4</sup>See Aghion et al. (2014) for a survey of the Schumpeterian growth literature.

<sup>&</sup>lt;sup>5</sup>Experimental evidence on such persistent differences in growth potential within the context of developing countries is also found in Banerjee et al. (2015).

Schoar (2006), Chen (2014) and Grobvosek (2015) study within-firm considerations where firms ("owners") need to hire managers subject to contractual frictions. In contrast to our theory, all these papers assume that firm productivity is constant, i.e., there is no interaction between the delegation environment and firms' endogenous growth incentives. Guner et al. (2015) and Roys and Seshadri (2014) present recent dynamic models of (managerial) human capital accumulation and economic development. In contrast to us, they do not focus on the implications of creative destruction for the resulting process of selection and firm dynamics. Finally, there is a large literature on management and the hierarchical structure of the internal organization of the firm; see Garicano and Rossi-Hansberg (2015) for a survey. This literature has a much richer microstructure of firms' delegation environment, but does not focus on the link between cross-country differences in the ease of delegation and the resulting properties of firm dynamics.

The remainder of the paper is organized as follows. In Section 2 we describe the theoretical model, where we explicitly derive the interaction between firms' delegation decisions and their endogenous growth incentives. We then take the model to the data. In Section 3 we calibrate the model to the US and Indian micro data. Section 4 contains our main counterfactual exercises to link the delegation environment to firms' life-cycle. In Section 5 we use cross-country data on the aggregate importance of managerial employees, their human capital, a country's the quality of legal institutions and its financial development to decompose the cross-country variation in the delegation environment into fundamental factors. Section 6 concludes. All proofs are contained the Appendix. An Online Appendix, which is available on our websites, contains further robustness checks and additional results.

# 2 Theory

We consider a model of firm dynamics in the spirit of Klette and Kortum (2004). This framework is a natural starting point in that it offers a tractable formalization of the firm-level growth process, which we can take to the micro data. In particular, it puts the role of selection and creative destruction at center stage. Firms spend resources to expand by stealing market share of their competitors. Producers that are unsuccessful in growing are being replaced by their more innovative rivals and leave the economy. Hence, the degree of selection is fully endogenous: Firms only shrink and exit if other producers expand. We augment this framework with two ingredients: (i) we assume that entrepreneurs are heterogeneous in their growth potential, and (ii) we explicitly allow for managerial services as an input in production and let entrepreneurs choose whether they want to delegate managerial tasks to outside managers to augment their own managerial time.

It is the interaction of these two ingredients in which we are primarily interested. The first item allows us to meaningfully speak about a process of selection, whereby high type-firms replace producers with little growth potential. The second item gives a specific role for managerial delegation to determine firms' expansion incentives and hence how quickly this process takes place. As we are mostly interested in the process of firm dynamics, we keep the demand side of the economy as

simple as possible by assuming a representative household with standard preferences.<sup>6</sup>

**Technology** We consider a closed economy, where a final good is a Cobb-Douglas composite of a measure one of intermediate goods and produced under perfect competition. Specifically,

$$ln Y_t = \int_0^1 \ln y_{jt} dj, \tag{1}$$

where  $y_{jt}$  is the amount of product j produced at time t. To save notation we will drop the time subscript t whenever it does not cause any confusion.

The production of intermediate goods is conducted by heterogeneous firms. Firms differ in the productivity with which they can produce different intermediate products. Because firms' outputs within product line j are perfect substitutes, each product is produced by a single firm f, which is the most efficient producer and we will refer to this firm as the producer of product j. Production requires both production workers and managers. In particular, firm f can produce good j according to

$$y_{jf} = q_{jf}\mu\left(e_{jf}\right)l_{jf},\tag{2}$$

where  $q_{jf}$  is the firm-product specific efficiency,  $l_{jf}$  is the number of workers employed for producing intermediate good j,  $e_{jf}$  denotes the amount of managerial services firm f allocates toward the production of good j and  $\mu(e_{jf}) \geq 1$  is an increasing function translating managerial services into productivity units.<sup>7</sup>

Technologies used for intermediate good production in (2) become obsolete through the introduction of newer and better technologies. In this economy, firms are defined as collections of leading-edge technologies. Figure 1 illustrates examples of two firms in the economy.

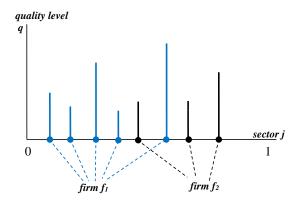


FIGURE 1: DEFINITION OF FIRMS

Notes: This figure depicts two examples of firms in this economy. While firm one owns five leading-edge technologies, firm two has three leading-edge technologies in its portfolio.

<sup>&</sup>lt;sup>6</sup>See Appendix A.1 for details.

<sup>&</sup>lt;sup>7</sup>We will specify the provision of managerial services below.

In this example, firm  $f_1$  owns five leading-edge technologies and  $f_2$  owns three. Firms can expand into a new market j' by introducing better versions of the technology of what the current incumbent in j' uses. We will describe the process of firm growth in Section 2.2 in more detail, when we turn to the dynamics. Before doing so, we now describe the static allocations, which determine firms' profits and hence their incentives to expand.

## 2.1 Static Production Decision and Demand for Managers

In this paper we focus on the interaction between managerial delegation and its impact on firm dynamics and the resulting endogenous selection process. Therefore we keep the static market structure as tractable as possible by assuming that there is a competitive fringe of potential producers that have access to the same technology as the leading firm and to a level of managerial services  $\mu^{fringe}$ , which we normalize to unity. Because the market leader faces a demand function with unitary elasticity, it will set its price equal to the marginal costs of its competitor, which is  $\frac{w}{q_j}$ , where w is the equilibrium wage. Letting e be the amount of managerial services employed by firm f for its own product f, total profits after paying for production workers  $l_f$  are given by

$$\pi_j(e) = p_j y_j - w l_j = \left(\frac{\mu(e) - 1}{\mu(e)}\right) p_j y_j = \left(\frac{\mu(e) - 1}{\mu(e)}\right) Y.$$
 (3)

Expression (3) stresses that firm f's profits in product line j depend only on the amount of managerial services that it allocates toward the production of product j. Intuitively, because managerial inputs increase physical productivity, more managerial inputs allow firms to sustain higher mark-ups over competing firms. Crucially, because firms' incentives to expand into new product lines are governed by the profits they will be able to earn, (3) shows that such incentives are determined by the ease with which firms expect to be able to acquire managerial inputs.

For analytical convenience, we are going to assume that  $\mu(e) = \frac{1}{1-e^{\sigma}}$ , where  $e \in [0,1)$  and  $\sigma < 1$ . This implies that firm f's profit in product line j is given by

$$\pi(e_j) = e_j^{\sigma} Y, \tag{4}$$

i.e., profits are a simple power function of managerial effort parameterized by the elasticity  $\sigma$ . Note that if the producing firm has no managerial services at its disposal, it is unable to outcompete the competitive fringe as  $\mu(0) = \mu^{fringe} = 1$  and  $\pi_j = 0$ .

Managerial services e can be provided both by the entrepreneur herself and outside managers. In particular, we assume that there is a measure 1 of individuals who can work either as production workers or outside managers. Letting the number of production workers and managers hired by firm f be  $l_{ft}$  and  $m_{ft}$ , respectively, labor market clearing requires that

$$1 = \int_{f \in F_t} (l_f(w_t) + m_f(w_t)) df, \tag{5}$$

where  $F_t$  is the total mass of producing firms. As we show in Appendix A.1, the above environment implies that aggregate output in this economy is given by

$$Y_t = Q_t \mathcal{M}_t L_t^D, \tag{6}$$

where  $L_t^D = \int_{f \in F_t} l_{ft} df$  denotes the amount of production workers,  $Q_t$  is the usual Cobb-Douglas composite of individual efficiencies

$$\ln Q_t \equiv \int_0^1 \ln q_{jt} dj,$$

and  $\mathcal{M}_t$  is an endogenous TFP term based on managerial effort, which summarizes the aggregate effects of the distribution of mark-ups in this economy. In particular,

$$\mathcal{M}_{t} = \left[ \int_{j=0}^{1} [1 - (e_{jt})^{\sigma}] dj \right]^{-1}.$$
 (7)

Furthermore, the equilibrium wage is given by  $w_t = Q_t$ .

Managerial Services and Delegation At the heart of our theory is the allocation of managerial services  $e_j$  as these determine profitability and therefore firms' incentives to grow. We assume that managerial services can be provided by the entrepreneur herself and by outside managers. In particular, we assume that each entrepreneur has a fixed endowment of managerial time T > 0, which she provides inelastically as managerial services into her firm. Empirically, T can for example represent both the entrepreneur's human capital and - in the context of developing economies - the size of the entrepreneur's family. If an entrepreneur owns a firm with n production units and decides to run her firm alone, then she will have  $e_j = T/n$  units of managerial services per production unit.<sup>8</sup> Equation (4) then directly implies that the normalized profit from each product line is  $\tilde{\pi}_j \equiv \pi_j/Y = (T/n)^{\sigma}$ . Hence, the normalized value of a firm run by its owner without any outside managers is simply given by

$$\tilde{V}^{self}(n) \equiv V^{self}(n)/Y = n\tilde{\pi}_j = n \times (T/n)^{\sigma} = T^{\sigma} n^{1-\sigma}.$$
 (8)

This expression has a simple but important implication: While the value of a firm is increasing in the number of products n, it does so at a decreasing rate. This is because the owner has a fixed time endowment T and runs into span of control problem as in Lucas (1978). This concavity of  $\tilde{V}^{self}(n)$  has important dynamic ramifications for firms' expansion incentives: As marginal returns are decreasing in n, the incentives to grow and to break into new markets decline as the size of the firm increases. To prevent this scarcity of managerial services from being a drag on growth incentives, the entrepreneur can decide to bring outside managers into the firm. It is this process that we refer to as delegation.

<sup>&</sup>lt;sup>8</sup>That she will want to spread her T units of managerial time equally across all product lines follows directly from the concavity of  $\pi$  in (4).

The cost of hiring an outside manager is the wage rate w. The benefit is that outside managers will add to the firm's endowment of available managerial time, which translates into higher profits. The demand for outside managers therefore stems from the net increase in managerial resources they bring to the firm. In particular, suppose that each outside manager adds  $\xi$  units of managerial services to the firm. If an owner of a firm with n product lines hires  $m_j$  managers per product line j, the amount of managerial services per line j is then given by

$$e_j = T/n + \xi \times m_j$$
.

The parameter  $\xi$ , which we refer to as the delegation efficiency, is a country-specific parameter, and can depend on various fundamentals. First,  $\xi$  can depend on the human capital of outside managers. Hence, the delegation efficiency might be low, simply because potential managers do not have the necessary skills to organize the production process efficiently. While rich countries might have plenty of people who are able to supervise dozens of production workers and deal with multiple input suppliers, such skills might be scarce in poor countries. Second,  $\xi$  could depend on the contractual environment. If contractual imperfections are severe, entrepreneurs might need to spend substantial amounts of their own time monitoring their managerial personnel. This reduces the net time gain each outside manager adds to the firm. Third,  $\xi$  could capture cultural factors like trust or social norms, which facilitate the delegation of decision power. Finally,  $\xi$  might depend on the level of financial development since more developed financial markets might give the entrepreneur the opportunity to incentivize her managers better. At this point, we are agnostic about the exact determinants of  $\xi$ . We will rather take it as a country-specific parameter and calibrate it directly within our model. It is only in Section 5 that we will dig deeper into its determinants and use cross-country data on human capital and contractual institutions to decompose it into various components.

While our analysis does not need to take a stand on how  $\xi$  depends on particular country characteristics, the following simple example of a micro-founded contractual game might be useful to fix ideas. None of the results in this paper depend on the exact details of this example. Those readers who are not interested in this particular micro-foundation can therefore take  $\xi$  as given and skip to the section "Optimal Delegation" below.

Example 1 (A Toy Example of Managerial Delegation) Suppose that both managers and entrepreneurs each have one unit of time at their disposal. While the latter can provide T units of effort during that time interval, managers can provide  $\eta$  units of effort. Hence,  $\eta$  can be thought of as a measure of the human capital of outside managers. Suppose that the provision of managerial effort is subject to contractual frictions. For simplicity, assume that the manager can decide to either provide effort, in which case his contribution to the firm's managerial services is  $\eta$ , or shirk, in which case he adds no usable services to the firm.

While the manager's effort choice is not contractible, the entrepreneur can monitor the manager to prevent him from shirking. If the entrepreneur spends s units of her time monitoring the manager, she will catch a shirking manager with probability s. Whenever the manager shirks and gets caught, the entrepreneur can go to court and sue the manager for the managerial wage w. In particular, the court (rightly) decides in the entrepreneur's favor with probability  $\kappa$ . Hence one can think of  $\kappa$  as parameterizing the efficiency of the legal system. Finally, the demand for shirking arises because shirking carries a private benefit bw, where b < 1.9

It is straightforward to characterize the equilibrium of this simple game. If the entrepreneur spends s units of her time monitoring the manager, the manager does not shirk if and only if

$$w \ge bw + w(1 - \kappa s)$$
,

where  $(1 - \kappa s)$  is the probability that the manager gets paid despite having shirked. Clearly the owner will never employ a manager without inducing effort. Hence, the owner will spend  $s = b/\kappa$  units of time monitoring the manager. The overall amount of managerial services in product line j is therefore given by  $b^{10}$ 

$$e_{j} = \frac{T}{n} - m_{j}s + m_{j}\eta = \frac{T}{n} + \left(\eta - \frac{b}{\kappa}\right) \times m_{j} = \frac{T}{n} + \xi\left(\kappa, \eta\right) \times m_{j}. \tag{9}$$

Hence,  $\xi$  measures precisely the net increase in managerial services through delegation. In particular, the delegation efficiency is increasing in managerial human capital  $\eta$  and in the state of the contractual environment  $\kappa$ , because monitoring and the strength of the legal system are substitutes. Note also that the whole purpose of delegation is to increase a firm's managerial resources, so that firms will never hire a manager if  $\xi(\kappa,\eta) \leq 0$ . Hence, whenever managers are sufficiently unproductive or the quality of legal systems is sufficiently low, firms will never want to hire outside managers because owners need to spend more of their own time to prevent the opportunistic behavior of managers than they gain in return.

**Optimal Delegation** Given the delegation efficiency  $\xi$ , we can solve the entrepreneur's static delegation problem. Consider a firm with n products. The owner maximizes the total profits of the firm by choosing the optimal number of outside managers. Given (4), the owner therefore solves the following maximization problem:

$$V(n) = \sum_{j=1}^{n} \max_{m_j \ge 0} \left\{ \left( \frac{T}{n} + \xi m_j \right)^{\sigma} Y - w m_j \right\}.$$
 (10)

<sup>&</sup>lt;sup>9</sup>The necessity for the private benefit being proportional to the wage arises in order to make the contract stationary. <sup>10</sup>Note that we do not require that s < T, i.e., we do not require the owner to perform the monitoring himself. We rather think of managerial efficiency units to be perfect substitutes within the firm, i.e., an owner can hire a manager to monitor other managers.

This expression is intuitive. The owner allocates T/n units of her time on each product line. In addition, by hiring  $m_j$  outside managers for product j, she can obtain an overall managerial efficiency of  $T/n + m_j \xi$  but has to pay mw to managers. The optimal delegation decision is characterized in the following proposition.

**Proposition 1** Consider the maximization problem in (10) and let  $\omega \equiv w/Y$  be the normalized wage. Define the equilibrium managerial efficiency  $\psi$  as

$$\psi \equiv \left(\frac{\sigma\xi}{\omega}\right)^{\frac{1}{1-\sigma}}.\tag{11}$$

Then the following is true:

- 1. Small firms with size  $n < n^*(\psi) \equiv \frac{T}{\psi}$  do not hire any outside managers and are run only by their owners,
- 2. The demand for outside managers per product line is given by

$$m(n) = \frac{1}{\xi} \times \max\left\{0, \left(\psi - \frac{T}{n}\right)\right\},\tag{12}$$

3. The optimal amount of managerial services per product line e(n) is given by

$$e(n) = \max\left\{\frac{T}{n}, \psi\right\},\tag{13}$$

4. The normalized value of an n-product firm  $\tilde{V} = \frac{V}{Y}$  is given by

$$\tilde{V}(n) = \begin{cases} T^{\sigma} \times n^{1-\sigma} & \text{if } n < n^*(\psi) \\ T\left(\frac{1}{\psi}\right)^{1-\sigma} \sigma + (1-\sigma)\psi^{\sigma} n & \text{if } n \ge n^*(\psi) \end{cases}$$
(14)

**Proof.** Follows directly from the first-order condition of (10).

Proposition 1 characterizes the demand for outside managers, both on the intensive and the extensive margin. First, small firms do not hire outside managers. In particular, as long as the amount of internal managerial time per production unit T/n is large relative to the endogenous equilibrium efficiency  $\psi$ , firms are optimally run entirely by their owners. Note that the equilibrium efficiency  $\psi$  is increasing in delegation efficiency  $\xi$  and decreasing in the (normalized) wage  $\omega$ . Second, similar comparative statics hold for the intensive margin as well: For given factor prices  $\omega$ , conditional on hiring, ample managerial resources by the entrepreneur herself (T) will reduce the demand for outside managers and a larger delegation efficiency  $(\xi)$  will induce firms to delegate more. Note also that larger firms hire more managers conditional on hiring: As T is a fixed factor, larger firms will have a higher demand for managerial effort per production unit, simply because

less and less of the entrepreneur's own time can be allocated to each individual production unit, which increases the marginal product of outside managers. Finally (13) directly focuses on the endogenous choice of managerial services  $e_j$  and summarizes the economic rationale of delegation in this model: To keep the intensity of managerial resources  $\frac{T}{n}$  from declining, firms can delegate decision power. The efficiency with which they can do so depends on  $\psi$ .

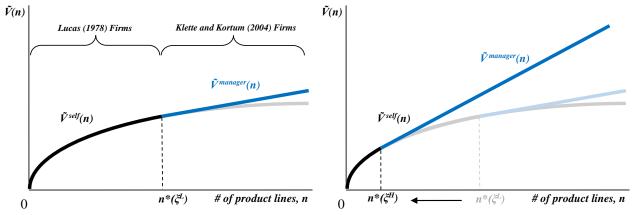
Note, in particular, that (13) implies that the option to delegate allows large firms to produce at the same managerial intensity as the marginal owner-run firm, i.e., a firm with  $\frac{T}{\psi}$  products. This property has a crucial implication: The value function is linear in firm size n once firms start delegating.<sup>11</sup> Hence, entrepreneurs can overcome the diminishing returns to expansion by delegating managerial tasks to outside managers. Moreover, the efficiency of the delegation environment directly parameterizes the slope of this value function, i.e., the incremental gain from firm growth. Specifically, the marginal return of adding an additional product to the firms' portfolio is given by

$$\tilde{V}'(n) = (1 - \sigma) \max\left\{ \left(\frac{T}{n}\right)^{\sigma}, \psi^{\sigma} \right\}. \tag{15}$$

As we formally prove below,  $\psi$  is increasing in  $\xi$ , even after taking into account the endogenous response of wages. This means that a higher delegation efficiency  $\xi$  leads to a higher equilibrium efficiency  $\psi$ , which in turn increases firms' incentive to grow and lowers the cutoff at which firms start to delegate  $n^*(\psi) = T/\psi$ .

FIGURE 2: VALUE FUNCTION

Figure 3: Increase in Delegation Efficiency  $\xi$ 



Notes: Figure 2 depicts the value function  $\tilde{V}(n)$  characterized in (14). Figure 3 depicts the change in the value function  $\tilde{V}(n)$  when the delegation efficiency  $\xi$  increases.

Figures 2 and 3 illustrate the final value function in (14). Consider first Figure 2. When a firm is run only by the owner, the firm runs into diminishing returns, as in Lucas (1978). By delegating authority, the firm can manage to keep the supply of managerial services growing and hence prevents the returns to growth from declining. Specifically, once the firm size hits the delegation cutoff

Note also that  $V^{manager}(n^*) = V^{self}(n^*) = T^{\sigma}(n^*)^{1-\sigma}$ , so that the value function is continuous.

 $n^*(\psi) = T/\psi$ , the value function becomes linear as in the baseline version of Klette and Kortum (2004). Figure 3 illustrates an increase in the delegation efficiency. If delegation becomes more efficient, e.g., through an increase in outside managers' human capital or through improvements in the contractual environment, both the delegation cutoff declines and the slope of the value function increases. In fact, Proposition 1 shows that the extensive margin of delegation and the marginal return to expansion are tightly linked: They both depend only on the endogenous equilibrium efficiency  $\psi$ . To study more formally how these static managerial returns shape the incentives for expansion and the degree of equilibrium selection, we will now embed the above static economy into a dynamic environment.

## 2.2 Dynamics, Firm Growth, and Selection

Our model is a model of creative destruction. Firms grow by stealing products from their competitors and decline in size if other producers replace them as the most productive producer of a particular product. Firms exit the economy when they lose their last product. New firms enter the economy by replacing existing firms as the producers of a particular product. Hence, aggregate growth, the existing firms' life-cycle and the resulting processes of exit and entry are all endogenous and linked to firms' growth incentives.

**Entry** In order to focus on the process of selection (or lack thereof) via product market competition and the expansion incentives of incumbent firms, we assume that an exogenous measure z of entrepreneurs enters the economy at each point in time. Importantly, entrants are heterogeneous in their growth potential and are either of high or low types. Formally, upon entry, each new entrant draws a firm type  $\theta \in \{\theta_H, \theta_L\}$  from a Bernoulli distribution, where

$$\theta = \begin{cases} \theta^H & \text{with probability } \alpha \\ \theta^L & \text{with probability } 1 - \alpha \end{cases}.$$

The firm's type  $\theta$  determines its innovation productivity or growth potential.

**Technology to Expand** Firms are endowed with a technology, which allows them to expand their scope of production. In particular, firms can spend resources to try to gain leadership in other markets. Formally, if a firm of type  $\theta$  with n products in its portfolio invests R units of the final good, it generates a flow rate of innovation equal to

$$X(R;\theta,n) = \theta \left[\frac{R}{Y}\right]^{\zeta} n^{1-\zeta},\tag{16}$$

i.e., with flow rate  $X(R; \theta, n)$  it improves the productivity of a randomly selected product and replaces the existing firm. Hence,  $\theta$  parameterizes the efficiency with which firms can expand and  $\zeta$  determines the convexity of the cost function. For simplicity we assume that  $\theta_L = 0$ , i.e., low-type

firms are stagnant and will never be able to grow. This polar case is conceptually useful because it stresses that low types are never supposed to grow. Hence, the sole difference in firm dynamics across countries will stem from the innovation incentives for high types and it will be high types' appetite for expansion that will determine the degree of selection, i.e., the time it takes for entering low-type firms to be replaced. The other terms in the innovation technology are the usual scaling variables required in many models of endogenous growth.<sup>12</sup>

Firms spend resources to expand and we define their maximization problem now. To simplify the exposition and computation, we now assume that all firms are run by short-lived owners who only maximize one-step-ahead profits (rather than the discounted sum of profits). After the current owner of the firm dies, the firm is taken over by an offspring of the exact same type  $\theta$ .<sup>13</sup> The optimal flow rate of expansion is therefore implicitly defined by

$$X_n = \arg\max_{X} \left\{ X \left[ V(n+1) - V(n) \right] - Y \left[ \frac{X}{\theta n^{1-\zeta}} \right]^{\frac{1}{\zeta}} \right\},\tag{17}$$

where V(n) is the firm's value function defined in Proposition 1 above. The first term is the expected profit of expanding with flow rate X and the last term is simply the cost function of innovation that is implied directly by (16).<sup>14</sup> Expression (17) has a simple solution. In particular, the optimal innovation rate per production unit,  $x_n = X_n/n$  is given by

$$x_n = \theta^{\frac{1}{1-\zeta}} \zeta^{\frac{\zeta}{1-\zeta}} \times \left( \frac{V(n+1) - V(n)}{Y} \right)^{\frac{\zeta}{1-\zeta}}.$$
 (18)

Naturally, the incentives to grow depend on the marginal returns of doing so, V(n+1) - V(n). It is this marginal return that links firms' innovation incentives to the delegation environment. Using (15) to approximate  $V'(n) \approx V(n+1) - V(n)$ , (18) implies that

$$x_n(\psi) = A \times \max\left\{ \left(\frac{T}{n}\right)^{\lambda}, \psi^{\lambda} \right\},$$
 (19)

 $<sup>^{12}</sup>$ Note that we scale firms' innovation costs by  $Y_t$ . We do so for two reasons. Because we denote innovation costs in terms of the final good, a growing scaling variable is required to keep the model stationary. We could have done so by using  $Q_t$  as a scale. As profits grow at rate  $Y_t$  (see (4)), this would have introduced scale effects in that firms' optimal innovation rate would depend on the size of the labor force and the average mark-up in the economy (see (6)). By using  $Y_t$  as a scale, we abstract from these general equilibrium feedbacks, which simplifies the exposition. However, we show explicitly in Section OA-1.1 in the Online Appendix that our theoretical results do not depend on this choice and that our calibration is unaffected by this choice. We also assume that firms' innovation costs depend on the number of products n to generate deviations from Gibrat's law solely through incomplete delegation. In particular, if the value function was linear (as in Klette and Kortum (2004)), the specification in (16) would imply that firm growth was independent of size.

