Reputational Concerns in Directed Search Markets with Adverse Selection*

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Abstract

This paper introduces reputation building in directed search with adverse selection. Seller types randomly determine the quality of the asset they hold, where both a seller's type and asset quality are private information. When an exchange occurs, the quality of the asset that a seller holds is revealed and the market updates its belief about a seller's type, which I refer to as reputation. Markets where sellers have a higher reputation have lower liquidity and higher prices. With reputational concerns, the downward liquidity distortions caused by adverse selection are exacerbated. Equilibrium selection is affected by the incentives sellers have to earn a higher reputation. Shocks to entry costs have larger effects when sellers can build a reputation through multiple matches with buyers.

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1 Introduction

The recent financial crisis and its effects have focused a lot of attention on the liquidity of financial markets and the effects of market participation on aggregates. In this regard, markets with search frictions and adverse selection can, to some extent, account for distortions in market participation. However, in the literature, information revealed during the course of a match between buyers and sellers does not have any repercussions for future payoffs. The question then naturally arises: how do incentives to hide and/or reveal information about the fundamental features of a market participant affect liquidity and equilibrium selection in directed search markets with adverse selection? To answer this question I construct a directed search model in which sellers build a reputation through their actions as well as the quality of their sales.

In this setting, reputation refers to the belief market participants have about a seller's underlying fundamental type. More specifically, this underlying type, which endures throughout a seller's life in the market, randomly determines the quality of the asset that a seller holds. When a seller and a buyer interact and a buyer learns the quality of the asset he has just purchased, he forms a belief about the seller's type, which I refer to as reputation. In a larger sense, one can think of reputation as a form of rating that outside observers attach to a particular seller given their observations about the quality of the asset that the seller has delivered in a successful transaction. I then enhance the usual definition of a submarket to include reputation so that when buyers post a price they also post the reputation of the sellers they are willing to buy from. In this enhanced version, search is directed not only in the price/liquidity dimension but also on the reputational one. Furthermore, the action of participating in a particular submarket reveals, albeit partially, some information about a seller's type.

This paper suggests that markets that attract more reputable sellers are less liquid and, because of the liquidity-price trade-off, more expensive. When a semi-pooling equilibrium occurs, sellers with a higher reputation hold a higher value asset in expectation. Therefore the price paid in the semi-pooling market is relatively high. Since in the lower separating market, price and tightness are unaffected by reputation, in markets that accept sellers of high reputation, the downward liquidity distortion is higher because of the higher incentives sellers have to imitate up.

The results indicate that in the presence of reputational concerns, the downward liquidity distortions caused by adverse selection are amplified. In the absence of reputational concerns, since agents holding a low quality asset tend to imitate up, the market tightness is distorted downward. When sellers are concerned about their reputation, they have an extra incentive to enter submarkets that deliver a higher reputation. This extra incentive causes the downward distortion to be more severe. It must be noted here that these distortions are present only when semi-pooling equilibria can be constructed. When there is full separation, reputation does not enter the mix since it does not affect the expectations buyers have about the type of asset they will receive in any particular market.

Another interesting implication of this paper is that timing matters for equilibrium selection; the multiplicity of equilibria means that if, in the future, sellers separate according to the type of asset they hold, semi-pooling equilibria are less likely in the present. This is because reputational concerns increase the incentive that low quality asset holding sellers have to imitate up, which, as explained above, increases the downward distortion in liquidity. Therefore, any shock that increases the equilibrium tightness in the semi-pooling market has to be larger in magnitude to cause an equilibrium shift in the case of reputational concerns. The converse is also true; if a semi-pooling equilibrium prevails for sellers when they are experienced, it is easier to maintain a semi-pooling equilibrium for new sellers.

A shock to the buyer's cost of entering the market has a higher negative effect on liquidity when reputational concerns are present. The intuition for this result is fairly clear. In the presence of reputational concerns, a seller holding a low quality asset can increase his reputation by entering the semi-pooling market. In a sense, this seller has an extra incentive to imitate up beyond the usual pricing incentive. When a shock in the cost of entry reduces the mass of buyers across submarkets, since the downward distortion in markets with reputational concerns must be larger in magnitude to account for this additional incentive, the total effect on liquidity is larger. This result suggests that markets where interactions between buyers and sellers reveal some information about the underlying type of a seller are more sensitive to shocks and can shut down easier than in the absence of reputational concerns.