<sup>&</sup>lt;sup>13</sup>Hence a firm's life-cycle is well defined: Firms will expand or shrink over time while being run by short-lived owners.

<sup>&</sup>lt;sup>14</sup>The probability of the firm *losing* a product does not feature in (17) because firms are assumed to be short-lived and that our continuous time formulation implies that events of expansion and destruction cannot happen simultaneously.

where  $A \equiv \left((1-\sigma)\theta^{1/\zeta}\zeta\right)^{\frac{\zeta}{1-\zeta}}$  and  $\lambda = \frac{\zeta\sigma}{1-\zeta}$  are constants and the equilibrium managerial efficiency  $\psi$  is constant in a stationary equilibrium.

Equation (19) is the crucial equation of this paper in that it explicitly expresses the incentives to expand as a function of the delegation environment, which is fully summarized by  $\psi$ . It is via (19) that delegation frictions will have dynamic implications. Consider Figure 4, which depicts the optimal innovation effort for firms of different sizes.

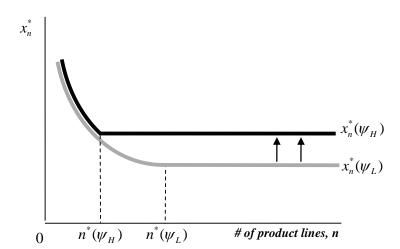


FIGURE 4: Expansion Incentives, Firm Size, and Equilibrium Delegation Efficiency  $(\psi)$ 

Notes: Figure 4 depicts the optimal expansion rate  $x_n(\psi)$  for different levels of  $\psi$ , where  $\psi_H > \psi_L$  (see (19)).

Focus first on the solid gray line, which corresponds to an environment where the equilibrium delegation efficiency is low  $(\psi_L)$ . The delegation cutoff is large and only very large firms (with n > $n_L^* = T/\psi_L$ ) start to delegate. Before delegation takes place, profits are characterized by decreasing returns, which induce declining innovation incentives. Hence, this is an economy where large firms will have a low incentive to grow and most firms will stay small. In the aggregate this implies that there is little creative destruction and a lack of selection as managerial frictions exclusively harm innovative entrepreneurs, which are the firms with the potential to be large. Stagnant producers that are small and never contemplated expanding were never planning to delegate decision power and hence are unaffected. Now suppose that the equilibrium delegation efficiency increases from  $\psi_L$ to  $\psi_H > \psi_L$ . This lowers the delegation cutoff to  $n_H^* = T/\psi_H$  and increases innovation incentives. Importantly, only large firms' expansion incentives will be affected. As innovative high types are more likely to be large, an efficient managerial delegation environment fosters expansion by high types and hence forces low-type firms to exit faster. Improvements in the efficiency of delegation therefore fosters selection and creative destruction by allowing good firms to become even larger at the expense of the inefficient firms. Empirically, this will manifest itself as a steeper increase in firm size as firms age.

The Stationary Firm-Size Distribution To study the aggregate consequences of selection, we need to characterize the endogenous firm-size distribution. We will focus on a stationary environment, where both the number of firms and the firm-size distribution is constant. Let us denote the (endogenous) number of high- and low-type firms by  $F^H$  and  $F^L$ , respectively, and let  $\nu_n^j$  be the (endogenous) share of firms of type j with n products. As there is a measure one of products, it is the case that

$$1 = F^{H} \sum_{n=1}^{\infty} n\nu_{n}^{H} + F^{L} \sum_{n=1}^{\infty} n\nu_{n}^{L} = F^{H} \sum_{n=1}^{\infty} n\nu_{n}^{H} + F^{L}, \tag{20}$$

where the second equality stems from the fact that there will not be any low types with more than one product since they never grow.

In a stationary equilibrium, firms' innovation incentives  $x_n$  are constant, i.e., they are a function of firm size but they are not time dependent. This implies that  $\psi$  is constant. Given this schedule of innovation intensities, we can construct the entire process of firm dynamics. In particular, let us denote the aggregate rate of creative destruction, i.e., the rate at which the producer of a given product is replaced, by  $\tau$ . Creative destruction can happen through entering firms or through the expansion of incumbent firms, whereby incumbents with n products expand at rate  $x_n$  (per product). Therefore,

$$\tau \equiv F^H \sum_{n=1}^{\infty} x_n n \nu_n^H + z. \tag{21}$$

We can determine the steady-state values for the number of firms and the distribution of high types from the economy-wide flow equations. These are given by the following set of equations:

STATE: OUTFLOW = INFLOW
$$F^{L}: \qquad F^{L}\tau = (1-\alpha)z$$

$$F^{H}\nu_{1}^{H}: \qquad F^{H}\nu_{1}^{H}\tau = \alpha z$$

$$\nu_{n>1}^{H}: \qquad \nu_{n}^{H}n\left[\tau + x_{n}\right] = \nu_{n-1}^{H}\left[n-1\right]x_{n-1} + \nu_{n+1}^{H}\tau\left[n+1\right]$$
(22)

The first line concerns the number of low-type firms in the economy. The left-hand side denotes the total number of low-type producers that exit the economy and the right-hand side shows the number of low-type one-product firms that enter the economy. The second line in (22) similarly ensures that the number of high-type firms is constant. Note that the total mass of one-product high-type firms is given by  $F^H \nu_1^H$ , a fraction  $\tau$  of which exit in each instant. Finally, the third line specifies the outflows and inflows for all high-type product lines with n > 1. The outflow from each product line can happen in two ways: Either the current producer of the product line will lose one of its n product lines at the total rate of  $n\tau$ , or it will come up with a new innovation at the rate  $X_n = nx_n$ , in which case the respective firm will expand into an (n+1)-product firm. Likewise, the inflow can occur in two ways: Either high-type firms with n-1 lines grow to being an n-line firm (which happens at the rate  $(n-1)x_{n-1}$ ) or firms with (n+1) products lose one product to another competitor (which happens at the rate  $(n+1)\tau$ ).

These flow equations imply that the number of low types is given by

$$F_L = \frac{(1-\alpha)z}{\tau}. (23)$$

Equation (23) stresses the importance of creative destruction: Holding the amount of entry constant, the number of surviving low types will be small whenever creative destruction is severe. As creative destruction is ignited by high types' expansion incentives embodied in  $x_n$  (see (21)), the abundance of small firms in poor countries is driven by the fact that transformative entrepreneurs might not be willing to grow, as expansion incentives decline quickly in size.

While the above discussion focused on the cross-sectional aspects of the innovation environment, the model also delivers a tractable theory of the firms' life-cycle. Within a small time interval dt, a firm with n products grows with probability  $nx_ndt$  and shrinks with probability  $n\tau dt$ . This implies that the expected (unconditional) growth rate is given by

$$g_n = x_n - \tau = A \times \max\left\{ \left(\frac{T}{n}\right)^{\lambda}, \psi^{\lambda} \right\} - \tau.$$
 (24)

Equation (24) shows precisely why limits to delegation are plausibly related to the shallow life-cycle profile in poor countries: If bottlenecks in managerial hiring cause the equilibrium efficiency  $\psi$  to be small, firms' optimal innovation incentives  $x_n$  are declining in size until firms start to delegate. This particular deviation from Gibrat's law implies that large firms will grow at a lower rate than small firms, so that age is less of a predictor of size.

Similarly, consider a low-type firm, which by construction has only a single product. As this firm loses its only product at rate  $\tau$ , the probability of that firm still being around at age a is simply given by

$$P[\text{survival until age } a|\text{low type}] = e^{-\tau \cdot a}.$$
 (25)

While all low-type firms exit the economy eventually, (25) stresses that this weeding-out process runs its course faster, the higher the rate of creative destruction  $\tau$ . The fact that stagnant firms in poor countries seem to survive for a long time is therefore consistent with the view that efficient firms generate too little creative destruction to drive them out of the market quickly. Finally, the rate of creative destruction is linked to the rate of aggregate growth via

$$g = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{Q}_t}{Q_t} = \ln(\gamma) \times \tau. \tag{26}$$

We now have all the ingredients in place to formally define and characterize a stationary equilibrium in this economy. The definition is standard and formally stated in Appendix A.2. In particular, we require that firms behave optimally, that the endogenous firm-size distribution is consistent with firms' innovation choices and that markets clear. There we not only show that there exists a unique equilibrium but, more interestingly, that our economy allows us to charac-

terize the stationary firm-size distribution in closed form. This makes the interaction between the delegation environment and the resulting firm-size distribution very concise.

**Proposition 2** The above economy has a unique stationary equilibrium. In this equilibrium high-types' innovation rates are given by

$$x_n(\psi) = A \times \max\left\{ \left(\frac{T}{n}\right)^{\lambda}, \psi^{\lambda} \right\},$$
 (27)

where  $A \equiv ((1-\sigma)\theta^{1/\zeta}\zeta)^{\frac{\zeta}{1-\zeta}}$  and  $\lambda = \frac{\zeta\sigma}{1-\zeta}$  are constants. Moreover, the firm-size distribution of high types is given by

$$\nu_n^H = \frac{n^{-1} \frac{\tau}{x_n} \prod_{j=1}^n \left(\frac{x_n}{\tau}\right)}{\sum_{s=1}^\infty s^{-1} \frac{\tau}{x_s} \prod_{j=1}^s \left(\frac{x_j}{\tau}\right)},\tag{28}$$

and the number of high- and low-type firms is given by

$$F^{H} = \frac{\alpha z}{\tau} \times \left[ \sum_{n=1}^{\infty} \frac{\tau}{n x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \right]$$
 (29)

$$F^L = \frac{(1-\alpha)z}{\tau},\tag{30}$$

the endogenous rate of creative destruction is given by

$$\tau = z \times \left[ \alpha \sum_{s=1}^{\infty} \prod_{j=1}^{s} \left( \frac{x_j}{\tau} \right) + 1 \right]. \tag{31}$$

Finally, the equilibrium efficiency  $\psi$  is implicitly defined from the labor-market clearing condition

$$1 = L^{P} + M^{D} = \frac{1}{\omega \mathcal{M}} + \sum_{n=1}^{\infty} m_{n} n F^{H} \nu_{n}^{H} + m_{n} F^{L},$$
 (32)

where  $m_n$  denotes the managerial demand given in Proposition 1 (see (12)).

### **Proof.** See Section A.2 in the Appendix.

Proposition 2 characterizes the equilibrium analytically and contains three main results. First, we find it useful to characterize the equilibrium firm-size distribution for a given equilibrium delegation efficiency  $\psi$ . This clarifies the margin through which the delegation environment matters: In our economy  $\psi$  affects the endogenous firm-size distribution only through high-types' innovation incentives. In particular, (28) - (31) depend only on the innovation schedule  $\{x_n(\psi)\}_n$ , which itself depends on  $\psi$  explicitly. Second, the equilibrium value for  $\psi$  can be calculated from the labor-market clearing condition. This is not surprising, as  $\psi$  is directly linked to the equilibrium wage

(see (11)). Finally, our main structural parameter of interest, the delegation efficiency  $\xi$ , affects firms' innovation incentives directly via  $\psi$  (see (11) and (27)). This implies that the variation in delegation efficiency  $\xi$  across countries will affect the demand for managers and hence the allocation of individuals into production workers and managers and then have implications for the implied process of firm dynamics. In our quantitative analysis, it is this property that will allow us to directly use the empirical variation in the occupational distribution to identify  $\xi$  and then gauge the aggregate implications.

The fact that  $\psi$  parameterizes the schedule of innovation intensities  $\{x_n(\psi)\}_n$  allows us to make tight predictions about the comparative statics. Let

$$\Phi_n(\psi) = F^L(\psi) + F^H(\psi) \sum_{j=1}^n n\nu_j^H(\psi)$$
(33)

be the share of products produced by firms with at most n products and let

$$\chi^{H}(\psi) = F^{H}(\psi) \sum_{n=1}^{\infty} n \nu_{n}^{H}(\psi) = 1 - F^{L}(\psi)$$
(34)

denote the share of products produced by high-type firms. Hence,  $\Phi_n(\psi)$  and  $\chi^H(\psi)$  are simple measures for the importance of large and innovative firms in the economy. We can then derive the following comparative static results.

**Proposition 3** Consider the economy above. The equilibrium delegation efficiency  $\psi$ , the aggregate managerial employment share and the economy-wide growth rate are increasing in the delegation efficiency  $\xi$ , i.e.,

$$\frac{\partial \psi}{\partial \xi} > 0, \qquad \frac{\partial M^D}{\partial \xi} > 0, \qquad \frac{\partial g}{\partial \xi} > 0.$$
 (35)

Furthermore, an increase in delegation efficiency  $\xi$  will

- 1. increase innovation incentives, i.e.,  $\frac{\partial x_n}{\partial \xi} \geq 0$  with inequality for  $n \geq \frac{T}{\psi}$ ,
- 2. increase the rate of creative destruction, i.e.,  $\frac{\partial \tau}{\partial \xi} > 0$ ,
- 3. reduce the number of low-type firms, i.e.,  $\frac{\partial F^L}{\partial \xi} < 0$ ,
- 4. increase the share of products produced by high-type firms, i.e.,  $\frac{\partial \chi^H}{\partial \xi} > 0$ ,
- 5. reduce the share of products produced by small firms, i.e.,  $\frac{\partial \Phi_n}{\partial \xi} < 0$  for all n,
- 6. increase average firm size, i.e.,  $\frac{\partial}{\partial \xi} \left( \frac{1}{F^L + F^H} \right) < 0$ .

#### **Proof.** See the Appendix.

Proposition 3 summarizes the economic effects of the delegation environment via firms' expansion activities and the endogenous adjustments through selection and creative destruction. The "prime-mover" of an improvement in delegation efficiency, i.e., an increase in  $\xi$ , is an increase in high-types' appetite for expansion. As high-type firms expand at the expense of their rivals, the rate of destruction  $\tau$  will increase. This will force low-type firms to exit at a faster rate (see (25)), thereby reducing both their number and the share of products they produce in equilibrium. Note also that this directly implies that the equilibrium growth rate q is increasing in the delegation efficiency. Furthermore, improvements in firms' delegation possibilities will affect the firm-size distribution in an asymmetric way: As small firms do not delegate decision power, large firms' expansion rates will increase relative to small firms and the distribution of firms' growth rates will more closely resemble Gibrat's law (see (24)). This implies that the firm-size distribution will shift toward larger firms and that average firm size increases. Hence, qualitatively, Proposition 3 is consistent with the main stylized facts about firms in rich and poor countries and provides a unifying mechanism to interpret this cross-country pattern: Firms in rich countries might have access to a better delegation environment via the abundance of managers with the appropriate human capital, a well-functioning system of contract enforcement or better complementary technologies, which raise the value of delegation. 15 Whether this mechanism is also quantitatively able to explain the differences across countries is the subject of the next sections.

## 3 Quantitative Exercise

We now take the model to the data to gauge the quantitative importance of cross-country differences in the delegation environment for the implied process of firm dynamics. Our strategy is as follows: After describing the main data sources used in the quantitative part, we first calibrate the model to the US and the Indian economy in Section 3.3. In particular, we target different moments of the process of firm dynamics of the US and Indian manufacturing sector and ensure that the model matches the life-cycle of the firms in both countries. The delegation environment, captured by  $\xi$ , is disciplined by matching the managerial employment share. In Section 4 we then consider the US counterfactual economy by changing  $\xi$  to the Indian level and study the resulting change in firms' life-cycle. We also compare this counterfactual exercise with an alternative strategy, where we recalibrate the delegation environment  $\xi$  to match the Indian managerial share while keeping all other parameters at the US level. The difference between these two counterfactuals is informative about complementarities between the delegation environment and other structural parameters. <sup>16</sup>

<sup>&</sup>lt;sup>15</sup>Bloom et al. (2012), for instance, use cross-country data to show that trust in the legal system correlates significantly with average firm size. Likewise Kumar et al. (1999) find a similar positive link between average firm size and the strength of the legal system among European countries. Laeven and Woodruff (2007) establish a causal link between the quality of legal institutions and average firm size using Mexican firms. La Porta et al. (1997) show that the sales of the 20 largest publicly traded companies as a share of GDP strongly correlates with the level of trust prevailing in a country.

<sup>&</sup>lt;sup>16</sup>These exercises are deliberately silent on the underlying source of variation in the delegation efficiency  $\xi$ . For instance, in Example 1,  $\xi$  depends on the level of managerial human capital  $\eta$  and the strength of the legal system

## 3.1 Data

Here we briefly describe the main data sources for our analysis. A detailed description with additional descriptive statistics is contained in Section C.1 in the Appendix.

US Data To calibrate our model to the US manufacturing sector, we rely on publicly available data from the Business Dynamics Statistics (BDS). The BDS is provided by the U.S. Census Bureau and compiled from the Longitudinal Business Database (LBD), which draws on the Census Bureau's Business Registry to provide longitudinal data for each plant with paid employees. The BDS uses a unified treatment of plants and firms. While a plant is a fixed physical location where economic activity occurs, firms are defined at the enterprise level such that all plants under the operational control of the enterprise are considered part of the firm. The BDS contains information on the cross-sectional relationship between age and size (which we refer to as the life-cycle), exit rates and exit rates by age conditional on size. The latter will be a crucial moment for identifying the importance of heterogeneous types in the US economy. We focus on the data from 2012.

We augment the firm- and plant-level data by additional information pertaining to the importance of managerial personnel in the US economy. We rely on two data sources. We first focus on individual level micro data from the US census, which contains detailed information on earning and occupational categories. This allows us to measure the importance of managers in both factor payments and employment in the manufacturing sector in the US. Second, we use the US Product and Income Accounts (NIPA) to measure corporate profits and employee compensation for US manufacturing firms, which will be helpful in identifying the elasticity of managerial effort  $\sigma$ . Finally, to be in line with the existing literature (e.g., Hsieh and Klenow (2014)), we will first focus on the life-cycle of plants in the main text and will conduct robustness checks using firm-level data in Section OA-2.1 in the Online Appendix.

Indian Data We use two data sources about Indian manufacturing plants. The first source is the Annual Survey of Industries (ASI) and the second is the National Sample Survey (NSS). Hence, the data are the same as those used in Hsieh and Klenow (2014) and Hsieh and Olken (2014). The ASI is an annual survey of manufacturing enterprises. It covers all plants employing ten or more workers using electric power and employing twenty or more workers without electric power. For an economy like India, the ASI covers only a tiny fraction of producers, as most plants employ far fewer than twenty employees. To overcome this oversampling of large producers in the ASI, we complement the ASI with data from the NSS, which (every five years) surveys a random sample of the population of manufacturing plants without the minimum size requirement of the ASI. While these producers are (by construction) very small, they account for roughly 76% of aggregate employment in the manufacturing sector. We merge the NSS data with the ASI using the sampling weights provided in

 $<sup>\</sup>kappa$ . From the micro-variation within a country, we cannot identify these individual components but only  $\xi$  itself. In Section 5.1 we use cross-country data to make progress in decomposing the delegation environment into these different components.

the data and focus on the year 2010, which is the latest year for which both data sets are available. In terms of the data we use, we mostly focus on the employment side. In particular, we draw on the information on age and employment to study the cross-sectional age-size relationship. For a more detailed description and some descriptive statistics, we refer to Section C.1 in the Appendix.

Data on Managerial Employment To measure managerial employment, we employ national census data from the IPUMS project, which provides micro data from a variety of countries. For each country we get a sample from the census, which has detailed information about individual characteristics. We observe each respondent's education, occupation, employment status, sex and industry of employment. We focus on male workers in the manufacturing industry working in private-sector jobs. We always use the most recent data available, which is 2004 in the case of India and 2010 in the case of the US.

To classify workers as managers in the sense of our model, we use information about workers' occupational status and their employment status. As our theory stresses the importance of delegating authority to *outside* managers, we classify employees as managers if they got assigned the occupational code "Legislator, Senior official and manager" and they are hired as wage workers instead of being, for example, family members of the firms' owner or the employer themselves. The latter distinction is important. To see this, consider Table 1.

Table 1: Outside Managers in India and the US

	US	India	
Share of managers (ISCO)	14.1	11.5	
Share of outside managers	12.6	1.5	
Distribution of Managers			
Self-employed	10.0%	56.2%	
Employer	0.0%	11.6%	
Wage worker	89.5%	12.0%	
Unpaid family worker	0.1%	19.2%	

Notes: The table contains the share of managers according to the occupation classification ISCO and the distribution of economic status conditional on being classified as a manager according to the occupational code. Outside managers are all managers, according to ISCO, who are hired as wage workers. The conditional distribution in the lower panel of Table 1 does not exactly sum up to unity as there are some additional worker class categories, which we do not display for brevity.

In the bottom panel we report the workers' economic status conditional on being classified as a manager from their occupational code. For the case of the US, roughly 14% of the labor force is classified as managers according to their occupational code and the vast majority, namely 90%, are wage workers and hence outside managers in the sense of our theory. This is very different in the case of India. Conditional on being in a managerial occupation, the share of outside managers is only 12%. In contrast, the vast majority of individuals working in managerial occupations are

either entrepreneurs themselves or unpaid family members. The latter is very much consistent with the findings in Bloom et al. (2013), who also argue that Indian firms acquire managerial services mostly from their owners or close family members. This pattern is very much the exception in the US.

## 3.2 Firm-Level Evidence on Managerial Employment in India

Before we turn to the quantitative exercise, we want to briefly mention some basic patterns of managerial employment in the Indian micro data contained in the ASI and NSS and how they relate to our theory. In order to save space, this analysis is relegated to Section OA-2.4 in the Online Appendix, where we discuss the empirical findings at length and report a variety of reduced-form regression results.

At the heart of our mechanism is the interaction between the static demand for managerial personnel and the dynamic growth incentives resulting from the ease with which managers can be hired. Our theory implies that the likelihood of hiring a manager is increasing in firm size n, increasing in the delegation efficiency  $\xi$  and decreasing in the entrepreneurs' time endowment T (see Proposition 1). In the Indian micro data, we observe whether the firm employs any managers. As we conceptualized T as time that is inelastically provided and does not require monitoring, we take family size, which is observable in parts of the Indian micro data, as inducing firm variation in T. As for the variation in  $\xi$ , we follow Bloom et al. (2012) and assume that the efficiency with which decisions can be delegated is increasing in the level of trust across Indian regions, which we can measure from the World Values Surveys (WVS) and link to the Indian firm-level data. Using these measures, we find that both firm size and the level of trust are strongly positively correlated and the size of the family strongly negatively correlated with the probability of Indian firms hiring managers.

Turning to the dynamic implications, our theory implies that managerial services increase firms' expansion incentives. In particular, firms' expansion incentives are increasing in both the efficiency of the delegation environment  $\xi$  and the owner's time endowment T. Moreover,  $\xi$  and T are substitutes, in that managerial resources within the firm (T) are particularly valuable if delegation to outside managers is difficult (see Proposition 2). In the Indian firm-level data we indeed find that firm size is positively correlated with both the level of trust and the size of the owner's family and that the effect of family size on firm size is particularly strong in regions where trust is low and delegation of decision power might be difficult.

Finally, we focus directly on the patterns of firm growth, which we can estimate using panel data from the ASI for the years 1998 - 2009. The main dynamic implication of our model is that limits to delegation induce a deviation from Gibrat's law in a particular way: Firm growth is declining in firm size, and the worse the delegation environment, the more it declines (see especially (24)). In the data we indeed find that larger firms grow substantially slower than small firms and that this is particularly true in regions where the level of trust is low.