This paper builds on the directed search literature starting with Peters (1991), Montgomery (1991) and Moen (1997) as well as the more recent literature that investigates the effects of private information on liquidity such as Guerrieri et al. (2010), Guerrieri and Shimer (2011) and Chang (2012). What is novel in this paper is the consideration of information about seller types being revealed during a successful match. Delacroix and Shi (2012) also build a directed search model in which a seller's actions send a signal to a buyer about the quality of the good a seller holds. However, in this paper sellers are ultimately concerned with their reputation and since it is buyers who post prices, seller's actions only reveal *ex-post* information about a seller's fundamental type. In the presence of semi-pooling equilibria as constructed by Guerrieri and Shimer (2011) as well as Chang (2012), reputational concerns determine outcomes since for some sellers there exist scenarios in which buyers overestimate the probability that a seller is of a higher type. In markets with full separation, these scenarios do not exist since buyers know with certainty the type of good they are to receive.

2 Environment

Consider a directed search market with a continuum of buyers and sellers. Time is continuous. Each seller is of type $i \in \{L, H\}$ with probability π and $1-\pi$ respectively. When a seller first enters he draws an asset from the set $S = \{s_1, s_2\}$, where $s_1 < s_2$ and the probability that a type i seller draws s_2 is given by λ^i, where $\lambda^L < \lambda^H$. In other words, high type sellers are more likely to draw the higher value asset. Buyers' and sellers' valuation of an asset is the same, but the type of asset a seller is holding and the type of the seller is private information. Each seller is endowed with only two draws from the set S, but must sell the first asset in order to be endowed with a second one. After having sold the second time, a seller exits the market and is immediately replaced by a new seller. The figure below depicts the timing of a seller's life in the market. There is a flow holding cost $c \in \{c_l, c_h\}$ where $c_l < c_h$ to the seller which is independent of a seller's type. The probability that a seller draws c_h is γ. Buyers pay a flow cost k to enter the market. Once a buyer exits the market he is immediately replaced by another.

Agents are risk neutral, infinitely lived with a discount rate of r. As is standard in directed (competitive) search markets, buyers post a price p and sellers direct their search towards their most preferred submarket. Each submarket is defined by the price p and

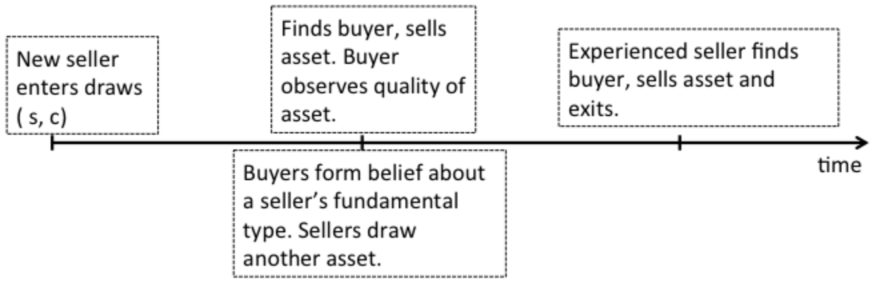

Figure 1: A Seller's timeline

the tightness (the ratio of buyers to sellers) denoted by $\theta(p)$.[1] In each submarket, matches are bilateral and random. The Poisson rate with which a seller matches with a buyer in submarket $(p, \theta(p))$ is $m(\theta(p))$ and the rate with which a buyer meets a seller is $q(\theta)$. By the definition of the market tightness, $m(\theta)$ is an increasing function of θ. Given the usual assumptions on the matching function, we have that $m(\theta) = \theta q(\theta)$.