## 3.3 Calibration

**Identification** We now discuss the identification of our model. The closed-form expression for the firm-size distribution allows us to give a precise discussion about the identification and we do so in detail in Section B of the Appendix. Here we provide a heuristic description about which moments in the data are informative about which parameters of the model. Naturally, we calibrate all parameters jointly. We will consider the model outlined above with one additional degree of freedom. We are particularly interested in the importance of selection, i.e., the process whereby high-type firms replace low-type firms in the economy. Recall that we parameterized the share of high-type firms upon entry by  $\alpha$  and that low and high types differed only in their cost of innovation  $\theta$ . We identify  $\alpha$  from age differences in the exit profile for firms of equal size. Our reasoning is the following: Without type heterogeneity, all dynamic moments such as firm growth and firm exit would depend only on firm size. Hence, age should not matter once firm size is controlled for. In the data, however, exit rates are strongly decreasing in firm age conditional on size, especially for the smallest firms (Haltiwanger et al. (2013)). We interpret this pattern as driven by selection, whereby the share of high-type firms within a given cohort increases as the cohort of firms ages. As high types are less likely to exit (conditional on size) because they have a higher likelihood of growing before they exit, the conditional exit rate is declining in age if selection is an important aspect. To be able to quantitatively match the data, however, we require an additional margin for high-type firms to exit less. In the data, conditional on size, old firms are far less likely to exit but have only moderately higher growth rates. To match this feature of the data, we therefore assume that high types have the opportunity to "defend their turf" conditional on a competing firm trying to expand into their product line. Formally, if the rate of creative destruction is  $\tau$ , high types only lose their product at rate  $\beta \tau$ , where  $\beta \leq 1$ . In the theoretical model, we implicitly assumed  $\beta = 1$ , for ease of exposition. In our quantitative exercise we will calibrate  $\beta$ . 17

With this additional degree of freedom we have eight parameters to calibrate:  $\sigma$ ,  $\xi$ ,  $\alpha$ ,  $\beta$ ,  $\theta$ , z,  $\gamma$ , and T.<sup>18</sup> The entry intensity z and the efficiency of innovation technology  $\theta$  are identified from two moments of the firm-size distribution, namely, the aggregate entry rate  $(M_1)$  and the implied life-cycle  $(M_2)$ , i.e. the average size of old firms relative to young firms. We always define old firms as firms of age 21-25 and young firms as firms younger than five years old. The parameters  $\alpha$  and  $\beta$  determine the degree of selection in the economy. The two moments we use to discipline these parameters are the aggregate employment share of old firms  $(M_3)$  and the exit rate by age conditional on size  $(M_4)$ .  $M_3$  is related to  $\beta$ , because  $\beta$  determines how quickly high-type firms, which are older on average, lose market share to their competitors.  $M_4$ , as discussed above, is informative about  $\alpha$ . In the model we focus on the exit rate of one-product firms that are between

<sup>&</sup>lt;sup>17</sup>See also Section B in the Appendix where we discuss this in more detail. There we also derive all our theoretical results for this more general case.

<sup>&</sup>lt;sup>18</sup>As we do not have data on spending on innovation, we do not attempt to estimate the curvature of the expansion cost function  $\zeta$ . Instead we follow the microeconomic literature on R&D spending, whose estimates imply a quadratic cost function, i.e.,  $\zeta = 2$ . See Akcigit and Kerr (2010), Acemoglu et al. (2013) and Bloom et al. (2015), who discuss this evidence in more detail.

21-25 years old relative to those that are younger than 5. By focusing on single-product firms, we not only control for firm size but we also look at the subset of firms, where selection is the strongest, both in the data and in the model. In the data we calculate this moment by focusing on the smallest available employment category. In the US case, the BLS reports employment by age and size only for a "1-4" employees bin. In the Indian case, we take firms with a single outside employee.

The parameters  $\sigma$ , T and  $\xi$  are directly related to managerial personnel and firm profitability. As  $\sigma$  is the elasticity of profits with respect to managerial services, we identify it from the share of managerial compensation relative to corporate profits  $(M_5)$ . The owner's time endowment T is directly related to firms' profitability, as it determines the optimal allocation of managerial services (see (13)), which in turn affects firms' mark-ups (see (4))). The overall delegation efficiency  $\xi$  will affect firms' demand for managers relative to production workers - recall that we showed in Proposition 3 that the aggregate managerial employment share is increasing in  $\xi$ . Hence, we calibrate T and  $\xi$  to match the average mark-up  $(M_6)$  and the aggregate managerial employment share  $(M_7)$ . Finally, there is the innovation step-size  $\gamma$ , which translates firms' innovation outcomes into aggregate growth. In particular, our model implies that aggregate productivity growth  $g_Q$  is given by  $g_Q = \tau \times \ln \gamma$ . As  $\gamma$  does not enter any other moment in the model, we can simply choose  $\gamma$  to match the aggregate growth rate  $(M_8)$ .

## 3.3.1 Calibrating the Model to the US and India and the Goodness of Fit

We now turn to the US economy and calibrate the model to the data on US manufacturing plants.<sup>19</sup> In Table 2 we report the targeted moments (Panel A) and the resulting parameters (Panel B). There we also report the main target for the respective parameters even though the parameters are calibrated jointly.

The first four moments concern the plant-level outcomes. The entry rate in the US manufacturing sector is 7.3%, the average employment of 21-25-year-old plants relative to 1-5 year-old-plants is 2.53 and these firms account for roughly 8.15% of aggregate employment. As for the conditional exit hazards, while young plants (i.e., plants of age 1-5) with 1-4 employees have an exit rate of 22.4% per year, plants of age 21-25 with an equal size have an exit rate of only 14%. Hence, small young firms are around 1.6 times as likely to exit as small old firms. The next three moments are informative about the managerial environment. In particular, managerial compensation in the US roughly amounts to 49% of total corporate profits gross of managerial payments 20 and the share of outside managers in the workforce is 12.6%. As for profitability, we target an average mark-up of 20%. 21 Finally, we target an aggregate rate of productivity growth of 1.72%, which is the average

<sup>&</sup>lt;sup>19</sup>We focus on plants instead of firms, both because the Indian data are only available at the plant level and to be consistent with the earlier literature, which also relied on plant-level data. However, in Section OA-2.3 in the Online Appendix, we repeat our quantitative analysis using the US firm-level data.

<sup>&</sup>lt;sup>20</sup>As managerial compensation accounts for roughly 23% of total labor compensation, this implies that aggregate profits net of managerial payments amount to roughly 20% of aggregate labor income.

<sup>&</sup>lt;sup>21</sup>This number is broadly consistent with Oberfield and Raval (2014), who report mark-ups between 25% and 40%

0.756

0.131

0.358

4.620

0.072

Table 2: Estimation for the US

	A. Moments To	argeted for the US	5	
$\underline{\text{Moment}}$			$\underline{\mathrm{Data}}$	$\underline{\text{Model}}$
$M_1$ . Entry rate		7.3%	7.3%	
$M_2$ . Mean employment for 21-25-year-old firms		2.5	2.5	
$M_3$ . Employment share of 21-25-year-old firms		8.1%	6%	
$M_4$ . Relative exit rate of small 21-25-year-old firms		1.6	1.6	
$M_5$ . Share of	of manager compensation		49.0%	49.0%
$M_6$ . Average mark-up		20.0%	20.0%	
$M_7$ . Share of managers in workforce		12.6%	12.6%	
$M_8$ . Aggreg	gate growth rate		1.7%	1.7%
	B. Parame	eters of the US		
Parameter	Interpretation	Target		$\underline{\text{Value}}$
ξ	Delegation efficiency	Managerial employment share 0.72		0.726
$\alpha$	Share of high type	Age vs exit pro	ofile	0.590

Managerial compensation

Empl. share of old firms

Life-cycle

Rate of entry

Share of non-managerial firms

Notes: In Panel A we report both the data moments (column 1) and the corresponding moments in the model (column 2). The plant-level moments  $(M_1 - M_4)$  stem directly from the 2012 micro data of US manufacturing plants reported in the BDS.  $M_2$  is the mean employment for 21-25-year-old firms relative to firms younger than 5.  $M_3$  is the fraction of workers employed by 21-25-year-old firms.  $M_4$  is the exit rate of 21-25-year-old firms relative to firms younger than 5 conditional on having 1-4 employees. The share of managerial compensation  $(M_5)$  is calculated from the US National Income and Product Accounts and the census micro data on managerial earnings available in the Census. The share of managers in the workforce  $(M_7)$  is also calculated from the US Census. The aggregate growth rate  $(M_8)$  is an estimate of the US growth rate from the Penn World Tables for the years 1970-2011. See Section C.1

Curvature of efficiency

Managerial endowment

High-type replacement

Innovativeness

Entry flow rate

T

β

 $\theta$ 

z

reported in column 2 of Panel A.

growth rate of TFP between 1970 and 2005 according to the Penn World Tables. Overall the model matches the moments exactly, with the exception of the aggregate employment share of old firms, which the model slightly underestimates. $^{22}$ 

in the Appendix for details. In Panel B we report the corresponding parameter estimates that yield the moments

As for the implied structural parameters, we estimate the share of high-type entrepreneurs entering the economy to be 0.59, which implies that even in the US more than 40% of entrepreneurs

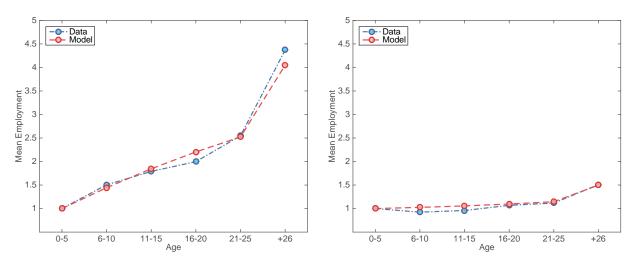
for the US manufacturing sector and De Loecker and Warzynski (2012), who find median mark-ups of 17%-28% for manufacturing firms in Slovenia.

<sup>&</sup>lt;sup>22</sup>One reason why the model predicts slightly too few old firms is that in our model growth is only driven by the extensive margin of adding products. Hence, the process of growth and the resulting exit hazard are tightly linked. If we allowed for growth on the intensive margin (e.g., through quality innovations within existing product lines as in Akcigit and Kerr (2010) or Garcia-Macia et al. (2015)), we could break this link.

never expand. The estimate of  $\beta$  implies that high types are quite successful at "shielding" themselves from being replaced. The fact that the model implies  $\beta < 1$  reflects the steep decline in the probability of exit: As older firms are more likely to be high types and older firms exit less often than young firms conditional on size without growing much faster, the model requires a mechanism of high types being less likely to be replaced. Finally, in Figure 5 we depict the plants' life-cycle in the data and in the model. The model essentially matches the life-cycle perfectly, even though we only targeted the relative size of 21-25-year-old plants relative to young plants.

FIGURE 5: LIFE-CYCLE OF US PLANTS

FIGURE 6: LIFE-CYCLE OF INDIAN PLANTS



Notes: The figure depicts the cross-sectional age-size relationship, i.e. average plant employment as a function of age. Figure 5 focuses on the US. The data correspond to the population of US manufacturing plants in 2012 and are taken from the BDS. The model corresponds to the US parametrization reported in Table 2. Figure 6 focuses on India. The data correspond to the Indian manufacturing plants in 2010 and are taken from the ASI and the NSS. The model corresponds to the Indian parameterization reported in Table 3.

We now turn to the calibration for the Indian economy. We follow the same strategy as for the US. However, in contrast to the US case, we do not explicitly target profitability and the share of managerial compensation, as the Indian data for informal firms in the NSS do not allow us to convincingly calculate profits. Therefore we keep T and  $\sigma$  constant at their respective US values when calibrating the model to the Indian data and calibrate only the six remaining parameters. We will, however, discuss the model's implication for these two non-targeted moments below and we provide additional robustness checks in Section D in the Appendix.

The results for the India calibration are contained in Table 3 below. The first four moments again pertain to the process of firm dynamics. While the entry rate and the employment share of old plants in India are in fact almost the same as in the US,<sup>23</sup> both the life-cycle and the age profile

 $<sup>^{23}</sup>$ At first glance it might be surprising that old firms, i.e., firms of ages 21-25, have roughly the same aggregate employment share. The reason is that the aggregate employment share of very old firms is much higher in the US. In the US (India) the share of firms older than 25 years is 55% (20%). See Sections OA-2.1 and OA-2.2 in the Online Appendix for details.

of exit rates differ markedly. While 21-25-year-old plants in the US are about 2.5 times as big as young plants, old plants in India are hardly bigger than young plants - on average they grew by 12% conditional on survival. One reason for this shallow profile is that there is less selection in India. In particular, in contrast to the US, young plants exit almost at the same rate as old plants.<sup>24</sup> In our model, this fact implies that the share of high types within a cohort does not strongly increase as the cohort ages, that is, the economy is characterized by little selection. Finally, the share of managerial employment is only 1.5% (see Table 1) and the rate of aggregate TFP growth between 1980 and 2005 is about 2.5%.

Table 3: Estimation for India

A. Moments Targeted for India			
Moment	$\underline{\text{Data}}$	$\underline{\mathrm{Model}}$	
$M_1$ . Entry rate	7.0%	7.0%	
$M_2$ . Mean employment for 21-25-year-old firms	1.1	1.1	
$M_3$ . Employment share of 21-25-year-old firms	8.1%	8.0%	
$M_4$ . Relative exit rate of 21-25-year-old firms	1.1	1.1	
$M_5$ . Share of managers in workforce	1.5%	1.5%	
$M_6$ . Aggregate growth rate of TFP	2.6%	2.6%	

B. Parameters of India				
<u>Parameter</u>	Interpretation	Target	<u>Value</u>	
ξ	Delegation efficiency	Managerial employment share	0.608	
$\alpha$	Share of high type	Age vs exit profile	0.129	
$\beta$	High-type replacement	Empl. share of old firms	0.435	
heta	Innovativeness	Life-cycle	1.142	
z	Entry flow rate	Rate of entry	0.080	
$\gamma$	Innovation step size	Aggregate growth rate	1.445	

Notes: In Panel A we report both the data moments (column 1) and the corresponding moments in the model (column 2). The first four moments stem directly from the 2010 micro data of Indian manufacturing plants, i.e., the ASI and the NSS.  $M_2$  is the mean employment for 21-25-year-old firms relative to firms younger than 5.  $M_3$  is the fraction of workers employed by 21-25-year-old firms.  $M_4$  is the exit rate of 21-25-year-old firms relative to firms younger than 5. The share of managers in the workforce ( $M_5$ ) is calculated from IPUMS. The aggregate growth rate ( $M_6$ ) is an estimate of the Indian growth rate from the Penn World Tables for the years 1980-2005. See Section C.1 in the Appendix for details. In Panel B we report the corresponding parameter estimates that yield the moments reported in column 2 of Panel A.

Column 2 of Panel A in Table 3 shows that we can calibrate the model to match the Indian moments exactly. The resulting parameters are contained in Panel B. In particular, the delegation efficiency  $\xi_{IND}$  is estimated to be substantially lower than the US level to successfully match the small share of outside managers. The other main differences between India and the US relate to

<sup>&</sup>lt;sup>24</sup>The Indian micro data do not have a panel dimension. Hence, we follow Hsieh and Klenow (2014) to calculate exit rates from the relative size of cohorts across different census years. This does not allow us to calculate exit rates conditional on size. Hence, in the data, we take the unconditional exit rates. Empirically, the vast majority of firms have less than 4 employees and firm size is not strongly related to age.

the importance of high-type firms ( $\alpha$ ) and the costs to expansion ( $\theta$ ). Specifically, merely 13% of entering firms in India have the ability to expand (compared to around 59% in the US) and their technology to expand is quite unproductive as  $\theta_{IND} < \theta_{US}$ .<sup>25</sup> In Figure 6 we again compare the entire life-cycle profile with the one observed in the data. As was the case for the US, the model essentially matches the observed life-cycle even though we only calibrated to the target moment of the 21-25-year-old firms in Table 3.

Non-targeted Moments The model is also broadly consistent with additional non-targeted moments. Consider first the aggregate importance of firms that do not delegate decision power. The model predicts that in India firms without any outside managers should have an employment share of 79%. As the Indian micro data contain information on managerial hiring, we can calculate this moment in the data. We find that firms without any managers account for 78% of aggregate employment.<sup>26</sup> In the US, the model predicts an employment share of firm without delegation of zero, i.e., in equilibrium all firms are hiring outside managers. This difference in the extensive margin of managerial hiring stems from the fact that delegation is more efficient in the US, i.e.,  $\xi_{US} > \xi_{IND}$ , which according to Proposition 3 implies that equilibrium delegation benefits in the US exceed their Indian counterpart, i.e.,  $\psi_{US} > \psi_{IND}$ .

Second, the model also makes predictions about the share of managerial compensation relative to aggregate profits.<sup>27</sup> The model implies this moment for India to be 5.7%. Because we do not have reliable information on profits for informal firms, we cannot directly calculate this moment in the entire Indian micro data. We can, however, look at the firms in the ASI that mostly do hire managerial personnel. For these firms, managerial compensation amounts to 18% of profits. As the firms in the ASI account for roughly 25% of employment, the implied moment in India would be about  $18\% \times 25\% = 4.5\%$  if the aggregate profits and employment were in the same proportion in the ASI and the NSS. The implied average mark-up for the Indian economy is about 26%. While this is slightly higher than in the US, this number is also not entirely off the mark.<sup>28</sup>

Finally, and most important, we can confront the implied dynamic evolution of a cohort with the data. This is a crucial moment for the aggregate degree of selection in the economy. The theory stresses that the Indian life-cycle profile in Figure 6 masks heterogeneity across firms, whereby some producers do grow, but not sufficiently to affect the aggregate life-cycle profile. To see this

Note also that the step-size of innovation is calibrated to be higher in India ( $\gamma_{IND} > \gamma_{US}$ ) as the Indian economy will endogenously have less creative destruction but a slightly higher rate of productivity growth. Recall that  $\gamma$  does not affect the process of firm dynamics in our model.

<sup>&</sup>lt;sup>26</sup>Note, however, that we cannot precisely distinguish whether these managers are outside managers or family members. Given the plant-level data, we do not think this to be a severe problem. Even if we were to assume that *no* informal firm in the NSS hires outside managers, we find an aggregate employment share of managerial firms of 80%, because only few firms in the NSS report hiring any managers at all.

<sup>&</sup>lt;sup>27</sup>Recall that we targeted this moment for the calibration to the US economy.

<sup>&</sup>lt;sup>28</sup>De Loecker et al. (2012), for example, find median mark-ups of 18% (at the firm-product level) and 40% (at the firm level) for large Indian manufacturing firms and Hsieh and Klenow (2009) use mark-ups between 25% and 50% for firms in the ASI. See also Section D in the Appendix, where we perform multiple robustness checks with regard to different calibration strategies to match mark-ups.

mechanism in the calibrated model and in the data, consider Figure 7, where we depict the share of small firms by age (relative to the share among the young firms) both in the model (solid line) and in the data (dashed line). The figure shows an important aspect of the shallow life-cycle profile in India: The average old firm is small in India because there are still ample tiny old producers. In particular, while (in the model) the share of one-product firms in the US declines to 40% by age 25, 90% of old firms in India still only have one product. Hence, the flat life-cycle in Figure 6 is due to a substantial amount of stagnant producers that do not grow and exit only very slowly. To see that these dynamics are very similar in the data, we superimpose the data on the share of small firms by age. In the US data, we only see the number of firms for different employment bins. The smallest employment category corresponds to 1-4 employees, which we therefore take as our definition of small firms in the US. In India, we see the entire micro data and hence we define small firms to be firms with a single employee.<sup>29</sup> Hence, the model generates a realistic pattern for the relative cohort size by age, which is in line with selection through creative destruction being much more important in the US.

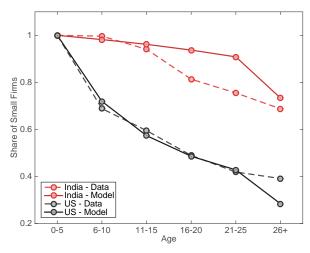


FIGURE 7: SHARE OF SMALL FIRMS (DATA VS MODEL)

Notes: Figure 7 shows the share of small firms by age in the model (solid lines) and the data (dashed lines). For the US we define small firms as all plants with 1-4 employees. For India we define small firms as all plants with a single hired employee. The data for the US correspond to the population of US manufacturing plants in 2012 and stem from the BDS. The data for India correspond to the Indian manufacturing plants in 2010 and are taken from the ASI and the NSS. The parameters for the respective models are contained in Tables 2 and 3.

## 3.3.2 Quantitative Implications

With the calibrated model at hand we can also give a structural interpretation to the selection dynamics displayed in Figure 7, by directly focusing on the share of high-type firms in a cohort.

<sup>&</sup>lt;sup>29</sup>Note also that the US data do not distinguish plant age for an age exceeding 26 years. Hence, we can only report the aggregated age category 26+.

Figure 8 plots this share both for the US and India. Two results stand out. First, the share of high-type firms in the US is significantly bigger among the entering cohort as  $\alpha_{US} > \alpha_{IND}$ . Second, high-type firms grow faster in the US, creating a much stronger selection force. While the share of high-type firms of age 20+ is essentially 100% in the US, high types are still in the minority among old plants in India: Even for 30-year-old plants, more than half of them are low types in India. How much of this lack of selection is due to the fact that there are simply very few high-type firms in India to begin with? The answer to this question is depicted by the light blue line in Figure 8, where we simulate a counterfactual cohort in the US economy, which starts with the initial type distribution of India, i.e., where the initial share of high types was  $\alpha_{IND}$ . It is clearly seen that the missing growth incentives of existing high-types in India are a key aspect of the selection dynamics. By the age of 25, the cohort would again be populated only by high-type firms despite the few high-type firms at the time of entry. Hence, as long as existing innovative firms have the right playing field, few of these firms might be enough to create quantitatively important selection dynamics by pushing out low-type, stagnant firms quickly.

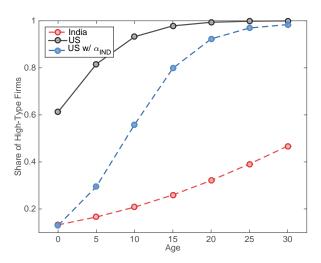


FIGURE 8: SHARE OF HIGH-TYPE FIRMS

Notes: Figure 8 shows the share of high-type firms by age for both the India calibration (Table 3) and the US calibration (Table 2). Additionally, we show the counterfactual share of high types by age if the initial share of high types in the US was given by its Indian counterpart (i.e.,  $\alpha_{IND}$ ) but the remaining parameters were equal to the US calibration.

Finally, we summarize some of the quantitative implications in Table 4. As suggested by Figure 8, high-type firms are of limited importance for the Indian economy. In the stationary distribution in the US, around 90% of firms are high types (compared to 59% at the time of entry) and they have a combined market share of 96% as they are bigger on average. In India high-type firms account for only 32% of firms and 40% of aggregate employment. The reason for these differences is, of course, that innovative firms in the US have a substantially higher average innovation rate than in India. These missing expansion incentives for high-type firms in India allow low-type firms

	US	India
Share of high-type firms	0.897	0.320
Employment share of high-type firms	0.961	0.397
Average innovation rate by high types	0.640	0.030
Employment share by one-product firms	0.174	0.789
Employment share of firms without managers	0	0.789
Average firm size (rel to US)	1	0.428
Rate of creative destruction	0.111	0.070

TABLE 4: STEADY-STATE COMPARISON OF THE US AND INDIA

Notes: The table contains different moments from the model. The average innovation rate by high types is defined as  $\sum_{n=1}^{\infty} x_n n \nu_n^H$  (see equation (21)). In column 1 we show the case of the US, i.e., the calibration reported in Table 2. In column 2 we report the case of India, i.e., the calibration reported in Table 3.

to survive, thereby shifting the Indian firm-size distribution to the left, causing a plethora of small firms. Not only do firms with a single product employ 80% of the workforce and the average firm is only 40% as large as in the US, but none of these single-product firms employ any managers, as the returns to delegation are low. This is very different in the US. Firms with a single product account for only a small share of economic activity and the high efficiency of outside managers in the US also implies that all firms in the US rely on outside managers to provide managerial services. The equilibrium summary statistic for these dynamic properties is the rate of creative destruction, which also declines by almost 40% in India relative to the US.

# 4 Counterfactual Analysis: Delegation Efficiency and the Life-Cycle

Given these calibrated economies, we can now turn to our main counterfactual exercise of interest: What would the life-cycle of US manufacturing plants look like in the case where the delegation efficiency  $\xi_{US}$  was equal to its Indian counterpart  $\xi_{IND}$ ? To answer this question, we will consider two counterfactual scenarios, which we will refer to as the *full* and the *partial* counterfactuals.

First, we are going to take the estimated delegation efficiency in Table 3 as a structural parameter and study how the life-cycle of US firms would change if they were subject to the Indian delegation environment ( $\xi_{IND} = 0.608$ ) instead of the one in the US ( $\xi_{US} = 0.726$ ). We will refer to the exercise as this full counterfactual, because  $\xi_{IND}$  stems from an entirely new calibration of the model using the Indian data, which differs from the US calibration in all parameters.

Second, as a comparison, we will also consider a *partial* counterfactual. Our calibration strategy for this second exercise is as follows: We showed analytically that there is a monotone positive relationship between delegation efficiency  $\xi$  and the endogenous managerial employment share.

<sup>&</sup>lt;sup>30</sup>Formally, the endogenous delegation cutoff  $n^*(\psi) = T/\psi$  is smaller than unity, the minimum firm size in the economy.

Empirically, the observed managerial shares in the US and India are 12.6% and 1.5%, respectively. Hence, for the US economy to generate a managerial employment share as observed in India while keeping all other parameters fixed at their US levels, the delegation efficiency had to decline from  $\xi_{US} = 0.726$  to  $\xi_{IND}^P = 0.560$ . Here the P superscript indicates that  $\xi_{IND}^P$  stems from a partial comparative statics exercise where all other parameters are held constant at the US level.

Conceptually, the second (partial) exercise is very similar to the quantitative strategy adopted by Buera et al. (2011) in the context of a model with credit market frictions. There, the sole variation across countries is assumed to lie in the quality of the financial system. To discipline the variation in financial systems across countries, Buera et al. (2011) target the cross-country variation in the debt-to-GDP ratio, which is an endogenous outcome of the model. In our case, we envision the source of variation stemming from delegation efficiencies and identify  $\xi$  from the cross-country variation in managerial employment shares, which is also an endogenous outcome of the model. The fact that  $\xi_{US} > \xi_{IND} > \xi_{IND}^P$  shows that other differences between the US and India help to account for some of the gap in aggregate managerial employment but that the model still requires a worse delegation environment in India to be able to match the data. We will come back to this point below in Section 5.

The implication of such changes in the delegation environment for the plants' life-cycle is contained in Figure 9. We depict both the observed life-cycle for the US and India and the implied life-cycle of reducing the delegation efficiency in the US from  $\xi_{US}$  to  $\xi_{IND}$  and  $\xi_{IND}^{P}$  respectively. It is seen that the implied life-cycle flattens considerably. While 21-25-year-old plants in the US are on average about 2.5 times as big as new entrants, they would only be 1.9-2 times as big if the delegation efficiency in the US was given by the one in India. In the Indian micro data, old firms are about 1.1 times as big as new entrants. Hence, depending on the age of the cohort, variation in the estimated efficiency of delegation accounts for about 30-40% of the empirically observed gap in life-cycle growth. As  $\xi_{IND} > \xi_{IND}^{P}$ , the full counterfactual effects are slightly smaller than the partial counterfactual effects. The quantitative magnitude of these differences, however, is relatively small.<sup>31</sup>

In Figure 10 and 11 we look at the underlying mechanisms for the aggregate pattern depicted in Figure 9. In Figure 10 we depict the share of surviving plants by age and show that a deterioration of the delegation environment slows down the process of shake-out, especially in the first 10-15 years in the life-span of a cohort. In Figure 11 we show that this increase in the rate of creative destruction affects firms asymmetrically. In particular, the exit rate is particularly high for low-type firms if the delegation environment is well-developed. To see this, Figure 11 depicts the share of high types by age and shows that a better delegation environment allows high-type firms to expand

 $<sup>^{31}</sup>$ To get a sense of the maximum quantitative magnitude of our mechanism, suppose that  $\xi=0$ , i.e., that managerial delegation was abolished. In that case, the predicted life-cycle growth by age 21-25 would be around 1.8. Hence, the counterfactual move to the Indian delegation efficiency is quite close to the upper bound the model can deliver. The reason why high-type firms still keep growing in the US even in the absence of managerial delegation is that their own managerial efficiency endowment T still allows for some expansion along the life-cycle. If T was smaller, the quantitative impact of a deterioration in the delegation environment would be bigger. This, however, would also imply counterfactually low mark-ups in the US calibration.

-US India US w/  $\xi_{\text{IND}}^{\text{P}}$ US w/  $\xi_{\mathrm{IND}}$ Mean Employment 0-5 6-10 16-20 21-25 +26 Age

FIGURE 9: THE DELEGATION ENVIRONMENT AND THE LIFE-CYCLE OF PLANTS

Notes: The figure depicts the cross-sectional age-size relationship, i.e., average plant employment as a function of age. The data for the US correspond to the population of US manufacturing plants in 2012 and are taken from the BDS. The data for India corresponds to manufacturing plants from India in 2010 and are taken from the ASI and the NSS. The model corresponds to the US parameterization reported in Table 2 with the delegation efficiencies given by  $\xi_{IND}=0.608$  and  $\xi_{IND}^{P}=0.560$ , respectively.

faster at the expense of low types.

0.8

Number of Firms 9.0

0.2

0

0

5

10

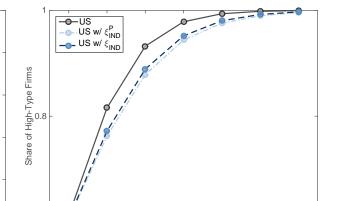
15 Age

20

25

30

FIGURE 10: SURVIVAL OF A COHORT



15 Age

20

25

30

5

10

FIGURE 11: THE SHARE OF HIGH-TYPE FIRMS

US w/  $\xi_{\text{IND}}^{\text{P}}$ US w/  $\xi_{\text{IND}}$ 

Notes: Figure 10 shows the size of an entering cohort by age. Figure 11 depicts the model's implication for the share of high-type firms by age. In both figures we show both the case of the US (i.e. the calibration reported in Table 2) and the counterfactuals with delegation efficiencies given by  $\xi_{IND} = 0.608$  and  $\xi_{IND}^P = 0.560$ .

Table 5 below finally contains some aggregate characteristics from the firm-size distribution to

measure the quantitative effect of this change in the delegation environment.

TABLE 5: THE EFFECTS OF THE DELEGATION ENVIRONMENT AT STEADY STATE

	US	US with $\xi_{IND}$	US with $\xi_{IND}^P$
Employment share by one-product firms	0.174	0.237	0.256
Employment share of firms without managers	0	0.237	0.525
Average firm size (rel to US)	1	0.765	0.713
Average innovation rate by high types	0.64	0.56	0.44
Rate of creative destruction	0.111	0.086	0.082
TFP growth rate	1.72%	1.34%	1.26%

Notes: The table contains different moments from the model. The average innovation rate by high types is defined as  $\sum_{n=1}^{\infty} x_n n \nu_n^H$  (see equation (21)). In column 1 we show the case of the US from Table 4. In columns 2 and 3 we report counterfactual results for the full exercise ( $\xi_{US} \to \xi_{IND} = 0.608$ ) and the partial exercise ( $\xi_{US} \to \xi_{IND}^P = 0.560$ ).

In the first column we report the numbers from the US for comparison (see Table 4). Columns 2 and 3 contain the details of the counterfactual exercises. The size of the average firm declines by 25-30% and the employment share of one-product producers increases by almost 40-50%. The reason for this reallocation is the decline in the average innovation intensity of high-type firms, which we report in row 4: the lower the delegation efficiency  $\xi$ , the less willing are high-type firms to expand. For the economy as a whole, this results in a decline of the rate of creative destruction. Note also that the delegation cutoff increases so that firms without outside managers now account for more than half of aggregate employment. The last row finally reports the growth implications: Because in our economy the rate of growth is proportional to the rate of creative destruction, the rate of growth declines by roughly 0.4-0.5 percentage point.

So far we only focused on counterfactuals that reduce the US delegation environment to the Indian level. Given the calibrated parameters for the Indian economy, we could also go the other way and ask what the Indian life-cycle would look like if the delegation environment were to improve to the US level. The difference between these two counterfactuals is informative about complementarities between the ease of delegation and other aspects of the economy. Intuitively, seamless delegation is probably of greater importance for the aggregate economy if dynamic firms are plentiful and their expansion technology is efficient. This is exactly what we find: While the rate of life-cycle growth for Indian firms would increase if the delegation environment were to improve, the quantitative effect is small.<sup>32</sup> In the US calibration, high types are abundant and the costs of innovation are low (i.e.,  $\alpha$  and  $\theta$  are high). Preventing these dynamic entrepreneurs from growing by subjecting them to the inefficient delegation environment of India is costly in terms of life-cycle growth. In contrast, in India transformative entrepreneurs are not only relatively scarce but also expand less efficiently (i.e.,  $\alpha$  and  $\theta$  are low). While there is a benefit to allowing these firms to sustain their expansion incentives through better delegation, the aggregate effects are more muted.

 $<sup>^{32}</sup>$ In particular, the average size of 21-25-year-old firms (relative to new entrants) would increase from 1.1 to 1.25. The full results of this exercise are available upon request.

### 5 Extensions and Robustness

In the final section of the paper, we discuss two extensions and the robustness of our results. Further details are contained in our accompanying Appendices.

### 5.1 Decomposing Delegation Efficiency

So far, we have shown that differences in delegation efficiencies  $\xi$  are quantitatively important to explain differences in the life-cycle of US and Indian plants. However, we did not need to take any stand on the exact determinants of the delegation environment. In this section, we extend our analysis one step further and attempt to understand the fundamental components of  $\xi$ .<sup>33</sup> In other words, we try to explicitly decompose this "country fixed effect" into different components using the structure of the theory. Our focus is on three widely studied fundamental differences across countries that the previous empirical growth and development literature has considered: the quality of legal institutions, human capital, and the degree of financial development.

Our empirical strategy is as follows. To shed light on the determinants of country c's delegation efficiency  $\xi_c$ , we assume that the delegation environment depends only on the three country characteristics mentioned above, i.e.,

$$\xi_c = \xi(HC_c, ROL_c, FD_c; \vartheta), \tag{36}$$

where  $HC_C$ ,  $ROL_c$  and  $FD_c$  are measures of human capital, the rule of law and financial development in country c and  $\vartheta$  denotes a vector of parameters of the function  $\xi(.)$ . Hence, (36) imposes a stable relationship between country fundamentals and the resulting delegation efficiencies and we will use our model to estimate  $\vartheta$ . In practice we impose the condition that  $\xi(.)$  is a linear function and we estimate  $\vartheta$  with GMM using data on managerial employment, the rule of law, human capital and financial development across countries.

To measure managerial employment we again draw on census information disseminated by IPUMS-International for a broad cross-section of countries and we adopt the exact same measurement choices as for the case of India and the US explained above. To measure the rule of law and the state of financial development, we use data from the World Bank Worldwide Governance Indicators and the World Bank Global Financial Development database. Finally, we extract information on human capital from the Penn World Tables.<sup>34</sup> We find that the three country characteristics are positively related to the delegation environment. In terms of the underlying data, this reflects the fact that they are positively correlated with the share of managerial employment.<sup>35</sup>

 $<sup>^{33}</sup>$ In this section we will only report the main results. See Section OA-2.5 in the Online Appendix for details.

<sup>&</sup>lt;sup>34</sup>See Section C.1 in the Appendix for further details.

 $<sup>^{35}</sup>$ In Section OA-2.7 in the Online Appendix, we also provide additional reduced-form evidence for this positive correlation. In particular, we consider various robustness checks using alternative measures. Additionally, we also show that there is a positive relationship between managerial employment and measures of trust, which is consistent with our cross-state results from India mentioned above. We do not explicitly include trust as a determinant of  $\xi_c$  in (36), because only 39 countries have data on both managerial employment and trust.

Given the estimated parameters  $\hat{\vartheta}$  we can use (36) to decompose the variation in inferred delegation efficiency  $\xi_c$  into its different components and gauge the implications for a plant's lifecycle. For instance, we can predict the counterfactual delegation efficiency in the US if it had the level of human capital of India by  $\xi(HC_{IND}, ROL_{US}, FD_{US}; \hat{\vartheta})$ . The predicted life-cycle implied by these delegation efficiencies can then be interpreted as the partial effect accounted for by changes in human capital. Using this intuition we can hence decompose the explained life-cycle difference between US and Indian plants into its different components.

The results are contained in Table 6 and Figures 12 and 13. Consider first Table 6, where we report the share of the differences in life-cycle growth accounted for by each margin by age of the cohort. In the first two rows we report the respective life-cycle profiles predicted by the estimated model for the US and India, i.e., based on  $\hat{\xi}_{US}$  and  $\hat{\xi}_{IND}$ .<sup>36</sup> In the remaining rows, we report the decomposition. Given our point estimates  $\hat{\vartheta}$ , differences in human capital explain roughly 54% of the predicted variation, differences in the rule of law account for about 41% and the remaining 5% are attributed to the financial system.<sup>37</sup> To quantify the uncertainty about these decompositions based on point estimates, we use a bootstrap procedure to provide confidence intervals for the decomposition results.<sup>38</sup> In Table 6 we also report the 25% and 75% quantiles of the respective distributions of explanatory power in brackets. Hence, while differences in human capital between the US and India account for 40% to 65% of the life-cycle differences of 21-25-year-old firms with 50% probability, the observed differences in financial development account for 1% to 9% of the observed differences.

In Figures 12 and 13 we depict these results graphically. The solid black line at the very top and the dashed red line at the very bottom in Figure 12 show the model's predictions for the US  $(\hat{\xi}_{US})$  and India  $(\hat{\xi}_{IND})$ , respectively. The three remaining dashed lines in between refer to the decompositions of the life-cycle. It is clearly seen that differences in the rule of law and human capital explain the vast majority of cross-country life-cycle differences due to delegation environment, with the level of financial development playing a minor role. In Figure 13 we depict the bootstrap distributions of the respective contributions of the three country characteristics around their point estimates. We focus on the cohort of firms of age 21-25. While both human capital and the legal environment account for the bulk of variation of life-cycle differences across countries, the sampling variation in the parameter estimates  $\vartheta$  introduces noise into the model's implications. In particular, given the data, we cannot rule out that legal institutions (or human capital) account for anything between 20 - 80% of the implied life-cycle differences. In order to learn more about

<sup>&</sup>lt;sup>36</sup>Note that the results for India are similar to our partial exercise (based on  $\xi^P$ ) above. If the parameterization in (36) would match the observed Indian managerial share perfectly, the results would be identical.

<sup>&</sup>lt;sup>37</sup>For the cohort of firms older than 25 years, we find the following: The model explains 89% of the variation and the respective explanatory power of the rule of law, human capital and financial development is 41.8%, 52.8% and 5.3%.

<sup>&</sup>lt;sup>38</sup>Intuitively, we generate 10.000 samples from our cross-sectional data across countries with replacement and redo our estimation and decomposition. Hence, the respective decompositions take into account the joint distribution of country fundamentals and are not based on the marginal effects. See Section OA-2.6 in the Online Appendix for details of the bootstrap procedure.

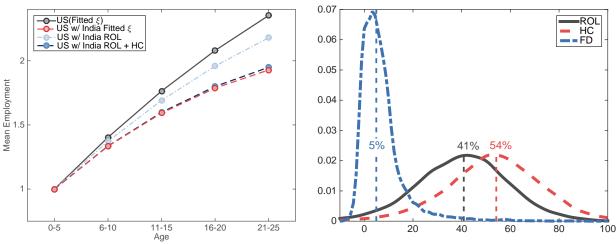
Table 6: Decomposition of Life-Cycle Differences by Cohort Age

		Age					
	6-10	11-15	16-20	21-25			
Predicted Life-cycle US	1.41	1.76	2.08	2.35			
Predicted Life-cycle India	1.34	1.59	1.79	1.92			
Human Capital	53.5	54.0	54.2	54.2			
	[38.8; 65.5]	[39.9; 65.7]	[40.4; 65.8]	[40.6; 65.7]			
Rule of Law	43.8	42.4	41.4	40.8			
	[30.6; 57.3]	[29.3; 55.2]	[28.5; 53.9]	[28.1; 53.0]			
Financial Development	2.60	3.61	4.39	4.93			
	[0.7.; 4.75]	[1.0; 6.7]	[1.2; 8.3]	[1.4; 9.4]			

Notes: This table reports the decomposition results of the differences in the life-cycle profile between the US and India. In the first row, we report the US life-cycle profile predicted by the model using  $\hat{\xi}_{US}$ . The second row corresponds to the Indian life-cycle profile predicted by the model using  $\hat{\xi}_{IND}$ . The remaining rows decompose the difference between rows 2 and 3 into the respective components. We report the decomposition based on the point estimates reported in Table OA-6 and the 25% and 75% quantiles of the bootstrapped distribution in brackets. We estimate the bootstrap distribution using 10.000 iterations. See Section OA-2.6 in the Online Appendix for details.

the exact institutional underpinnings of the observed differences in firm dynamics, one would need more detailed micro data and credible exogenous variation in the institutional environment firms participate in and we leave this for further research.

FIGURE 12: DECOMPOSING THE LIFE CYCLE FIGURE 13: BOOTSTRAPPED CONTRIBUTIONS



Notes: Figure 12 shows the decomposition of the US lifecycle into the different components of delegation efficiency. The decomposition is done based on (36) and the estimated  $\hat{\vartheta}$ . Figure 13 shows the distribution of the contributions of the three country characteristics to explain the predicted difference in plants' life-cycle between India and the US. We focus on the cohort of firms of age 21-25. We depict the point estimates (which refer to to the column "Age 21-25" in Table 6) as dashed lines. We estimate the distribution of these marginal effects using a bootstrap procedure with 10.000 iterations. See Section OA-2.6 in the Online Appendix for details.

## 5.2 The Importance of the Delegation Environment $\xi$

In this section we discuss in what sense other differences besides the delegation environment could explain both the low managerial share and the shallow life-cycle in India.

From Firm Size to Managerial Demand Consider first the low managerial employment share in India. Although our calibration yields a lower value for  $\xi$  to explain the low managerial employment share in India, still, it is natural to wonder whether the model is able to explain this "lack" of hiring outside managers simply from differences in the firm-size distribution. Because our model predicts a complementarity between firm size and managerial hiring, it seems intuitive that the aggregate demand for managerial personnel will be low whenever the economy is populated mainly by small firms, even if the delegation efficiency in India was similar to the one in the US.

To analyze this possibility, we kept the delegation efficiency at the US level ( $\xi_{US}$ ), but recalibrated the five parameters pertaining to the process of firm dynamics, i.e., ( $\alpha, z, \beta, \theta, \gamma$ ), to match all the Indian moments in Table 3 except the managerial employment share ( $M_5$ ). Hence, this calibration asks the following question: Could alternative theories that directly affect the firm-size distribution without directly affecting the demand for managerial personnel explain the low managerial employment share in India? The answer is a clear "no." While the model matches the four firm-level moments perfectly, the equilibrium share of outside managers is 13.9%, which is slightly higher than in the US.<sup>39</sup> The reason is the general equilibrium adjustment of wages. Precisely because there are few large firms in India, wages will fall to clear the labor market. This, however, will increase firms' incentives to hire managers, holding firm size constant. In the aggregate, the new equilibrium employment share of managers will therefore not fall substantially. To explain the absence of outside managers in India, it therefore has to be the case that the delegation efficiency in India is lower than that in the US.

Alternative Mechanism for the Shallow Life-Cycle Consider now the importance of other margins to explain the observed life-cycle differences between the US economy and India. As for our main counterfactual exercise, we studied the implications for the life-cycle of US firms if we were to change the rate of entry (z), the share of high types  $(\alpha)$  or the innovation efficiency  $(\theta)$  in the US to its Indian counterpart. Figure 14 presents the resulting life-cycle patterns. As seen from the figure, neither the cross-country differences in entry nor the share of innovative firms can account for the differences in plants' life-cycle. While increases in the entry rate will shift resources from old, large incumbents to new, small entrants and hence reduce the life-cycle profile, the entry rate in India is not very different from that in the US. Hence, the quantitative effect is negligible. As for the share of high-type firms  $\alpha$ , old firms would be even bigger if there were as few high types in the US as there are in India. The reason is that a higher share of innovative types implies more competition between high types, which makes it harder for old firms to grow large.

<sup>&</sup>lt;sup>39</sup>The resulting structural parameters for that calibration are  $(\alpha, z, \beta, \theta, \gamma) = (0.136, 0.079, 0.445, 0.910, 1.44)$ .

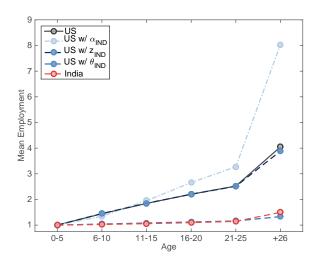


FIGURE 14: THE LIFE-CYCLE: OTHER MARGINS

Notes: Figure 14 shows the plants' life-cycle in the US, India and the counterfactual US economy with the Indian share of high-type firms  $(\alpha_{IND})$ , the Indian rate of entry  $(z_{IND})$  and the Indian expansion technology  $(\theta_{IND})$ .

The case of expansion efficiency  $\theta$  is different. While the implied life-cycle would indeed be quite close to the one observed in India, differences in the productivity of expansion cannot explain both the facts on life-cycle growth and managerial employment patterns. In the data we see that the Indian economy is characterized by (i) a shallow life-cycle profile, (ii) a low managerial employment share and (iii) a large number of producers who do not use outside managers. While a less efficient delegation environment  $\xi$  in India will qualitatively readily imply all three of these facts, higher costs of innovation will counterfactually predict that more firms will actually use managers. The reason is simple. If it was only the case that Indian entrepreneurs were inefficient in expanding their market share, there would be fewer large firms, which would put downward pressure on managerial wages. The marginal firm would therefore be more willing to hire managerial personnel. Hence, if the US and India differed only in their expansion technology  $\theta$ , we would expect the share of firms without any managers to be higher in the US. This is clearly counterfactual. To match the data on life-cycle growth and managerial employment, one requires variation in the efficiency of delegation  $\xi$ .

### 5.3 Robustness of Quantitative Results

In Section D in the Appendix and Section OA-2.3 in the Online Appendix we discuss in detail three robustness checks for our counterfactuals. In Section D, we reestimate the model allowing for differences in owner efficiency T and the elasticity of managerial effort  $\sigma$  between India and the US. We provide four specifications, where we target different values for average mark-ups in India. These calibrations give results very similar to our baseline counterfactual results reported in Section 4 above. There we found that the India delegation efficiency  $\xi_{IND}$  would reduce the rate of life-cycle

growth to about 2 for firms of age 21-25 (see Figure 9). For our alternative specifications, we find slightly larger results: A decline in the delegation efficiency to the level observed in India would reduce the life-cycle to about 1.85. There we also report the sensitivity of our results with respect to the share of managerial compensation - a moment that is quite noisy in the data. Varying the managerial compensation share between 40% and 55% of aggregate profits (recall that we target 49% for our baseline analysis) does not change the impact of changes in the delegation environment on the implied life-cycle.

In Section OA-2.3 in the Online Appendix we redo our analysis using the data for firms (instead of plants). As we cannot link plants to firms in the Indian economy, we have to assume that all firms in India own a single plant. This is probably accurate for the vast majority of producers in India. For the US, focusing on firms changes the following three micro moments: the entry rate declines (because many entering plants are new plants of existing firms), the employment share of firms of age 21-25 declines (because very old firms are much bigger than old plants because they have multiple plants) and the relative exit rate of old and young firms (conditional on size) increases slightly. Surprisingly, the life-cycle is not too different for firms of age 21-25; the most striking differences between firms and plants occur only for very old firms, which are older than 25 years. As far as the counterfactuals are concerned, the implications for the model calibrated to firm-level data are very similar to the ones reported in Figure 9 above.

### 6 Conclusion

This paper studies the reasons behind the stark differences in firm dynamics across countries. We focus on manufacturing plants in India and analyze the stagnant firm behavior. We show that the stagnant life-cycle behavior in India could be explained by the lack of firm selection, wherein firms with little growth potential survive because innovative firms do not expand sufficiently to push them out of the economy. Our theory stresses the role of imperfect managerial delegation as a major cause of the insufficient expansion by the firms with growth potential. We show that if the delegation efficiency in a country is low, firms will quickly run into decreasing returns. This in turn will reduce the incentives to grow. Quantitatively, such limits to delegation can explain an important fraction of the difference in life-cycle growth between the US and India. Finally, we decompose the delegation efficiency into different components and we show that improvements in the degree of contract enforcement and improvements in human capital can raise the expansion incentives and increase the degree of creative destruction.

Our findings emphasize the importance of managerial delegation for firm selection. Our analysis also highlights the fact that many low-type small firms in developing countries are able to survive due to lack of competition. An important next step in this research agenda is to incorporate these findings into the study of industrial policies in developing countries. For instance, many regulations that support and facilitate the survival of small firms might have undesired consequences once the heterogeneity in firm types are taken into account. The quantitative implications of industrial

policies for firm selection and firm dynamics are first-order issues that await future research.

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# **Appendices**

# A Theoretical Appendix

### A.1 Static Equilibrium

On the demand side, we have a representative household with standard preferences

$$U_0 = \int_0^\infty \exp(-\rho t) \ln C_t dt,$$

where  $\rho > 0$  is the discount factor. Given the unitary intertemporal elasticity of substitution, the Euler equation along the balanced growth path is simply given by  $g = r - \rho$ , where g is the growth rate of the economy and r is the interest rate.

Now consider the equilibrium in the product market. At each point in time, each product line j is populated by a set of firms that can produce this good with productivity  $\left[q_{jt}^f\right]_f$ , where f identifies the firm. As firm f sets a price equal to  $p_{jf}=q_{jf}^{-1}w_t$  we get that

$$ln(Y_t) = \int_0^1 ln(y_{jt})dj = \int_0^1 ln(p_{jt}y_{jt})dj - \int_0^1 ln(p_{jt})dj = ln(Y_t) - ln(w_t) + \int_0^1 ln(q_{jt})dj$$

which implies  $w_t = Q_t$ .

The production function (see (2)) also implies that

$$ln(L_t^P) = \int_0^1 ln(l_{jt})dj = \int_0^1 ln(y_{jt})dj - \int_0^1 ln(q_{jt})dj - \int_0^1 ln(\mu(e_{jt}))dj,$$

so that

$$L_t^P = \frac{Y_t}{Q_t \mathcal{M}_t} = \frac{1}{\mathcal{M}_t} \frac{1}{\omega_t},$$

where  $\omega_t = \frac{w_t}{Y_t}$  and  $\mathcal{M}$  is defined in (7).

### A.2 Proof of Proposition 2

To prove Proposition 2 it is useful to work with the delegation cutoff

$$n^* = \frac{T}{\psi},\tag{37}$$

which is simply a transformation of  $\psi$ . A stationary equilibrium in this economy is defined in the usual way.

**Definition 1** A stationary equilibrium consists of firms' demand schedules for managers and production workers  $[m_j, l_j]$ , firms' innovation rates  $[x_j]$ , measures of low- and high-type firms  $(F^L, F^H)$ ,

a distribution of high-type firms across products  $\nu_n^H$  and a delegation cutoff  $n^*$ , such that

- 1.  $[l_j, m_j, x_j]$  are consistent with firms' profit maximization problem,
- 2.  $(F^L, F^H)$  and  $[\nu_n^H]_n$  are consistent with firms' optimal innovation rates  $[x_j]$  and the law of motion (22),
- 3.  $n^*$  is consistent with labor market clearing, i.e.

$$1 = \sum_{n=1}^{\infty} l(n)nF^{H}\nu_{n}^{H} + l(1)F^{L} + \sum_{n>n^{*}}^{\infty} m(n)nF^{H}\nu_{n}^{H},$$

where  $l(n)n = l_{j(n)}$  if firm j has n products in its portfolio and for m(n) similarly.

We now prove the existence and uniqueness of a stationary equilibrium in our economy. We proceed in two steps. First, we will argue that there is a unique stationary distribution for a given  $n^*$ . Then we will show that there is a unique  $n^*$  consistent with labor market clearing, taking the dependence of the stationary distribution on  $n^*$  into account.

Step 1: The Stationary Distribution Given  $n^*$  The stationary distribution is fully determined by  $[x_n]$  and the parameters  $(\alpha, z)$ . The innovation intensities  $[x_n]$  are given by (see (19))

$$x_n = A \times \max\left\{n^{-\lambda}, (n^*)^{-\lambda}\right\},\,$$

where A and  $\lambda$  are constants given in Proposition 2. Hence, given  $n^*$ ,  $[x_n]$  are known. We will now construct the stationary distribution.

The stationary distribution is described by equations (21) and (22) and the requirement that  $\nu_n^H$  be a proper distribution

$$\sum_{n=1}^{\infty} \nu_n^H = 1. \tag{38}$$

We need to find  $F^H$ ,  $F^L$ ,  $\left[\nu_n^H\right]_{n=1}^{\infty}$ . Let  $\nu_1^H$  and  $\tau$  be given. From (22) we get  $F^L$ ,  $F^H$  and  $\left[\nu_n\right]_{n=2}^{\infty}$ . Then we can use (22) and (38) to find  $\tau$  and  $\nu_1^H$ . We now solve explicitly for these objects.

**Lemma 1** The distribution of high types takes the form

$$\nu_n^H n = \frac{\prod_{j=1}^n x_j}{\tau^n} \frac{\tau}{x_n} \nu_1. \tag{39}$$

**Proof.** Substituting (39) in (22) shows that if  $\nu_n^H$  satisfies (39), it satisfies all the flow equations in (22).

Hence, we can use (38) to get

$$1 = \sum_{n=1}^{\infty} \nu_n^H = \sum_{n=1}^{\infty} \frac{\prod_{j=1}^n x_j}{\tau^n} \frac{\tau}{x_n} \frac{1}{n} \nu_1^H = \nu_1^H \sum_{n=1}^{\infty} \frac{1}{n} \frac{\tau}{x_n} \prod_{j=1}^n \left(\frac{x_j}{\tau}\right),$$

so that

$$\nu_1^H = \left(\sum_{n=1}^{\infty} \frac{1}{n} \frac{\tau}{x_n} \prod_{j=1}^n \left(\frac{x_j}{\tau}\right)\right)^{-1}.$$
 (40)

Hence, (39) implies that

$$\nu_n^H = \frac{1}{n} \frac{\prod_{j=1}^n x_j}{\tau^n} \frac{\tau}{x_n} \frac{1}{\sum_{n=1}^\infty \frac{1}{n} \frac{\tau}{x_n} \prod_{j=1}^n \left(\frac{x_j}{\tau}\right)}.$$
 (41)

Using (40) we get from (22) that

$$F^H = \frac{\alpha z}{\tau} \times \left[ \sum_{n=1}^{\infty} \frac{1}{n} \frac{\tau}{x_n} \prod_{j=1}^n \left( \frac{x_j}{\tau} \right) \right] \quad \text{and} \quad F^L = \frac{(1-\alpha)z}{\tau}.$$

Hence, we only need to determine  $\tau$ , which we get from (21) as

$$\tau = \sum_{n=1}^{\infty} n x_n \nu_n^H F^H + z = \left[ \sum_{n=1}^{\infty} \alpha \left( \prod_{j=1}^n \left( \frac{x_j}{\tau} \right) \right) + 1 \right] z. \tag{42}$$

Note that given z > 0, (i) as  $\tau \to \infty$ , the LHS of (42) is greater than the RHS and (ii) as  $\tau \to \min\{x_j\}_j$ , the RHS is greater than the LHS. Together with the continuity of (42), this implies that there exists a finite  $\tau > \min\{x_j\}_j$  which solves (42). Moreover, the LHS is increasing in  $\tau$ , and the RHS is decreasing in  $\tau$ . Hence, there is a unique  $\tau$ . Note also that this solution is consistent with the fact that the total product space is of measure one. This proves the first part of Proposition 2, in particular (28)-(31).

Step 2: Uniqueness of  $n^*$  Now we will argue that there is a unique  $n^*$ , which is consistent with labor market clearing, i.e., (32). Using (6) and (7) we get the labor market clearing condition

$$1 = L_D + M_D = \frac{1}{\omega_t \mathcal{M}_t} + M_D, \tag{43}$$

where  $L_D$  and  $M_D$  are the total demand for production workers and managerial workers, respectively. First note that

$$M_D(n^*) = \sum_{n=1}^{\infty} m_n(n^*) \varphi_n(n^*) = \sum_{n \ge n^*}^{\infty} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n(n^*),$$
 (44)

where  $\varphi_1 = F^L + F^H \nu_1^H$  and  $\varphi_n = F^H n \nu_n^H$  for n > 1. Now consider  $L_D$ . From the definition of  $n^*$  (see (37)) and (11) we get that

$$\omega = (n^*)^{1-\sigma} \frac{\xi \sigma}{T^{1-\sigma}},$$

so that

$$L_D(n^*) = \frac{1}{\omega_t \mathcal{M}_t} = \frac{T^{1-\sigma}}{\sigma \xi} (n^*)^{-(1-\sigma)} \times \frac{1}{\mathcal{M}_t}, \tag{45}$$

where

$$\mathcal{M}_t(n^*) = \left[\sum_{n=1}^{\infty} \left(1 - e_n(n^*)^{\sigma}\right) \varphi_n(n^*)\right]^{-1}.$$
(46)

Hence, (43), (44), (45) and (46) imply that

$$1 = \frac{T^{1-\sigma}}{\sigma\xi} (n^*)^{-(1-\sigma)} \left[ \sum_{n=1}^{\infty} (1 - e_n (n^*)^{\sigma}) \varphi_n (n^*) \right] + \sum_{n>n^*}^{\infty} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n (n^*).$$
 (47)

This is one equation in one unknown  $n^*$ . In Section OA-1.2 in the Online Appendix we show that as long as  $\frac{1-T}{\xi} > 1$  (47) has a unique solution and that the RHS of (47) is strictly decreasing in  $n^*$ .<sup>40</sup>

# A.3 Proof of Proposition 3

We are going to prove the different parts of the Proposition in turn. First, we show that  $\psi$  is increasing in  $\xi$ . Recall that  $n^* = \frac{T}{\psi}$ , where  $n^*$  is implicitly defined in (47). We showed that the RHS of (47) is decreasing in  $n^*$ . Hence  $n^*$  is decreasing in  $\xi$ , which implies that  $\frac{\partial \psi}{\partial \xi} > 0$ . Then we turn to the comparative static results of Proposition 3:

- 1. Obvious from the definition of  $x_n$  in (19).
- 2.  $\tau$  is uniquely defined by (see (42))

$$\tau = \left[ \sum_{n=1}^{\infty} \alpha \left( \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \right) + 1 \right] z. \tag{48}$$

As  $[x_n]_{n\geq n^*}$  is strictly decreasing in  $n^*$  and  $[x_n]_{n< n^*}$  is not a function of  $n^*$ , (48) directly implies that  $\tau$  is decreasing in  $n^*$  and hence increasing in  $\xi$  and  $\psi$ .

- 3. Follows directly from the fact that  $\tau$  is increasing in  $\xi$  and  $\psi$  and from  $F^L = \frac{(1-\alpha)z}{\tau}$ .
- 4. Follows directly from  $\chi^{H}\left(n^{*}\right)=1-F^{L}\left(n^{*}\right)$  (see 34).
- 5. From (33) and we get that

$$\Phi_n(n^*) = \sum_{j=1}^n \varphi_j(n^*) = F^L + \sum_{j=1}^n F^H \nu_j^H j.$$

 $<sup>^{40}</sup>$ The condition  $\frac{1-T}{\xi} > 1$  is a sufficient condition. It is satisfied in our calibration.

Using Proposition 2 we get

$$\Phi_n(n^*) = \frac{(1-\alpha)z}{\tau} + \sum_{j=1}^n \frac{\alpha z}{\tau} \frac{\tau}{x_j} \prod_{r=1}^j \left(\frac{x_r}{\tau}\right) = \frac{z}{\tau} \left[ 1 + \alpha \sum_{j=1}^{n-1} \prod_{r=1}^j \left(\frac{x_r}{\tau}\right) \right]. \tag{49}$$

Moreover, note that for any  $n_1^*$  and  $n_2^*$ , we have

$$\lim_{n \to \infty} \Phi_n\left(n_1^*\right) = \lim_{n \to \infty} \Phi_n\left(n_2^*\right) = 1 \tag{50}$$

and  $\Phi_0(n_1^*) = \Phi_0(n_2^*) = 0$ .

Now consider  $n_1^* < n_2^*$ . We are going to show that  $\Phi_n(n_2^*) > \Phi_n(n_1^*)$  for all n. Define the function

$$u_n(n^*) \equiv \frac{x_n(n^*)}{\tau(n^*)},\tag{51}$$

which satisfies the following property: there is  $n_1^* < \overline{n} < n_2^*$  such that

$$u_n(n_2^*) > u_n(n_1^*) \text{ for } n < \overline{n}$$
  
 $u_n(n_2^*) < u_n(n_1^*) \text{ for } n \ge \overline{n}.$  (52)

To see why, note first that  $x_n(n_1^*) = x_n(n_2^*)$  for all  $n \leq n_1^*$ . Because  $\tau(n_2^*) < \tau(n_1^*)$ , (51) implies that  $u_n(n_2^*) > u_n(n_1^*)$  for  $n \leq n_1^*$ . Now suppose there was no  $\overline{n}$ , such that  $u_n(n_2^*)$  and  $u_n(n_1^*)$  were to cross. Then  $u_n(n_2^*) > u_n(n_1^*)$  for all n. (48), however, would then imply that  $\tau(n_2^*) > \tau(n_1^*)$ , which is a contradiction. Hence,  $\overline{n}$  exists. Because both  $u_n(n_2^*)$  and  $u_n(n_1^*)$  are constant for  $n \geq n_2^*$ , it has to be that case the  $n_1^* < \overline{n} < n_2^*$ . Using (51), (49) implies that

$$\Phi_n(n^*) = \frac{z}{\tau(n^*)} \left[ 1 + \alpha \sum_{j=1}^{n-1} \prod_{r=1}^{j} u_r(n^*) \right].$$
 (53)

Because  $\tau(n_2^*) < \tau(n_1^*)$ , (52) implies that

$$\Phi_n\left(n_2^*\right) > \Phi_n\left(n_1^*\right) \text{ for all } n \le \overline{n}.$$
(54)

To show that (54) holds for all n, it suffices to show that  $\Phi_n(n_2^*)$  and  $\Phi_n(n_1^*)$  never cross. We proceed by contradiction. Suppose there was  $\tilde{n} > \overline{n}$  such that

$$\Phi_{\tilde{n}-1}(n_2^*) > \Phi_{\tilde{n}-1}(n_1^*) \text{ and } \Phi_{\tilde{n}}(n_2^*) \le \Phi_{\tilde{n}}(n_1^*).$$
 (55)

(53) implies that

$$\Phi_n(n^*) = \Phi_{n-1}(n^*) + \frac{z}{\tau(n^*)} \prod_{r=1}^{n-1} u_r(n^*).$$
 (56)

Hence, (55) and (56) yield

$$\frac{z}{\tau(n_{1}^{*})} \prod_{r=1}^{\tilde{n}-1} u_{r}(n_{1}^{*}) - \frac{z}{\tau(n_{2}^{*})} \prod_{r=1}^{\tilde{n}-1} u_{r}(n_{2}^{*}) = \Phi_{\tilde{n}}(n_{1}^{*}) - \Phi_{\tilde{n}-1}(n_{1}^{*}) - (\Phi_{\tilde{n}}(n_{2}^{*}) - \Phi_{\tilde{n}-1}(n_{2}^{*}))$$

$$= \Phi_{\tilde{n}}(n_{1}^{*}) - \Phi_{\tilde{n}}(n_{2}^{*}) + \Phi_{\tilde{n}-1}(n_{2}^{*}) - \Phi_{\tilde{n}-1}(n_{1}^{*})$$

$$> 0,$$

so that

$$\frac{z}{\tau(n_1^*)} \prod_{r=1}^{\tilde{n}-1} u_r(n_1^*) > \frac{z}{\tau(n_2^*)} \prod_{r=1}^{\tilde{n}-1} u_r(n_2^*). \tag{57}$$

Now consider  $n > \tilde{n}$  and define

$$\Delta_n(n^*) \equiv \Phi_n(n^*) - \Phi_{n-1}(n^*) = \frac{z}{\tau(n^*)} \prod_{r=1}^{n-1} u_r(n^*).$$

Then,

$$\Delta_{n}(n_{1}^{*}) - \Delta_{n}(n_{2}^{*}) = \frac{z}{\tau(n_{1}^{*})} \prod_{r=1}^{n-1} u_{r}(n_{1}^{*}) - \frac{z}{\tau(n_{2}^{*})} \prod_{r=1}^{n-1} u_{r}(n_{2}^{*})$$

$$= \frac{z}{\tau(n_{1}^{*})} \prod_{r=1}^{\tilde{n}-1} u_{r}(n_{1}^{*}) \prod_{r=\tilde{n}}^{n-1} u_{r}(n_{1}^{*}) - \frac{z}{\tau(n_{2}^{*})} \prod_{r=1}^{\tilde{n}-1} u_{r}(n_{2}^{*}) \prod_{r=\tilde{n}}^{n-1} u_{r}(n_{2}^{*})$$

$$> \left\{ \frac{z}{\tau(n_{1}^{*})} \prod_{r=1}^{\tilde{n}-1} u_{r}(n_{1}^{*}) - \frac{z}{\tau(n_{2}^{*})} \prod_{r=1}^{\tilde{n}-1} u_{r}(n_{2}^{*}) \right\} \prod_{r=\tilde{n}}^{n-1} u_{r}(n_{1}^{*}) > 0,$$

where the first inequality follows from (52) and the last inequality follows from (57). Hence, for all  $n > \overline{n}$ 

$$\Phi_{n}(n_{1}^{*}) - \Phi_{n}(n_{2}^{*}) = \Phi_{\overline{n}}(n_{1}^{*}) - \Phi_{\overline{n}}(n_{2}^{*}) + \sum_{j=\overline{n}}^{n} \left[ \Delta_{n}(n_{1}^{*}) - \Delta_{n}(n_{2}^{*}) \right] > 0.$$

Furthermore,  $\Phi_n(n_1^*) - \Phi_n(n_2^*)$  is strictly increasing so that

$$\lim_{n \to \infty} \left[ \Phi_n \left( n_1^* \right) - \Phi_n \left( n_2^* \right) \right] > 0.$$

This violates (50). Hence,  $\Phi_n(n_2^*) > \Phi_n(n_1^*)$  for all n.

6. The fact that average firm size is increasing in  $\xi$  and  $\psi$  follows directly from the fact that  $\Phi_n(n^*)$  is increasing in  $n^*$  as the product space has size one.

# B Identification of the Model

We will now discuss the identification of our model. The analytical results for the stationary firmsize distribution makes our identification approach transparent. In total there are 8 parameters to identify

$$(T, \theta, \sigma, \xi, \alpha, \gamma, z, \beta)$$

In Proposition 2 we showed that the process of firm dynamics and the stationary distribution is fully determined by the share of high types  $\alpha$ , the entry rate z and the expansion schedule  $[x_n]_n$ . We will first show that we can prove a similar result in our extended model, where high-type firms have a different exit probability parameterized by  $\beta$ .

Suppose that high types lose their products only with rate  $\beta$  conditional on some other firm innovating in their product line. As innovations are undirected, the probability of a given innovation being successful is given by

$$\Gamma = 1 - \chi + \beta \chi,\tag{58}$$

where  $\chi$  is the share of products populated by high-type firms. Hence, the optimal degree of innovation effort is given by (see (17))

$$X_n = \arg\max_{X} \left\{ X\Gamma \left[ V_t(n+1) - V_t(n) \right] - Y_t \left[ \frac{X_n}{\theta n^{1-\zeta}} \right]^{\frac{1}{\zeta}} \right\}.$$

The optimal innovation rate per product line,  $x_n = X_n/n$  is then given by

$$x_n = (1 - \sigma)^{\frac{\zeta}{1 - \zeta}} \Gamma^{\frac{\zeta}{1 - \zeta}} \theta^{\frac{1}{1 - \zeta}} \zeta^{\frac{\zeta}{1 - \zeta}} \times \max \left\{ \left( \frac{T}{n} \right)^{\frac{\zeta \sigma}{1 - \zeta}}, \psi^{\frac{\zeta \sigma}{1 - \zeta}} \right\}.$$
 (59)

Using again the delegation cutoff  $n^* = \frac{T}{\psi}$ , (59) reduces to

$$x_n = A \times \max\left\{n^{-\lambda}, (n^*)^{-\lambda}\right\},\tag{60}$$

where

$$A = [(1 - \sigma)\Gamma\zeta T^{\sigma}\theta^{\frac{1}{\zeta}}]^{\frac{\zeta}{1-\zeta}}$$
 (61)

$$\lambda = \frac{\zeta \sigma}{1 - \zeta}.\tag{62}$$

Hence, (60) fully parameterizes firms' innovation effort as a function of A (which is endogenous due to the dependence on  $\Gamma$ ),  $n^*$  and  $\sigma$ , i.e.,  $x_n = x_n(A, \sigma, n^*)^{41}$  Given (60), let the *net* rates of innovation, entry and destruction be  $(\hat{\tau}^H, \hat{\tau}^L, \hat{x}_n^H, \hat{z})$ . These are given by  $\hat{\tau}_H = \beta \tau$ ,  $\hat{\tau}_L = \tau$ ,  $\hat{x}_n^H = \Gamma \times x_n^H$  and  $\hat{z} = \Gamma \times z$ , where  $\chi = F^H \sum_{n=1}^{\infty} \nu_n^H n = 1 - F_L$  and  $\Gamma$  is given in (58). The (gross)

<sup>&</sup>lt;sup>41</sup>Recall that we set  $\zeta$  exogenously.

rate of creative destruction is given by

$$\tau = F^H \sum_{n=1}^{\infty} \nu_n^H x_n n + z. \tag{63}$$

As the flow equations in (22) still apply, Lemma 1 yields

$$\nu_n^H = \frac{n^{-1} \frac{\hat{\tau}_H}{\hat{x}_n^H} \prod_{j=1}^n \left(\frac{\hat{x}_j^H}{\hat{\tau}_H}\right)}{\sum_{s=1}^\infty s^{-1} \frac{\hat{\tau}_H}{\hat{x}_s^H} \prod_{j=1}^s \left(\frac{\hat{x}_j^H}{\hat{\tau}_H}\right)}.$$
 (64)

This determines the distribution of high types as a function of  $(\hat{x}_n^H, \hat{\tau}^H)$ . Using (22) we get that

$$\chi = 1 - F^L = 1 - \frac{(1 - \alpha)\hat{z}}{\hat{\tau}^L} = 1 - \frac{(1 - \alpha)\Gamma z}{\tau}.$$

Finally, we have the definition of  $\tau$  (see (63)) that

$$\tau = \sum_{n=1}^{\infty} x_n F^H \nu_n^H n + z = z \left[ \alpha \sum_{n=1}^{\infty} \left( \frac{\prod_{j=1}^n (\Gamma x_j)}{(\beta \tau)^n} \right) + 1 \right].$$

Hence, given the gross rates  $(z, x_n)$  and the parameters  $(\alpha, \beta)$  we get three equations in the unknowns  $(\tau, \Gamma, \chi)$  which are

$$\tau = z \left[ \alpha \sum_{n=1}^{\infty} \left( \frac{\prod_{j=1}^{n} (\Gamma x_j)}{(\beta \tau)^n} \right) + 1 \right]$$
 (65)

$$\Gamma = 1 - \chi + \beta \chi \tag{66}$$

$$\chi = \frac{\alpha \Gamma z}{\beta \tau} \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{\prod_{j=1}^{n} (\Gamma x_j)}{(\beta \tau)^n} \right) \right]. \tag{67}$$

These determine  $(\tau, \Gamma, \chi)$  and hence  $(\hat{\tau}^H, \hat{\tau}^L, \hat{x}_n^H, \hat{z})$ , which then fully determine  $(F^H, F^L, \nu_n^H)$ . Hence, the entire distribution of firms and the number of firms boil down to solving three equations in three parameters (65)-(67), which we get in closed form as a function of "fundaments"  $(z, x_n)$ .

To understand our identification strategy, fix  $n^*$  and suppose that  $\sigma$  was given. The above then shows that from the firm-level data we can at most identify the four objects  $(\alpha, z, A, \beta)$ . We do so using the following four pieces of information (and for concreteness we also report the main targets with an " $\rightarrow$ "):

### 1. The entry rate

$$M_1 = \frac{\hat{z}}{F^L + F^H} = \frac{\Gamma z}{F^L + F^H} \to z$$

2. The relative size of firms age 21-25 relative to young firms

$$M_2 = \frac{\sum_{a=21}^{25} \bar{l}_a \frac{N_a}{\sum_{a=21}^{25} N_a}}{\sum_{a=0}^{5} \bar{l}_a \frac{N_a}{\sum_{a=0}^{5} N_a}} \to A,$$

where  $\bar{l}_a$  and  $N_a$  denote average employment and the number of firms of age a.

3. The ratio of aggregate employment of old firms relative to young firms

$$M_3 = \frac{\sum_{a=21}^{25} \bar{l}_a N_a}{\sum_{a=0}^{5} \bar{l}_a N_a} \to \beta$$

4. The relative exit rate of young versus old firms conditional on size, i.e.

$$M_4 = \frac{\text{exit}(a = 21 - 25|n = 1)}{\text{exit}(a = 1 - 5|n = 1)} \to \alpha$$

These four moments will identify the parameters, because there is a unique distribution of firm size given  $(n^*, \alpha, z, A, \beta)$ . That z is informative about the entry rate is intuitive. Similarly, A determines the level of innovation effort, which hence governs the speed at which firms grow. Hence, it is informative about the slope of the life-cycle. As  $\beta$  effectively controls the size of old cohorts (as it determines the speed with which high-type firms exit), it is related to the aggregate importance of old cohorts in the economy. Finally, the exit hazard conditional on size is informative about the degree of selection. If there was no type heterogeneity, the exit rate would only be a function of size. To the extent that older firms are positively selected, they are less likely to exit conditional on size. The ex-ante heterogeneity  $\alpha$  determines how strong this effect can be.

We then use two moments related to managerial occupations, namely, the managerial employment share and the compensation of managers relative to corporate profits, and average mark-ups to identify  $\sigma$ ,  $\xi$  and T. Consider first  $\sigma$ . The total compensation for managerial personnel relative to aggregate profits is

$$\frac{wM_D}{\Pi} = \frac{\sum_{n=1}^{\infty} wm_n \varphi_n\left(n^*\right)}{\sum_{n=1}^{\infty} \pi_n \varphi_n\left(n^*\right)} = \frac{\sum_{n=1}^{\infty} \omega m_n \varphi_n\left(n^*\right)}{\sum_{n=1}^{\infty} \tilde{\pi}_n \varphi_n\left(n^*\right)}.$$

Using that  $m_n = T\xi^{-1} \times \max\left\{0, (n^*)^{-1} - (n)^{-1}\right\}$  and  $\tilde{\pi}_n = \left(T \max\left\{n^{-1}, (n^*)^{-1}\right\}\right)^{\sigma}$ , we get that

$$\frac{wM_D}{\Pi} = \sigma \frac{\sum_{n=1}^{\infty} (n^*)^{1-\sigma} \left( \max\left\{0, \frac{1}{n^*} - \frac{1}{n}\right\} \right) \varphi_n\left(n^*\right)}{\sum_{n=1}^{\infty} \left( \max\left\{\frac{1}{n}, \frac{1}{n^*}\right\} \right)^{\sigma} \varphi_n\left(n^*\right)}.$$
 (68)

Hence, conditional on  $n^*$  and the firm-size distribution, which fully determines  $\varphi_n(n^*)$ , (68) only depends on  $\sigma$ . Now the firm-size distribution does depend on  $\sigma$  and hence we need to iterate on (68), taking the dependence of  $\varphi_n$  on  $\sigma$  into account. Note that neither T nor  $\xi$  enter in (68). Hence, we can calibrate  $\sigma$  independently of T and  $\xi$ . To finally determine the parameters T and  $\xi$  and

the endogenous variable  $n^*$ , recall that managerial demand  $M_D(n^*)$  (which equals the managerial employment share) is given by

$$M_D(n^*) = \sum_{n>n^*}^{\infty} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n(n^*), \qquad (69)$$

and the total demand for production workers is given by

$$L_D(n^*) = \frac{T^{1-\sigma}}{(n^*)^{1-\sigma}\xi\sigma} \left[ 1 - T^{\sigma} \sum_{n} \max\{n^{-\sigma}, (n^*)^{-\sigma}\} \varphi_n(n^*) \right].$$
 (70)

Additionally, the average mark-up in the economy (see Section OA-1.3 in the Online Appendix for the derivation) is

$$\overline{MU} = \left[ \sum_{n=1}^{n^*-1} \frac{1}{1 - \left(\frac{T}{n}\right)^{\sigma}} \frac{\varphi_n(n^*) n^{-1}}{F_H + F^L} + \sum_{n=n^*}^{\infty} \frac{1}{1 - \left[1 - \sigma + \frac{n^*}{n}\sigma\right] \left(\frac{T}{n^*}\right)^{\sigma}} \frac{\varphi_n(n^*) n^{-1}}{F_H + F^L} \right]$$
(71)

Given empirical moments for mark-ups and managerial employment, we can solve for  $(T, \xi, n^*)$  from (69), (71) and the labor market clearing condition  $1 = L_D(n^*) + M_D(n^*)$ . Finally, we can use the calibrated A and (61) to find the deep primitive parameters  $\theta$  as

$$\theta = A^{1-\zeta} \left( \frac{1}{(1-\sigma)\Gamma\zeta T^{\sigma}} \right)^{\zeta}. \tag{72}$$

We can then solve for the step-size  $\gamma$  to fit the aggregate growth rate of TFP as  $g_Q = \ln(\gamma) \Gamma \tau$ .

# C Empirical Appendix

### C.1 Data

In this section we provide more information about our data sources.

Plant- and Firm-level Information for the US We use data from the Business Dynamics Statistics (BDS). BDS is a product of the US Census Bureau. The BDS data are compiled from the Longitudinal Business Database (LBD). The LBD is a longitudinal database of business plants and firms covering the years between 1976 and 2012. We focus on the manufacturing sector in 2012. The data are publicly available at http://www.census.gov/ces/dataproducts/bds/.

For our analysis, we utilize the following four moments from the US data: (i) the cross-sectional relationship between age and size, which we refer to as the life cycle, (ii) the aggregate employment share by age, (iii) the exit rate as a function of age *conditional on size* and (iv) the rate of entry. For our main analysis we focus on plants. In Section OA-2.3 in the Online Appendix we replicate our results at the firm level. The BDS reports both aggregate employment and the number of

plants by age. This allows us to calculate the first two moments. The BDS also directly reports both entry and exit rates for each size-age bin. The entry rate at the plant level is calculated as the number of new plants at time t relative to the average number of plants in t and t-1. Similarly, the exit rate at the plant level is calculated as the number of exiting plants in t relative to the average number of plants in t and t-1. The corresponding information is also reported at the firm level. In particular, the BDS reports the number of exiting firms for different size-age bin. Note that all plants owned by the firm must exit for the firm to be considered an exiting firm. As for firm entry, we treat firms of age 0 as an entering firm. Because a firm's age is derived from the age of its plants, this implies that we treat firms as entering firms only if all their plants are new. In Section OA-2.1 in the Online Appendix we provide detailed descriptive statistics about the dynamic process at both the firm- and plant level.

Plant-Level Information for India As explained in the main body of the text, we construct a representative sample of the Indian manufacturing sector by combining data from the Annual Survey of Industries (ASI) and the National Sample Survey (NSS), which - every five years - has a special module to measure unorganized manufacturing plants. We use cross-sectional data from 2010. In contrast to the US, both the ASI and NSS are based on plants and we cannot link plants to firms. With the majority of employment being accounted for by very small producers, multi-plant firms are unlikely to be important for the aggregate in India. Firms in the NSS account for 99.2% of all plants and for 76% of manufacturing employment. In Section OA-2.2 in the Online Appendix we provide more detailed descriptive statistics and additional results concerning the process of firm dynamics of ASI and NSS plants.

Data on Managerial Compensation from NIPA We identify  $\sigma$  from the share of managerial compensation in aggregate profits before managerial payments; see (68). To measure this moment we are going to use two data sources. From NIPA we can retrieve a measure of aggregate profits in the manufacturing industry. Specifically, we start with aggregate corporate profits, which are directly measured in NIPA. The BEA's featured measure of corporate profits -profits from current production - provides a comprehensive and consistent economic measure of the income earned by all US corporations. As such, it is unaffected by changes in tax laws, and it is adjusted for non- and misreported income. We then add to this measure nonfarm proprietors' income in the manufacturing sector, which provides a comprehensive and consistent economic measure of the income earned by all US unincorporated nonfarm businesses. This measure of aggregate profits does not coincide with  $\Pi = \sum_i \pi_i$  in the model, because the data are net of managerial payments.

To measure managerial wages, we augment the information in NIPA from information in the census. While NIPA reports compensation for workers, managerial payments are not directly recorded in NIPA. To calculate the managerial wage bill, we therefore use the US census data. In the census we have micro data on labor compensation and occupations at the micro level. Hence, we calculate the *share* of managerial payments in the total wage bill and apply that share to the

aggregate compensation data in NIPA. According to the census, managerial compensation amounts to roughly 20% of total wages. Recall that the managerial employment share in the US is about 12% so that managerial wages are relatively high. We then calculate the share of managerial compensation (CSM) in aggregate profits net of managerial wages as

$$CSM = \frac{\text{Managerial Compensation}}{\text{Corporate Profits} + \text{Nonfarm Proprietor's Income} + \text{Managerial Compensation}},$$

where "Managerial Compensation" is simply 20% of the total labor compensation in NIPA. We also calculate a second measure of CSM, where we do not include "Nonfarm Proprietor's Income." We calculate CSM before the Great Recession, because we were concerned about corporate profits being very low during the financial crisis. CSM is quite volatile. It ranges from 65% in 2001 to 33% in 2006. For our calibration we focus on the average across the years 2000 - 2007, which is 49%. If we do not include "Nonfarm Proprietor's Income", the numbers are very similar and only slightly larger, ranging from 69% in 2001 to 35% in 2006. Hence, it is not essential for us to take "Nonfarm Proprietor's Income" into account.

Data on Managerial Employment To measure managerial employment in different countries across the world, we employ national Census data from the IPUMS project. We focus on data for 2000 onward, extract data from 69 countries and take the most recent data for each country. For each country we get a sample from the census, which has detailed information about personnel characteristics. In particular we observe each respondent's education, occupation, employment status, sex and industry of employment. We focus on male workers in the manufacturing industry working in private-sector jobs. We do this because female labor force participation differs substantially across countries and our model does not feature any unemployment. We drop all countries from the sample for which we do not have occupational information according to the consistent measure of the International Standard Classification of Occupations (ISCO).

The list of occupations according to ISCO is contained in Table 7. To qualify as a manager in the sense of our theory, two characteristics have to be satisfied. First, the respective individual has to work as a "Legislator, senior official and manager." In order to focus on managers, which are agents of a firm owner, i.e., outside managers, we *also* require workers to be wage workers and not working on their own account or to be unpaid family members. This information is also contained in the IPUMS census data in the variable "worker type." As we showed in Table 1 above, it is important to take these differences into account as poor countries have a higher share of people working on their own account (or as a family member) *conditional* on being classified as a manager according to ISCO.

Cross-Country Data For our cross-country analysis in Section 5.1, we augment these data sources with additional country characteristics. As our baseline measure for human capital at the country level, we take the information straight out of the Penn World Tables Version 8.0, which

Table 7: List of occupations according to ISCO

Legislators, senior officials and managers	Plant and machine operators and assemblers
Professionals	Elementary occupations
Technicians and associate professionals	Armed forces
Clerks	Other occupations, unspecified or n.e.c.
Service workers and shop and market sales	Response suppressed
Skilled agricultural and fishery workers	Unknown
Crafts and related trades workers	NIU (not in universe)

Notes: Table 7 contains the occupational categories available in the IPUMS data. A necessary condition for someone to be classified as an outside manager is to be assigned the occupational title "Legislators, senior officials and managers." See the main body of the text for the additional requirements.

is based on the Barro-Lee data set. Our measure of financial development stems from the World Bank Global Financial database. Specifically, we measure financial development by the share of private credit in GDP. This is not only the standard measure of financial development, but it is also available for the largest set of countries in our sample. However, other measures of financial development, i.e., the number of bank branches or the share of bank deposits relative to GDP, yield similar results. As for the quality of legal institutions, we rely on the measure for the "Rule of Law," which is reported in the Worldwide Governance Indicators distributed by the World Bank (see www.govindicators.org). It "captures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence" and is measured using survey data from firms and individuals.

While these are the three country characteristics we exploit for our decomposition exercise in Section 5.1, we provide additional reduced-form evidence for the cross-country variation in managerial employment shares in Section OA-2.7. There we also use information on GDP per capita and capital intensities from the Penn World Tables, cross-country information on trust, which we take from the World Value Surveys, and an alternative measure of human capital, which we calculate directly from the IPUMS data. More specifically, concerning the measurement of trust, we focus on answers to the question "Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?" and average over the waves of the World Values Survey between 1981 and 2014 to get an aggregate measure of trust at the country level. As for human capital, while the IPUMS data do not contain information on the years of schooling directly, educational attainment is reported in four categories: (i) less than primary school, (ii) primary school completed, (iii) secondary school completed and (iv) university completed. In order to calculate aggregate human capital stocks, we assign these categories 2, 6, 12 and 16 years of schooling, respectively. We then translate years of schooling into human capital units by assuming a return to schooling of 13.4% for the first 4 years, 10.1% for the next 4 years and 0.068% for each additional year. This is the standard specification adopted in the development accounting literature (see, e.g., Hall and Jones (1999) and Caselli (2005)). We calculate this measure of human capital both at the country level for the whole economy and for the manufacturing sector.

# D Quantitative Analysis: Robustness of Counterfactuals

In this section we discuss the robustness of our results with regard to our calibration strategy. In particular, we now explicitly recalibrate T and  $\sigma$  for the Indian economy. The results are contained in Table 8.

Table 8: Robustness of Counterfactuals

	US		India				
				Recalib	orate T	Recalibr	ate $T$ and $\sigma$
	Baseline		Baseline	(I)	(II)	(III)	(IV)
			Stru	ctural I	Paramet	ers	
ξ	0.726		0.608	0.615	0.611	0.003	0.000
$\sigma$	0.756		0.756	0.756	0.756	0.182	0.109
T	0.131		0.131	0.099	0.072	0.000	0.000
$\alpha$	0.590		0.130	0.133	0.136	0.136	0.136
heta	4.620		1.142	1.246	1.371	0.621	0.599
				Mom	ents		
Mark-up	1.200		$1.262^{\dagger}$	1.201	1.152	1.234	1.229
Managerial empl. share	0.126		0.0149	0.0149	0.0150	0.0148	0.015
Managerial compensation	0.490		$0.058^{\dagger}$	$0.074^\dagger$	$0.095^\dagger$	0.060	0.061
Empl. share of non-managerial firms	0.000		$0.790^{\dagger}$	$0.794^{\dagger}$	$0.000^{\dagger}$	$0.000^{\dagger}$	$0.000^{\dagger}$
$Counterfactual \ life-cycle$							
US with $\xi^{IND}$			2.043	2.065	2.054	1.880	1.880
India with $\xi^{US}$			1.219	1.221	1.236	1.571	1.633

Notes: This table contains robustness results for the counterfactuals reported in the main text. Columns 1 and 2 report the baseline results for the US and India respectively. In columns 3 - 6 we consider four alternative calibrations for the Indian economy. In specifications (I) and (II) we recalibrate T to match average mark-ups of 20% and 15%, respectively. In specifications (III) and (IV) we recalibrate T and  $\sigma$  to match average mark-ups of 20% and 15%, respectively, and a managerial compensation share (relative to aggregate profits) of 4%. The respective non-targeted moments are denoted with "†". The last two rows contain the counterfactual life-cycle profile for firms of age 21-25 relative to new entrants. The case "US with  $\xi^{IND}$ " considers the US economy (column 1) with the respective Indian  $\xi$  (row 1). The case "India with  $\xi^{US}$ " considers the respective Indian economy (column 2-6) with the respective US  $\xi$  (recolumn 1).

In columns 1 and 2 we report our baseline results for comparison (see Tables 2 and 3 and Figure 9). As seen in column 2, the implied mark-up for the Indian economy is 26%, i.e., slightly higher than the targeted mark-up for the US. In columns 3 and 4 we therefore allow T to differ across countries to match a mark-up of 20% (column 3) and 15% (column 4) in India. This reduces T relative to the US and increases the efficiency of innovation  $\theta$  slightly relative to the baseline.

 $<sup>^{42}</sup>$ Note that the reduction in T in column 4 is sufficiently strong for all Indian firms to actually hire managers despite their low delegation efficiencies, i.e., the employment share of non-managerial firms drops to zero. While we

Importantly, the implied delegation efficiency  $\xi$  hardly changes. This directly implies that the implied counterfactuals for plants' life-cycle are essentially identical to the ones of our baseline calibration: While US plants grow by a factor of 2.5 by the time they reach age 21-25, they would only increase by a factor of about 2 if they had the Indian delegation efficiency. The reverse exercise of increasing Indian delegation efficiency to the US level is also very similar to the baseline. In columns 5 and 6 we not only allow T to differ but we also reestimate  $\sigma$  to target the share of managerial compensation in India. As discussed in the main text, the Indian data make it hard to know that number precisely because we do not have good data on profits of firms in the NSS. In the main text we argued that the appropriate share of managerial compensation might be on the order of 4%. Hence, in columns 5 and 6 we estimate the model targeting both a mark-up of 20% and 15% respectively and a share of managerial compensation of 4%. Some comments are in order: First, note that the model requires both a very small  $\sigma$  and a small T. Because the managerial share is determined by  $\frac{T}{\xi}$  (see (69)), this requires the implied delegation efficiency  $\xi$  to be essentially zero as otherwise the incentives for firms to hire managers would be too high. 43 Nevertheless, the counterfactual implications of reducing the US delegation environment, i.e., essentially outlawing managerial personnel, are qualitatively not too different from the baseline. In contrast, the Indian economy would now benefit much more from having a delegation environment akin to the US level.

In Table 9 we report another robustness check, namely, with respect to the parameter  $\sigma$ . Not only is  $\sigma$  an important structural parameter, but we argued in Section C.1 that the data, which identify  $\sigma$ , namely, the share of managerial compensation relative to aggregate profits, is quite noisy. To see the importance of this uncertainty for our counterfactual results, we redid the main analysis of this paper for different target values of this moment. For our benchmark results we took the average value of  $\frac{wM}{\Pi} = 0.49$ . In Table 9 we redo our analysis for three alternative values of  $\frac{wM}{\Pi}$ . Our calibration strategy is the same as for our benchmark results. In particular, we estimate T and  $\sigma$  for the US but keep them at the US level when we recalibrate the model to the Indian data. In the first panel we report the new structural parameters. The most important change is, of course, the elasticity of the managerial return function  $\sigma$ : The higher the share of managerial compensation, the higher  $\sigma$ . In order to still match the data on mark-ups, the level T also changes. And as the aggregate managerial employment share depends only on  $\frac{T}{\xi}$ , the level of delegation efficiency  $\xi$  will also be different. The share of high types  $\alpha$  in contrast is only little affected. Concerning the moments, we see that we match the data on mark-ups, managerial employment share and managerial compensation perfectly for the US. For the Indian case, we only target the share of managers (which we also match perfectly). The implications for mark-ups and managerial compensation are very similar to our benchmark results and hence broadly consistent with the Indian micro data. In the last two rows we again report the counterfactual results of changes in the delegation environment  $\xi$  on a plant's life-cycle. For the case of the US, these results are very

do not target this moment explicitly, this makes column 3 preferable as a calibration. Importantly, the implications for the counterfactual exercises are very similar.

<sup>&</sup>lt;sup>43</sup>Note that every firm in fact does hire a manager in that calibration.

Table 9: Robustness of Counterfactuals: Different targets for  $\sigma$ 

	$\frac{wM}{\Pi} = 0.40$		$\frac{wM}{\Pi} = 0.45$		$\frac{wM}{\Pi} =$	0.55
	US	India	US	India	US	India
		Stru	ctural	Param	eters	
ξ	0.334	0.152	0.560	0.445	0.895	0.778
$\sigma$	0.428	0.428	0.599	0.599	0.980	0.980
T	0.010	0.010	0.068	0.068	0.228	0.228
$\alpha$	0.636	0.136	0.650	0.134	0.530	0.125
heta	3.769	0.877	4.726	0.897	15.000	3.947
			Mor	nents		
Mark-up	1.200 1.155 1.200 1.241 1.2			1.200	1.289	
Managerial empl. share	0.126	0.015	0.126	0.015	0.126	0.015
Managerial compensation	0.400	0.091	0.450	0.062	0.550	0.053
Empl. share of non-managerial firms	0.000	0.000	0.000	0.794	0.168	0.784
	Counterfactual life-cycle					
US with $\xi^{IND}$	1.9	080	2.198		1.923	
India with $\xi^{US}$	1.2	243	1.1	.89	2.0	78

Notes: This table contains robustness results for the empirical moment of the share of managerial compensation relative to aggregate profits. For our benchmark results we took the average value of  $\frac{wM}{\Pi}=0.49$ . Here we redo our analysis for three alternative values of  $\frac{wM}{\Pi}$ . Our calibration strategy is the same as that for our benchmark results. In particular, we keep T and  $\sigma$  constant when we recalibrate the model to the Indian data. In the first panel, we report the implied structural parameters. In the second panel, we report the implied moments. The last two rows contain the counterfactual life-cycle profile for firms of age 21-25 relative to new entrants. The case "US with  $\xi^{IND}$ " considers the US economy with the respective Indian  $\xi$ . The case "India with  $\xi^{US}$ " considers the respective Indian economy with the respective US  $\xi$ .

similar to our baseline. For the case of India, this is also the case except for the case of  $\sigma = 0.98$ . If the managerial return function is almost linear, Indian firms would benefit much more from an increase in the delegation environment.

# Online Appendix

# OA-1 Online Appendix - Theory

# OA-1.1 Using $Q_t$ as a scale for the innovation cost function

In the main body of the paper we parameterized the cost of innovation by  $C(X) = Y_t \left[ \frac{X_t}{\theta n^{1-\zeta}} \right]$  (see (16)). In this section we discuss how our results would change if we had scaled by  $Q_t$  instead of  $Y_t$ . Using (18) and  $Y_t = Q_t \mathcal{M}_t L_t^D$  we get that

$$x_n^Q = \theta^{\frac{1}{1-\zeta}} (\Gamma \zeta)^{\frac{\zeta}{1-\zeta}} \times \left( \frac{V_t(n+1) - V_t(n)}{Q_t} \right)^{\frac{\zeta}{1-\zeta}} = \theta^{\frac{1}{1-\zeta}} (\Gamma \zeta)^{\frac{\zeta}{1-\zeta}} \times \left( \frac{Y_t}{Q_t} \frac{V_t(n+1) - V_t(n)}{Y_t} \right)^{\frac{\zeta}{1-\zeta}}$$
$$= \left( \mathcal{M}_t L_t^D \right)^{\zeta/(1-\zeta)} \times x_n$$

where  $x_n$  is given in (18). As  $\mathcal{M}_t$  and  $L_t^D$  are constant in a stationary equilibrium, innovation incentives are given by (see (19))

$$x_n^Q(\psi, \mathcal{M}_t L_t^D) = A(\mathcal{M}_t L_t^D) \times \max \left\{ \left(\frac{T}{n}\right)^{\lambda}, \psi^{\lambda} \right\}$$

where  $A(\mathcal{M}_t L_t^D) = \left[\mathcal{M}_t L_t^D (1-\sigma)\theta^{1/\zeta}\zeta\right]^{\frac{\zeta}{1-\zeta}}$ . Hence, the innovation schedule now depends on two endogenous variables  $\psi$  and  $\mathcal{M}_t L_t^D$ . Our identification strategy outlined in Section B is entirely unaffected by such changes. In particular, because  $A(\mathcal{M}_t L_t^D)$  is still constant in equilibrium, we will get the exact same structural parameters  $z, \alpha, T, \xi, \beta, \sigma$  regardless of our scale variables. In particular,  $\mathcal{M}_t L_t^D = \frac{Y_t}{Q_t} = \frac{1}{\omega_t} = \frac{1}{\sigma \xi} \left(\frac{T}{n^*}\right)^{1-\sigma}$ , so that  $\mathcal{M}_t L_t^D$  can be calculated given the calibrated parameters. It is therefore only when we identify  $\theta$  from the calibrated reduced-form parameter A that the scaling matters. In particular, we now get

$$\theta = A^{1-\zeta} \left( \frac{1}{(1-\sigma)\mathcal{M}_t L_t^D \Gamma \zeta T^{\sigma}} \right)^{\zeta}$$
 (OA-1)

instead of (72).

# OA-1.2 Uniqueness of $n^*$ in (47)

Here we show that there exists a unique  $n^*$ , which solves the labor market clearing condition (47). We first show that there exists at least one  $n^*$ , which is consistent with (47).

**Lemma 2** There is at least one  $n^*$ , which is consistent with (47).

**Proof.** Consider (47) and suppose that  $n^* \to \infty$ . Then

$$\lim_{n^* \to \infty} \left[ \sum_{n \ge n^*}^{\infty} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n \left( n^* \right) \right] \to 0$$

Note also that

$$(n^*)^{-(1-\sigma)} \left[ \sum_{n=1}^{\infty} (1 - e_n (n^*)^{\sigma}) \varphi_n (n^*) \right]$$

$$< (n^*)^{-(1-\sigma)} \left[ \sum_{n=1}^{\infty} \varphi_n (n^*) \right] = (n^*)^{-(1-\sigma)}$$

Hence, the RHS of (47) goes to 0 as  $n^* \to \infty$ . Now consider  $n^* \to T < 1$ . Then

$$\lim_{n^* \to T} \left[ (n^*)^{-(1-\sigma)} \left( \sum_{n=1}^{\infty} (1 - e_n (n^*)^{\sigma}) \varphi_n (n^*) \right) \right] = 0$$

as  $e_n(n^*) \to 1$ . And,

$$\lim_{n^* \to T} \left[ \sum_{n=1}^{\infty} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n \left( n^* \right) \right] = \frac{1}{\xi} - \frac{T}{\xi} \left[ F^L + F^H \right] > \frac{1 - T}{\xi}$$

Provided that  $\frac{1-T}{\xi} > 1$ , which is a sufficient condition, the RHS of (47) goes to a value that is greater than 1, as  $n^* \to T$ . Therefore, as (47) is continuous, there is at least one  $n^*$  consistent with (47).

Now consider the uniqueness of  $n^*$ . We have to show that the function

$$H(n^*) = \frac{T^{1-\sigma}}{\sigma \xi} (n^*)^{-(1-\sigma)} \left[ \sum_{n=1}^{\infty} (1 - e_n (n^*)^{\sigma}) \varphi_n (n^*) \right] + \sum_{n^*}^{\infty} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n (n^*)$$
 (OA-2)

is increasing in  $n^*$ . Note that, by using (13), we can write (OA-2) as

$$H\left(n^{*}\right) = \frac{1}{\xi\sigma} \left(\frac{T}{n^{*}}\right)^{1-\sigma} \left\{1 + \sum_{n=1}^{\infty} \phi_{n}\left(n^{*}\right) \varphi_{n}\left(n^{*}\right)\right\}$$

where

$$\phi_{n}(n^{*}) = \sigma\left(\frac{T}{n^{*}}\right)^{\sigma} \times \max\left\{1 - \frac{n^{*}}{n}, 0\right\} - \max\left\{\left(\frac{T}{n}\right)^{\sigma}, \left(\frac{T}{n^{*}}\right)^{\sigma}\right\}$$

$$= \begin{cases} -\left(\frac{T}{n}\right)^{\sigma} & \text{if } n \leq n^{*} \\ -\left(\frac{T}{n^{*}}\right)^{\sigma}\left(\sigma\frac{n^{*}}{n} + 1 - \sigma\right) & \text{if } n \geq n^{*} \end{cases}, \tag{OA-3}$$

so that

$$\frac{\partial \phi_n \left(n^*\right)}{\partial n^*} = \begin{cases} 0 & \text{if } n \le n^* \\ \left(\frac{T}{n^*}\right)^{\sigma} \sigma^{\frac{(1-\sigma)}{n^*}} \left(1 - \frac{n^*}{n}\right) & \text{if } n \ge n^* \end{cases}$$
(OA-4)

Hence,

$$\frac{\partial H\left(n^{*}\right)}{\partial n^{*}} = -\frac{1-\sigma}{\xi\sigma} \left(\frac{T}{n^{*}}\right)^{1-\sigma} \frac{1}{n^{*}} \left\{1 + \sum_{n=1}^{\infty} \phi_{n}\left(n^{*}\right)\varphi_{n}\left(n^{*}\right)\right\} + \frac{1}{\xi\sigma} \left(\frac{T}{n^{*}}\right)^{1-\sigma} \frac{\partial \sum_{n=1}^{\infty} \phi_{n}\left(n^{*}\right)\varphi_{n}\left(n^{*}\right)}{\partial n^{*}}$$

$$= -\frac{1}{\xi\sigma} \left(\frac{T}{n^{*}}\right)^{1-\sigma} \left[\underbrace{\frac{1-\sigma}{n^{*}} \left\{1 + \sum_{n=1}^{\infty} \left[\phi_{n}\left(n^{*}\right) - \frac{n^{*}}{1-\sigma} \frac{\partial \phi_{n}\left(n^{*}\right)}{\partial n^{*}}\right]\varphi_{n}\left(n^{*}\right)\right\}}_{A} - \underbrace{\sum_{n=1}^{\infty} \phi_{n}\left(n^{*}\right) \frac{\partial \varphi_{n}\left(n^{*}\right)}{\partial n^{*}}}_{B}\right]$$

Consider first term A and define

$$b_n(n^*) \equiv \phi_n(n^*) - \frac{n^*}{1 - \sigma} \frac{\partial \phi_n(n^*)}{\partial n^*}$$

. From (OA-4) we get that

$$b_{n}(n^{*}) = \begin{cases} -\left(\frac{T}{n}\right)^{\sigma} & \text{if } n \leq n^{*} \\ -\left(\frac{T}{n^{*}}\right)^{\sigma} \left(\sigma \frac{n^{*}}{n} + 1 - \sigma\right) - \frac{n^{*}}{1 - \sigma} \left(\frac{T}{n^{*}}\right)^{\sigma} \sigma \frac{(1 - \sigma)}{n^{*}} \left(1 - \frac{n^{*}}{n}\right) & \text{if } n > n^{*} \end{cases}$$
$$= -\max\left\{ \left(\frac{T}{n}\right)^{\sigma}, \left(\frac{T}{n^{*}}\right)^{\sigma} \right\}.$$

Hence,

$$A = \frac{1-\sigma}{n^*} \left\{ 1 - \sum_{n=1}^{\infty} \max\left\{ \left(\frac{T}{n}\right)^{\sigma}, \left(\frac{T}{n^*}\right)^{\sigma} \right\} \varphi_n\left(n^*\right) \right\} > \frac{1-\sigma}{n^*} \left(1 - T^{\sigma}\right)$$

$$> 0$$
(OA-5)

Now consider term B. Note that (OA-3) implies that

$$-T^{\sigma} \le \phi_n(n^*) < -(1-\sigma)\left(\frac{T}{n^*}\right)^{\sigma} < 0$$

with  $\phi$  being increasing in n. Because B is a weighted average of  $\phi_n(n^*)$  and an increase in  $n^*$  will shift  $\varphi_n(n^*)$  upward in a stochastic dominance sense (see Section A.3, for the proof)

$$B = \sum_{n=1}^{\infty} \phi_n(n^*) \frac{\partial \varphi_n(n^*)}{\partial n^*} < 0$$
 (OA-6)

(OA-5) and (OA-6) imply that  $\frac{\partial H(n^*)}{\partial n^*}<0$  as required.

### OA-1.3 Measuring profitability

We are going to use a measure of profitability to discipline on the parameters. For brevity we refer to it as the mark-up. We are going to measure the profitability of firm i as

$$MU_i \equiv \frac{\text{Rev}_i}{\text{Costs}_i}$$
 (OA-7)

i.e. revenue over total costs.<sup>44</sup> To derive (OA-7) in the model, note that  $\text{Rev}_i = Y \times n_i$ , where  $n_i$  denotes the number of products of firm i. Total costs are

$$Costs_{i} = wl(n_{i}) n_{i} + wm(n_{i}) n_{i} = [l(n_{i}) + m(n_{i})] wn_{i}$$

The demand for production workers is given by

$$l\left(n\right) \;\; = \;\; \frac{y\left(n\right)}{q\mu\left(e\left(n\right)\right)} = \frac{p\left(n\right)y\left(n\right)}{q\mu\left(e\left(n\right)\right)p\left(n\right)} = \frac{Y}{q\mu\left(e\left(n\right)\right)\frac{w}{q}} = \frac{Y}{w}\frac{1}{\mu\left(e\left(n\right)\right)}.$$

The demand for managers is given by

$$m(n) = \frac{1}{\xi} \max\left\{0, \psi - \frac{T}{n}\right\}. \tag{OA-8}$$

Mark-ups of firms without a manager For firms with  $n < n^*$ , we have that m(n) = 0. Hence,

$$MU_{NM} = \frac{Yn}{wl(n)n} = \frac{Yn}{nY\frac{1}{\mu(e(n))}} = \mu(e(n)) = \frac{1}{1 - e(n)^{\sigma}} = \frac{1}{1 - (\frac{T}{n})^{\sigma}}.$$
 (OA-9)

As expected, holding n fixed, profitability is increasing in T.

Mark-ups of firms with a manager From the managerial demand (OA-8) we get that for firms with  $n > n^*$ 

$$\operatorname{Cost}_{i}^{M} = wl(n) n + wn \frac{1}{\xi} \left( \psi - \frac{T}{n} \right) = \frac{1}{\mu(e(n))} Yn + wn \frac{1}{\xi} \psi - w \frac{T}{\xi}.$$

Using that  $\psi = \frac{T}{n^*}$ , we get

$$\operatorname{Cost}_{i}^{M} = \frac{1}{\mu(e(n))} Yn + wn \frac{1}{\xi} \frac{T}{n^{*}} - w \frac{T}{\xi}$$

$$MU_i^{CES} = \frac{p_i^{CES}}{MC_i} = \frac{\text{Rev}_i}{\text{Costs}_i} = \frac{\rho}{\rho - 1},$$

where  $\rho$  is the elasticity of substitution.

 $<sup>^{44}</sup>$ Note that this measure coincides with the standard measure of mark-ups in a monopolistic-competition-CES economy, as

Hence

$$MU_{M} = \frac{Yn}{\frac{1}{\mu(e(n))}Yn + wn\frac{1}{\xi}\frac{T}{n^{*}} - w\frac{T}{\xi}} = \frac{1}{1 - e(n)^{\sigma} + \omega\frac{1}{n^{*}}\frac{T}{\xi} - \frac{1}{n}\omega\frac{T}{\xi}}$$

Recalling that  $\omega = \sigma \xi \left(\frac{n^*}{T}\right)^{1-\sigma}$ , we get

$$MU_{M} = \frac{1}{1 - e(n)^{\sigma} + \sigma \xi \left(\frac{n^{*}}{T}\right)^{1 - \sigma} \frac{1}{n^{*}} \frac{T}{\xi} - \frac{1}{n} \sigma \xi \left(\frac{n^{*}}{T}\right)^{1 - \sigma} \frac{T}{\xi}}$$
$$= \frac{1}{1 - e(n)^{\sigma} + \sigma \left(\frac{n^{*}}{T}\right)^{1 - \sigma} \frac{T}{n^{*}} - \frac{1}{n} \sigma \left(\frac{n^{*}}{T}\right)^{1 - \sigma} T}$$

Also  $e(n) = \psi = \frac{T}{n^*}$ , so that

$$MU_M = \frac{1}{1 - \left[1 - \sigma + \frac{n^*}{n}\sigma\right] \left(\frac{T}{n^*}\right)^{\sigma}}.$$
 (OA-10)

Now let  $\rho_n$  be the firm-size distribution, i.e.

$$\rho_n = \begin{cases} \frac{F^L + \nu_1^H F^H}{F^L + F^H} & \text{if } n = 1\\ \frac{\nu_n^H F^H}{F^L + F^H} & \text{if } n > 1 \end{cases}$$

where  $\nu_n^H$  is the distribution of high-types characterized in Proposition 2. The average mark-up in the economy is then

$$\overline{MU} = \sum_{n=1}^{n^*-1} \frac{1}{1 - \left(\frac{T}{n}\right)^{\sigma}} \rho_n\left(n^*\right) + \sum_{n=n^*}^{\infty} \frac{1}{1 - \left[1 - \sigma + \frac{n^*}{n}\sigma\right] \left(\frac{T}{n^*}\right)^{\sigma}} \rho_n\left(n^*\right).$$

# OA-2 Online Appendix - Empirical Analysis

# OA-2.1 Firms vs. plants in the US manufacturing sector

In this section we compare the process of firm-dynamics across US manufacturing firms and plants. Table OA-1 provides some summary statistics about the size-distribution of firms and plants in the US. The average manufacturing firm in the US has 51 employees, while the average plant only 43. It is also the case that large firms have multiple plants (firms with more than 1000 employees have on average 13) so that large firms account for half of total employment. There is less concentration at the plant level in that plants with more than 1000 employees account for less than one-fifth of aggregate employment in manufacturing in the US.

We now turn to the implied dynamics. Because we focus on cross-sectional data, the information on firm (plant) age is crucial for us. For plants, the definition of age is straightforward. Birth year is defined as the year a plant first reports positive employment in the LBD. Plant age is computed

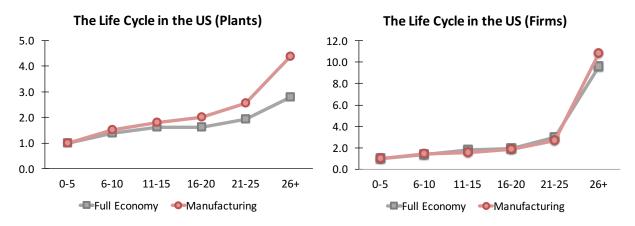
TABLE OA-1: DESCRIPTIVE STATISTICS: US MICRO DATA

		Fi	rms				Plants		
Size	No.	Avg.	Agg.	No. of	$\operatorname{Exit}$	No.	Avg.	Agg.	$\operatorname{Exit}$
		Employment	Share	plants	rate		Employment	Share	$_{\mathrm{rate}}$
1-4	86936	2.30	1.65	1.00	13.22	93038	2.31	1.78	16.50
5-9	48178	6.68	2.66	1.00	3.46	54281	6.73	3.02	4.20
10-19	37942	13.80	4.33	1.01	2.66	45803	14.01	5.30	3.10
20-49	32555	30.92	8.31	1.05	2.27	44085	31.90	11.62	2.40
50-99	13516	67.94	7.58	1.21	2.03	21582	71.54	12.75	1.90
100-249	8914	139.90	10.30	1.61	1.59	16476	155.76	21.20	1.00
250 - 499	3167	280.96	7.35	2.47	0.92	5444	348.72	15.68	0.50
500-999	1720	503.49	7.15	3.94	0.29	2120	677.19	11.86	0.30
1000+	2423	2531.92	50.67	12.68	0.25	984	2068.2	16.81	0.30
Aggregate	235351	51.44	100		6.53	283813	42.66	100	7.3

*Notes*: This table contains summary statistics for US manufacturing firms and plants in 2012. The data are taken from the BDS.

by taking the difference between the current year of operation and the birth year. Given that the LBD series starts in 1976, the observed age is by construction left censored at 1975. In contrast, firm age is computed from the age of the plants belonging to that particular firm. A firm is assigned an initial age by determining the age of the oldest plant that belongs to the firm at the time of birth. Firm age accumulates with every additional year after that. In Figure OA-1 we show the cross-sectional age-size relationship for plants (left panel) and firms (right panel) in the US.

FIGURE OA-1: LIFE CYCLE OF PLANTS AND FIRMS IN THE US



Notes: The figure contains the cross-sectional age-size relationship for plants (left panel) and firms (right panel) in the US. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

Not surprisingly, the life-cycle is much steeper for firms, especially for +26-year-old ones, as

firms grow both on the intensive margin at the plant level and the extensive margin of adding plants to their operation.

In Figure OA-2 we show the aggregate employment share of plants and firms of different ages. As suggested by the life-cycle patterns in Figure OA-1, old firms account for the bulk of employment in the US. However, the relative importance of old plants/firms is somewhat less pronounced because of exit, i.e., while the average firm/plant grows substantially by age conditional on survival, many firms/plants have already exited by the time they would have been 20 years old. Nevertheless, firms (plants) older than 25 years account for 76% (53%) of employment in the manufacturing sector.

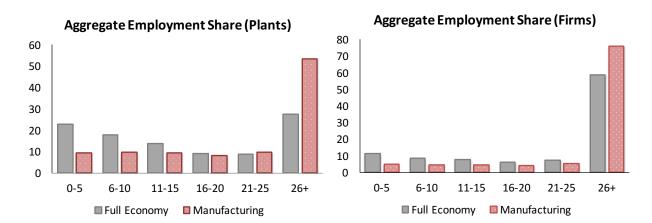


FIGURE OA-2: THE EMPLOYMENT SHARE BY AGE OF PLANTS AND FIRMS IN THE US

*Notes*: The figure contains the aggregate employment share of plants (left panel) and firms (right panel) in the US as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

This pattern of exit is depicted in Figure OA-3. There we show annual exit rates for firms and plants as a function of age. The declining exit hazard is very much suggestive of a model of creative destruction, whereby firms and plants grow as they age (conditional on survival) and exit rates are lower for bigger firms/plants.

An important moment for us is the age-specific exit rate conditional on size. It is this moment that will identify the importance of selection. In a model without heterogeneity, size will be a sufficient statistic for future performance, so that age should not predict exit conditional on size. However, if the economy consists of high- and low-type entrepreneurs, old firms are more likely to be composed of high types conditional on size. Hence, the size-specific exit rate by age is monotone in the share of high types by age. In Figure OA-4 we report this schedule for both plants and firms. The data show a large degree of age-dependence (conditional on size). The schedules for small firms and plants look almost identical. This is reassuring because small firms are almost surely single-plant firms, so that a firm-exit will also be a plant-exit and vice versa.

Exit Rates in the US (Firms) Exit Rates in the US (Plants) 14.0 14.0 12.0 12.0 10.0 10.0 8.0 8.0 6.0 6.0 4.0 4.0 2.0 2.0 0.0 0.0 0-5 6-10 16-20 21-25 0-5 6-10 21-25 11-15 26+ 11-15 16-20 26+ -Full Economy Manufacturing -Full Economy Manufacturing

FIGURE OA-3: THE EXIT RATES OF PLANTS AND FIRMS IN THE US BY AGE

*Notes*: The figure contains the exit rates of plants (left panel) and firms (right panel) in the US as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for both the manufacturing sector and the entire economy.

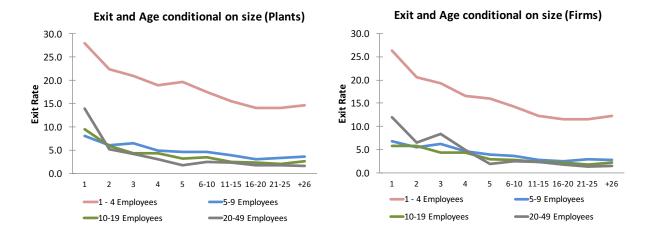


FIGURE OA-4: SIZE-DEPENDENT EXIT RATES OF PLANTS AND FIRMS IN THE US BY AGE

Notes: The figure contains the conditional exit rates by size of plants (left panel) and firms (right panel) in the US as a function of age. The data are taken from the BDS and we focus on the data for 2012. We depict the results for the manufacturing sector.

### OA-2.2 Plants in the Indian Manufacturing Sector

In this section we provide more descriptive evidence about the underlying process of firm dynamics in the manufacturing sector in India. Table OA-2 contains descriptive statistics for our sample of Indian manufacturing plants. For comparison, we organize the data in the same way as in the left

panel of Table OA-1, which contains the results for manufacturing plants in the US. It is clearly seen that the plant-size distribution in India is concentrated on very small firms. The average plant has less than 3 employees and more than 50% of aggregate employment is concentrated in plants with at most 4 employees. Such plants account for 93% of all plants in the Indian manufacturing sector.

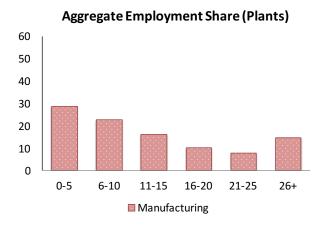
	Table $OA-2$ :	Descriptive	STATISTICS:	Indian	Micro	Data
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Size	No.	Avg. Employment	Aggregate Employment Share
1-4	15957296	1.56	54.76
5-9	843091	6.26	11.61
10-19	243868	12.98	6.96
20-49	70834	29.22	4.55
50-99	23242	69.89	3.57
100-249	14898	149.31	4.89
250 - 499	4701	346.69	3.58
500-999	2283	683.86	3.43
1000 +	1232	2452.65	6.65
Aggregate	17161444	2.65	100.00

*Notes*: This table contains summary statistics for plants in the Indian manufacturing sector in 2010. The data are taken from the ASI and the NSS. To calculate the number of firms, we use the sampling weights provided in the data.

Figure OA-5 reports the aggregate employment share by age for Indian manufacturing plants and is hence comparable to Figure OA-2 for the US.

FIGURE OA-5: THE EMPLOYMENT SHARE BY AGE OF PLANTS IN INDIA



Notes: The figure contains the aggregate employment share of manufacturing plants in India as a function of age. The data are taken from the ASI and the NSS and we focus on the data for 2010. We combine the two data sets using the sampling weights provided in the micro data.

It is clearly seen that the aggregate importance of old firms is very small in India. While firms, that are older than 25 years account for 55% of employment in the US, the corresponding number is less than 20% in India. This is a reflection of the shallow life-cycle in India and not of there being fewer old firms in the Indian economy.

### OA-2.3 Firm-Level Estimation

So far we have focused solely on plant-level data. We did so to (a) be consistent with the results reported in Hsieh and Klenow (2014) and (b) to ensure comparability between the US and India since we cannot link individual plants to specific firms in the Indian data. In this section, we show that this choice has no substantial implications for our conclusions regarding the counterfactual implications of a change in the delegation environment. Our strategy is exactly the same as in the main part of the paper. First we calibrate the model to the US data at the firm level. The results of that exercise are contained in Table OA-3.

Table OA-3: Estimation for US Firms

A. Moments Targeted for US Firms							
Moment	$\underline{\mathrm{Data}}$	$\underline{\text{Model}}$					
$M_1$ . Share of manager compensation	0.49	0.49					
$M_2$ . Aggregate growth rate	0.0172	0.0172					
$M_4$ . Share of managers in workforce	0.1261	0.1261					
$M_5$ . Entry rate	0.065	0.0694					
$M_6$ . Mean employment for 21-25-year-old firms	2.64	2.6127					
$M_7$ . Relative exit rate (age:21-25 to age:1-5 ratio )	1.64	1.603					
$M_8$ . Employment share of 21-25-year-old firms	0.0554	0.0547					
$M_9$ . Average mark-up	1.20	1.20					

B. Parameter	Calibra	tion for	US Firms
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<u>Parameter</u>	Interpretation	Target	$\underline{\text{Value}}$
ξ	Delegation efficiency	Managerial employment share	0.683
$\sigma$	Curvature of efficiency	Managerial compensation	0.713
T	Managerial endowment	Share of non-managerial firms	0.119
$\alpha$	Share of high type	Age vs exit profile	0.639
$\beta$	High-type replacement	Empl. share of old firms	0.268
$\theta$	Innovativeness	Life-cycle	5.99
z	Entry flow rate	Rate of entry	0.08
γ	Innovation step size	Aggregate growth rate	1.16

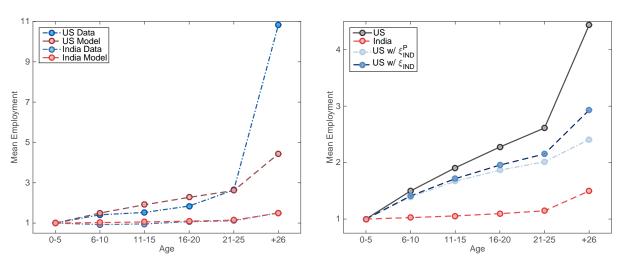
Notes: In Panel A we report both the data moments (column 1) and the corresponding moments in the model (column 2). The share of managers in the workforce is calculated from IPUMS. The aggregate growth rate is an estimate of the US growth rate from the Penn World Tables for the years 1970-2011. The share of manager compensation is calculated from the US Flow of Funds. All remaining moments stem directly from the 2012 micro data of US manufacturing firms reported in the BDS. In Panel B we report the corresponding parameter estimates that yield the moments reported in column 2 of Panel A.

Again the model is able to match the calibrated moments quite well. The main difference between plants and firms at the horizon of age 21-25 is the life-cycle, the aggregate employment share and the relative exit rate. The life-cycle is slightly steeper, the employment share is lower (because very old firms are much bigger than very old plants) and the relative exit rate of young firms is higher than that of plants, because old firms exit less frequently than older plants. The parameters consistent with the moments in Panel A are contained in Panel B. Again, these are not too different from our findings at the plant level. The efficiency of innovation  $\theta$  is estimated to be higher (reflecting the slightly steeper life-cycle profile), the delegation environment  $\xi$  is estimated to be a little lower (because the share of large firms is bigger than the share of large plants, which increases managerial demand) and the replacement rate of high types,  $\beta$ , is lower (which is required to make the employment share of very old firms large and hence that of firms of age 21-25 not too big).

Among these moments, the main moment of interest is the life-cycle, which is depicted in Figure OA-6.

FIGURE OA-6: LIFE CYCLE OF US FIRMS

FIGURE OA-7: THE DELEGATION ENVIRONMENT



Notes: The figures depict the cross-sectional age-size relationship, i.e., average firm employment as a function of age. Figure OA-6 depicts the firm-level data and the calibrated model. The data for the US correspond to the population of US manufacturing firms in 2012 and are taken from the BDS. The data for India correspond to manufacturing plants from India in 2010 and are taken from the ASI and the NSS. The model corresponds to the US parameterization reported in Table OA-3. Figure OA-7 contains the counterfactual exercise where the delegation efficiencies are given by  $\xi_{IND}^{P} = 0.50$  and  $\xi_{IND} = 0.57$  respectively.

While plants grow 2.5 times relative to their entry size by age 25, this number is 2.6 for US firms. Note the very steep increase in the size of very old firms. While plants older than 26 years are about 4.5 times as big as newly entering plants, old firms are 11 times as big as their newly born counterparts. Figure OA-6 shows that our model matches the life-cycle of firms well, except for this last age bin. The results of the counterfactual exercise of changing the US delegation

environment to the one calibrated in India are contained in Figure OA-7. Again we report the results based on both the "full exercise" (which uses the calibrated  $\xi$  from the Indian calibration) and the "partial exercise," where we recalibrate  $\xi$  using the US parameters to match the Indian managerial employment share. Comparing Figure OA-7 to Figure 9 in the main text, we can see that the results are very comparable.

#### OA-2.4 Reduced-Form Evidence: Variation across firms

In Section 3.1 we reported some basic patterns on the relationship between managerial hiring and firm size in the Indian micro data and how it related to our theory. This section describes this analysis in more detail.

The two most important parameters in our theory relate to the entrepreneur's time endowment T and the quality of the delegation environment  $\xi$ . In particular, we show that firms' demand for managerial personnel and their resulting incentives to expand are parameterized by the delegation environment  $\xi$  through the endogenous delegation return  $\psi = \left(\frac{\sigma \xi}{\omega}\right)^{1/(1-\sigma)}$ . More specifically, we show in Proposition 1 that firms start to hire outside managers if  $n \geq T/\psi$ , that their total supply of managerial services (per product line),  $e_n$ , is given by  $e_n = \max\{\frac{T}{n}, \psi\}$  and that the resulting expansion incentives are  $x_n = A \times \max\{\left(\frac{T}{n}\right)^{\lambda}, \psi^{\lambda}\}$  (see (19)). To test these predictions we need proxies for T and  $\xi$  and micro data on firm size, firm-growth and managerial hiring. While in many ways imperfect, the Indian data allow us to do so.

In the theory, we referred to T as the owner's time endowment. We assumed that this time endowment had a comparative advantage within the plant - it could neither be sold on the market, nor was there any need to monitor. The NSS data for 1995 contain information on the size of the family of the firm's owner. As long as family members require less monitoring time than outside managers, we can think of family size as inducing variation in the time endowment T. As for the quality of the delegation environment  $\xi$ , we will rely on the variation in trust across Indian states. The Indian micro data contain information about the state in which the respective firm or individual is located. Additionally, we extract information on the general level of trust between people at the state level from the World Value Surveys. While this variable is not directly aimed at eliciting the (perceived) quality of the prevailing legal environment, it fits well into our theoretical framework as long as trust reduces the required time the owner needs to spend to incentivize outside managers. See also Bloom et al. (2012), who also use this variable to proxy the efficiency with which decisions can be delegated by this variable.

In Table OA-4 we look at some of the equilibrium relationships of the theory.

<sup>&</sup>lt;sup>45</sup>Recall that we showed in Proposition 3 that  $\psi$  is increasing in  $\xi$  taking the general equilibrium adjustment of wages into account.

<sup>&</sup>lt;sup>46</sup>The World Values Survey is a collection of surveys based on representative samples of individuals and provides an index of trust in different regions of India. The primary index we use is derived from the answers to the question "Generally speaking, would you say that most people can be trusted, or that you can not be too careful in dealing with people?" Following Bloom et al. (2012) and La Porta et al. (1997), the regional trust index is constructed as the percentage of people providing the answer "Most people can be trusted" within the state where the firm is located. This is the most common measure of trust used in the literature.

TABLE OA-4: MANAGERIAL HIRING, FIRMS SIZE AND GROWTH IN INDIA

	Dependent Variable							
	Manager > 0	ln en	npl (Manage	er > 0	ln e	empl	Employme	ent growth
ln empl	0.040***							
	(0.003)							
ln HH size	-0.003**		0.657**	0.556**		0.201***		
	(0.001)		(0.265)	(0.247)		(0.034)		
Trust	0.012*	0.457*	2.467*		0.151***	0.240		
	(0.006)	(0.262)	(1.464)		(0.033)	(0.178)		
$\ln\mathrm{HH}\mathrm{size}\times\mathrm{Trust}$			-1.259*	-0.905		-0.020		
			(0.718)	(0.678)		(0.094)		
ln assets			0.125***	0.113***		0.054***		
			(0.012)	(0.013)		(0.002)		
ln empl							-0.394***	-0.441***
							(0.007)	(0.029)
$\ln \text{ empl} \times \text{Trust}$								0.134*
								(0.081)
age, rural controls	Y	Y	Y	Y	Y	Y	N	N
State FE	N	N	N	Y	N	N	Y	Y
Firm FE	N	N	N	N	N	N	Y	Y
N	178,999	21,498	2,298	2,298	204,820	178,024	152,808	152,808
$R^2$	0.04	0.11	0.51	0.57	0.12	0.22	0.23	0.23

Notes: Robust standard errors in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.10. The data are taken from the ASI and NSS in 1995. We use the sampling provided in the micro data to combine these data sets. All regressions control for 2-digit fixed effects, the age of the plant and a dummy variable for the plant to be in a rural area. "In empl" denotes the (log of) total employment at the plant. "In HH size" denotes the (log of) the size of the household of the plant's owner. This variable is only available for the NSS data. Whenever this variable is included in the regression, we therefore have to exclude the plants in the ASI from the analysis. "Trust" is the measure of trust at the state level, which we take from the World Value Surveys. "In assets" denote the (log of) assets at the plant level. The dependent variables are: an indicator of managerial hiring (column 1), log employment conditional on managerial hiring (columns 2 - 4), log employment (columns 5-6) and the growth of employment (columns 7-8). In columns 7-8 we focus on the panel dimension from the ASI, for the years 1998 - 2009. This allows us to calculate growth rates at the firm level and control for firm fixed effects.

In column 1 we focus on the extensive margin of managerial hiring. According to Proposition 1 the likelihood of hiring a manager should be increasing in firm size and delegation efficiency and declining in the owner's time endowment. Empirically, we indeed find that large firms and firms in states with favorable trust measures are more likely to hire outside managers, while firms with larger families abstain from hiring outside managerial personnel holding firm size constant.

These static determinants of managerial hiring have dynamic implications relating to firms' expansion incentives and hence long-run firm size. In particular, conditional on hiring managers, growth incentives and hence long-run firm size are increasing in delegation efficiency. Column 2 shows that firms in high-trust regions are indeed relatively large conditional on age. The theory also implies that delegation efficiency  $\xi$  and the owner's time endowment T are substitutes, i.e., we should expect a tighter link between family size and firm size in low-trust regions. Columns 3

and 4 show that this is the case.<sup>47</sup> Similar to Bloom et al. (2013), we also find a tight relationship between firm size and family size. We interpret this correlation as family members substituting for the scarcity of available outside managers. Furthermore, the coefficient on the interaction term is negative, which means that the positive relationship between firm size and family size is weaker in regions where trust is higher and hence delegation is more efficient.<sup>48</sup> In column 4, we replicate these results with state fixed effects to control for all time-invariant regional characteristics.

In columns 5 and 6 we redo the analysis of columns 2 and 3 for the whole sample of firms, i.e., we do not condition on delegation. Again we find a positive correlation between delegation efficiency and firm size (column 5) and between the size of the family and firm size (column 6). Note that the effect of trust for the entire sample of firms is much weaker. This is consistent with our theory, which implies that delegation efficiency only matters for the firms that actually delegate. For firms without outside managers (i.e., firms with  $n < n^*(\psi)$ ), growth incentives are only determined by the owner's time endowment T.

In columns 7 and 8 we finally turn to the firm growth and firm size relationship, for which we use the panel data from the ASI for the years 1998 - 2009. The main dynamic implication of our model is that limits to delegation induce a deviation from Gibrat's law in a particular way: not only is growth declining in size, but particularly so, the worse the delegation environment, i.e., the lower  $\xi$ . To study this implication, we regress the growth rate of firm employment on log firm size and and its interaction with the regional trust index. Moreover, we include firm fixed effects to solely focus on the effect of firm size on growth along a firm's expansion path. In column 7 we show that there is a strong negative relationship between growth and size. In column 8 we show that this negative correlation is less pronounced if the efficiency of the delegation environment is high. This is consistent with firms overcoming declining returns through the delegation of decision power.

Finally, we replicated the entire analysis of Table OA-4, which controlled for 2-digit sector fixed effects, with 3-sector fixed effects. The results are contained in Table OA-5. It is seen that all results are very similar. The only exception are the results in columns 2 - 4, which are conditioned on managerial hiring and hence have a small sample size. While all point estimates are of the same sign, they are not significantly different from zero. This is especially true for columns 3 and 4, where we have to rely on the NSS data because of the information for family size.

#### OA-2.5 Details for Section 5.1: Decomposing Delegation Efficiency

In this section we provide some additional details for the decomposition exercise discussed in Section 5.1. In (36) we assumed country c's delegation efficiencies to be a parametric function of the quality of legal institutions, human capital and the degree of financial development. Because our final

<sup>&</sup>lt;sup>47</sup>Recall that we have information on family size only for firms in the NSS and not the ASI. This explains the drop in sample size between columns 2 and 3.

<sup>&</sup>lt;sup>48</sup>We also control for the assets of the firm as both family size and the level of regional trust could be correlated with the supply of capital to the firm.

m	Managerial Hif	D C	·	T	D
		DINIC HIDNE SIZE	- A VII A ( ' D ( ) ( ) ( ) II I	VI I VII VI V •	RADITOTALISE
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				Dependent	Variable			
	Manager > 0	ln en	npl (Manage	er > 0	ln e	empl	Employme	ent growth
ln empl	0.040***							
	(0.003)							
ln HH size	-0.004***		0.254	0.247		0.198***		
	(0.001)		(0.228)	(0.220)		(0.031)		
Trust	0.012*	0.267	0.305		0.098***	0.139		
	(0.006)	(0.165)	(1.174)		(0.030)	(0.165)		
ln HH size $\times$ Trust			-0.277	-0.150		0.008		
			(0.578)	(0.555)		(0.087)		
ln assets			0.117***	0.108***		0.053***		
			(0.013)	(0.013)		(0.002)		
ln empl							-0.394***	-0.441***
							(0.007)	(0.029)
$\ln \text{ empl} \times \text{Trust}$								0.134*
								(0.081)
age, rural controls	Y	Y	Y	Y	Y	Y	N	N
State FE	N	N	N	Y	N	N	Y	Y
Firm FE	N	N	N	N	N	N	Y	Y
N	178,999	21,498	2,298	2,298	204,820	178,024	152,808	152,808
$R^2$	0.05	0.43	0.64	0.68	0.29	0.22	0.23	0.23

Notes: Robust standard errors in parentheses. \*\*\* p<0.01, \*\* p<0.05, \* p<0.10. The data are taken the ASI and NSS in 1995. We use the sampling provided in the micro-data to combine these datasets. All regressions control for 3-digit fixed effects, the age of the plant and a dummy variable for the plant to be in a rural area. "In empl" denotes the (log of) total employment at the plant. "In HH size" denotes the (log of) the size of the household of the plant's owner. This variable is only available for the NSS data. Whenever this variable is included in the regression, we therefore have to exclude the plants in the ASI from the analysis. "Trust" is the measure of trust at the state level, which we take from the World Value Surveys. "In assets" denote the (log of) assets at the plant level. The dependent variables are: an indicator of managerial hiring (column 1), log employment conditional on managerial hiring (columns 2 - 4), log employment (columns 5-6) and the growth of employment (columns 7-8). In columns 7-8 we focus on the panel dimension from the ASI, for the years 1998 - 2009. This allows us to calculate growth rates at the firm level and control for firm fixed effects.

dataset has information for only 53 countries, we adopt a linear parameterization  $\xi(.)$ , i.e.

$$\xi(HC_c, ROL_c, FD_c; \vartheta) = \vartheta_0 + \vartheta_{HC} \times HC_c + \vartheta_{ROL} \times ROL_c + \vartheta_{FD} \times FD_c,, \tag{OA-11}$$

and estimate the four parameters  $\vartheta = (\vartheta_0, \vartheta_{HC}, \vartheta_{ROL}, \vartheta_{FD})$  using GMM. More precisely, we estimate  $\vartheta$  by making our model match the cross-country data on managerial employment shares as closely as possible given the data on human capital, the rule of law, and financial development. Because we do not have plant-level micro data for the entire cross-section of countries, all other structural parameters are kept at the US benchmark. This is unlikely to be quantitively important, because the comparison between the full and partial counterfactuals above did not show large differences in the resulting life-cycles; see Figure 9. Because the two counterfactuals are especially close for firms less than 25 years old, we will not focus on firms older than 26 for this exercise.

 $<sup>^{49}</sup>$ We report the respective numbers for this cohort of old firms below when we discuss Table 6.

Table OA-6 contains the results and shows that all of our country characteristics are positively related to the delegation environment, although our measure of financial development only insignificantly so. In terms of the underlying data, this reflects the fact that they are positively correlated with the share of managerial employment.<sup>50</sup>

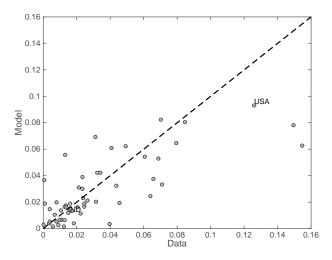
Table OA-6: Delegation Efficiency,  $\xi$ 

	$\vartheta_0$	$\vartheta_{HC}$	$\vartheta_{ROL}$	$\vartheta_{FD}$
Point estimate	0.4398	0.0458	0.0250	0.0083
Std error	(0.0433)	(0.0202)	(0.0121)	(0.0088)

Notes: The table contains the estimates of  $\vartheta$  in (OA-11). The standard errors are calculated using a bootstrap procedure with 10.000 iterations. See Section OA-2.6 in the Online Appendix for details.

In Figure OA-8 we compare the model's predictions with the data.

FIGURE OA-8: MANAGERIAL SHARES AROUND THE WORLD: DATA VERSUS MODEL



Notes: The figure shows the predicted managerial share given  $\xi(HC_c, ROL_c, FD_c; \hat{\vartheta})$  on the y-axis and the actual data on the x-axis. The parameter estimates  $\hat{\vartheta}$  are given in Table OA-6.

The model captures a sizable part of the cross-sectional variation.<sup>51</sup> Note that according to Proposition 3, there is a monotone relationship between  $\xi$  and the managerial employment share.

 $<sup>^{50}</sup>$ In Section OA-2.7, we also provide additional reduced-form evidence for this positive correlation. In particular, we consider various robustness checks using alternative measures. Additionally, we also show that there is a positive relationship between managerial employment and measures of trust, which is consistent with our cross-state results from India mentioned above. We do not explicitly include trust as a determinant of  $\xi_c$  in (36), because only 39 countries have data on both managerial employment and trust.

 $<sup>^{51}</sup>$ A regression of the actual managerial shares on the predicted managerial shares from (OA-11) has a coefficient of 1.03 with an  $R^2$  of 0.59.

Hence, in principle the model is able to explain the data perfectly with sufficient flexibility in  $\xi_c$  - a particular example is the partial counterfactual exercise for India above. Figure OA-8 shows that the three country characteristics we consider do a good job of capturing this variation.<sup>52</sup>

Given the estimated parameters  $\hat{\vartheta}$  we can use (OA-11) to decompose the variation in inferred delegation efficiencies  $\xi_c$  into its different components and gauge the implications for the plant's life-cycle. For instance, we can predict the counterfactual delegation efficiencies in the US if it had the level of human capital of India by

$$\hat{\xi}(HC_{IND}, ROL_{US}, FD_{US}) = \hat{\vartheta}_0 + \hat{\vartheta}_{HC} \times HC_{IND} + \hat{\vartheta}_{ROL} \times ROL_{US} + \hat{\vartheta}_{FD} \times FD_{US}. \quad (OA-12)$$

The predicted life-cycle based on  $\hat{\xi}(HC_{IND}, ROL_{US}, FD_{US})$  can then be interpreted as the partial effect of changes in human capital on the resulting life-cycle. Using this intuition and (OA-12) we can hence decompose the explained life-cycle difference between US and Indian plants into its different components. This is the content of Table 6 in the main body of the text. To do so, we focus on the predicted life-cycle profile for the US and India given their observables, i.e.  $\hat{\xi}_c = \hat{\xi}(ROL_c, HC_c, FD_c)$  for c = US, IND. Note that if both the US and India were exactly on the 45° line in Figure OA-8, we had  $\hat{\xi}_{US} = \xi_{US}$  and  $\hat{\xi}_{IND} = \xi_{IND}^P$  from above. Because the linear relation imposed in (OA-11) slightly underestimates the delegation efficiency in the US, the implied life-cycle for the US is slightly lower than in the data. In both Table 6 and Figure 12, we therefore decompose the predicted life-cycle differences into the three different components, which by construction - explain the entirety of the predicted difference between India and the US.

### OA-2.6 Bootstrap Procedure used in Section 5.1

Because our sample has only C=53 countries, and is hence relatively small, we estimate the distribution of our estimated parameters  $\vartheta$  and our decomposition results with the following bootstrap procedure:

- 1. From our cross-country data set, we draw B samples of size C with replacement.
- 2. For each sample b = 1, 2, ..., B we estimate the parameters  $\hat{\vartheta}^{(b)}$  and perform the decomposition exercise reported in Table 6, which yields the share explained by human capital for age group a,  $s_{HC}(a)^{(b)}$ , and for the other characteristics respectively.
- 3. For each j = HC, ROL, FD, this gives us a distribution of parameters  $\left\{\hat{v}_{j}^{b}\right\}_{b=1}^{B}$  and explained shares  $\left\{s_{j}\left(a\right)^{(b)}\right\}_{b=1}^{B}$ , from which we can estimate confidence intervals standard errors.

In Figure OA-9, we depict the bootstrap distribution of the parameters  $\vartheta_j$  and we also include the respective point estimates. The distribution for the decompositions of the life-cycle for firms of age 21-25 are contained in Figure 13 in the main text.

<sup>&</sup>lt;sup>52</sup>The two countries on the right of the US are the Netherlands and the UK, both of which have a very high managerial employment share in the data.

30 40 25 35 30 25 15 20 0.01 <sup>ϑ</sup>FD 0.02 0.04 0.06 0.08 0.02 0.04 0.08 -0.01 0.02

Figure OA-9: Bootstrap distribution of  $\hat{\vartheta}$ 

Notes: The three panels depict the bootstrap distribution for the parameters  $\vartheta_j$ . From left to right, we display the case of  $\vartheta_{HC}$ ,  $\vartheta_{ROL}$  and  $\vartheta_{FD}$  respectively. The dashed line refers to the respective point estimate reported in Table OA-6. We use 10.000 bootstrap iterations.

#### OA-2.7 Reduced-Form Evidence: Variation across countries

In this section we provide additional reduced-form evidence for the cross-country variation underlying our decomposition exercise in Section 5.1. There we used the model to explain the variation in a country's managerial employment shares by three observable characteristics: the rule of law, the country's stock of human capital and the degree of financial development. In Table OA-7 we analyze this variation in a regression context, where we consider regressions of the form

$$M_c = \vartheta_0 + \vartheta_{HC} \times HC_c + \vartheta_{ROL} \times ROL_c + \vartheta_{FD} \times FD_c + o_c'\delta + u_c, \tag{OA-13}$$

where  $M_c$  is the managerial employment share in country c, HC, ROL and FD are as defined above and  $o_c$  denotes additional controls. Columns 1 - 4 contain the respective bivariate regressions and the specification that is closest to our structural decomposition. The results imply that there is a robust positive correlation between managerial employment shares and our three country characteristics. In column 5 we control for GDP per capita (GDP pc). While this does not add much in terms of explanatory power, the collinearity between the four dependent variables and the small sample render our estimates insignificant (except for the rule of law, which is still marginally significant). Somewhat surprisingly even GDP pc does not have a significant positive effect.<sup>53</sup> The respective point estimates are all of the expected sign. Finally, columns 6 - 9 consider different measures of human capital, which we extract directly from the Census. In column 6 we measure aggregate human capital stocks from IPUMS using the information contained in education attainment (which, recall, is coarser than years of schooling) and in column 8 we calculate this measure from the population of manufacturing workers. In columns 7 and 9 we proxy human capital

<sup>&</sup>lt;sup>53</sup>When we include log GDP pc individually, we find positive significant effects for the rule of law and financial development and an insignificant positive coefficient for human capital. There is a significantly positive relationship between GDP pc and managerial employment.

TABLE OA-7: Managerial Employment Shares: Cross-Country Variation

	Dep. Variable: Managerial employment share								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ROL	0.026*** (0.006)			0.015* (0.008)	0.014* (0.007)	0.008 (0.006)	0.007 (0.006)	0.008 (0.006)	0.007 (0.006)
Human Capital (Barro-Lee)	,	0.041*** (0.008)		0.015* (0.008)	0.007 (0.013)	,	,	,	,
$\ln(\text{Credit/GDP})$		, ,	0.024*** (0.005)	0.008* (0.004)	0.004 $(0.004)$	0.005 $(0.005)$	0.005 $(0.005)$	0.005 $(0.005)$	0.005 $(0.005)$
ln(GDP pc)			,	,	0.007 (0.006)	0.005 $(0.005)$	0.007 (0.005)	0.005 $(0.005)$	0.007 (0.005)
Human Capital (IPUMS)					,	0.010 (0.009)	,	,	,
College share (IPUMS)						,	0.062 $(0.095)$		
Human Capital, Manu. (IPUMS)							,	0.010 $(0.009)$	
College share, Manu. (IPUMS)								` '	0.062 $(0.095)$
$\frac{N}{R^2}$	57 0.50	53 0.38	$\frac{55}{0.47}$	$\frac{53}{0.57}$	$\frac{53}{0.58}$	$\frac{53}{0.59}$	$\frac{53}{0.59}$	53 0.59	53 0.59

Notes: The table contains the results from the regression (OA-13). Robust standard errors are reported in parentheses. \*, \*\* and\*\*\* denotes significance at the 10%, 5% and 1% level, respectively. ROL denotes the rule of law as measured by the Worldwide Governance Indicators distributed by the World Bank. Human capital is either taken from the Penn World Tables (Barro-Lee) or calculated directly from the IPUMS data (IPUMS) according to the procedure detailed above. "Human capital, Manuf." considers only manufacturing workers. GDP pc is taken from the Penn World Tables (Version 8.0). "Credit/GDP" denotes total private credit relative to GDP and is taken from the World Bank Global Financial database. "College share" is calculated from IPUMS and denotes the share of people with a college degree either for the entire economy or the manufacturing sector.

by the share of college graduates, in case managerial human capital is particular reliant on higher education. These exercises are qualitatively similar to column 5. Given the data, there is little variation once GDP pc is controlled for. The point estimates of our three country characteristics, however, are all of the expected sign.

In Table OA-8 we explicitly analyze the cross-country relationship between a country's managerial employment share and the level of distrust as measured from the World Value Surveys. We report these results separately, because we have information on both trust and managerial employment only for about 40 countries. In column 1 we show the simple bivariate correlation between distrust and managerial employment, which is negative and significant. Columns 2 - 4 include the three country characteristics used in our decomposition exercise. While the point estimate of distrust remains negative, it is no longer significant once we control for the rule of law or the state of financial development. In the last columns, we include all variables and also control for GDP pc. While all point estimates are of the expected sign, there is too little variation in the data to identify any significant correlations.

TABLE OA-8: MANAGERIAL EMPLOYMENT SHARES AND TRUST

	Dep. Variable: Managerial employment share					
	(1)	(2)	(3)	(4)	(5)	
Distrust	-0.130**	-0.055	-0.089**	-0.009	-0.032	
	(0.056)	(0.037)	(0.042)	(0.058)	(0.039)	
ROL		0.023***			0.014	
		(0.008)			(0.009)	
Human Capital (Barro-Lee)			0.041***		0.011	
			(0.010)		(0.017)	
ln(Private Credit/GDP)				0.024***	0.002	
				(0.008)	(0.007)	
ln(GDP pc)					0.008	
					(0.008)	
$\overline{N}$	42	41	39	40	39	
$R^2$	0.23	0.50	0.45	0.44	0.57	

Notes: The table contains the results from the regression (OA-13) where we also include a measure of distrust as an independent variable. Robust standard errors are reported in parentheses. \*, \*\* and\*\*\* denotes significance at the 10%, 5% and 1% level, respectively. Distrust is measured directly from the World Values Surveys. ROL denotes the rule of law as measured by the Worldwide Governance Indicators distributed by the World Bank. Human capital is taken from the Penn World Tables. GDP pc is taken from the Penn World Tables (Version 8.0). "Credit/GDP" denotes total private credit relative to GDP and is taken from the World Bank Global Financial database.