2.1 Complete Information

Consider the case in which buyers have complete information on the type of asset a seller is holding.[2] Denote by $V_j(p, \theta)$ the value of a seller holding his j^{th} asset, where $j \in \{1, 2\}$. Then the model follows the previous literature where buyers post a price p having rational expectations about the equilibrium level of tightness $\theta(p)$ in each submarket. It is important to note here that submarkets are also defined by the length sellers have been in the market. So one can think of this categorization as a simple rule; when buyers post a price p in a market for new sellers, that implies that only new sellers are allowed to enter. New sellers then determine which submarket to enter by maximizing their expected profit. Given that in the benchmark case there are no informational asymmetries, a seller holding a type s asset solves the following problem:

$$\max_{\theta,p} rV_1(s, p, \theta(p)) = s_i - c_j + m(\theta(p))(p - V_1(s, p, \theta(p)) + E[V_2(s', p', \theta'(p'))]) \tag{2.1}$$

[1]This definition will be amended to include reputation in the later parts of the chapter.

[2]Note that in this case information about a seller's type is irrelevant since that does not affect a buyers payoff.

$$\max_{\theta,p} rV_2(s,p,\theta(p)) = s_i - c_j + m(\theta(p))(p - V_2(s,p,\theta(p))) \tag{2.2}$$

where $E[V_2(s^{'},p,\theta(p))|s]$ is the expected value to the seller of receiving the second asset of random quality $s^{'}$.

On the other side of the market, a buyers' utility is given by:

$$rU(p,\theta(p)) = -k + q(\theta)(\frac{s}{r} - p - U(p,\theta(p))) \tag{2.3}$$

Given that buyers face the free entry condition, in any equilibrium all buyers must be indifferent between entering the market or not, which implies that the price tightness relationship is expressed by:

$$p = \frac{s}{r} - \frac{k\theta(p)}{m(\theta(p))} \tag{2.4}$$

Note that in any equilibrium, the set of prices P and market tightness Θ must be exhaustive, in the sense that any deviation in the posting price $p^{'} \notin P$ is not profitable for a buyer.[3]

Solving the maximization problem for both sellers holding the first and the second asset we get the first best solution benchmark for the market tightness:

$$\begin{aligned} \theta_1^{FB} &: \quad \frac{c_j + rE[V_2(s^{'},p^{'},\theta^{'}(p^{'}))]}{k} = \frac{r + m(\theta) - \theta m^{'}}{m^{'}} \\ \theta_2^{FB} &: \quad \frac{c_j}{k} = \frac{r + m(\theta) - \theta m^{'}}{m^{'}} \end{aligned}$$

The first best solution in both cases is increasing in the ratio of the holding costs (c/k). Also note that due to perfect information, the quality of the asset does not determine market tightness. However, in markets with adverse selection this outcome cannot be sustained because sellers holding low quality assets have an incentive to pretend to be a higher type.

2.2 Markets with Incomplete Information

Given that sellers' types determine the quality of the asset that they hold, interactions between buyers and sellers are informative for the buyers. Once a buyer takes possession of an asset, its quality is immediately apparent and, if the seller of the asset was new in the market and is poised to receive another asset, the buyer forms an expectation about the type of the seller. Denote this expectation by μ, which represents the conditional probability that a seller is of type H, and by ϕ the conditional expectation that a seller

[3] On this see Burdett, Shi and Wright (2001).

holds s_2 given μ. The derivation of these probabilities will be discussed more in detail below. I assume that an expectation about a particular seller is public information. More specifically, when a seller that has already sold an asset in the market is searching for buyers, the market participants' beliefs about his type are common. One can consider this type of belief to be a form of reputation, since a higher μ implies that the seller has a higher likelihood of holding the higher value asset s_2.

Since the length a seller has been on the market and the market's belief about that seller type (μ) are common, submarkets here are defined not only by the usual parameters θ and p, but also by length in the market and reputation μ. So when a buyer sets up a market by posting a price, he specifies both length in the market and reputation of the type of sellers he wishes to attract. So in its entirety, a submarket is defined by four parameters $(p, \theta(p), \mu, j)$ where $j \in \{1, 2\}$ represents the new and experienced sellers respectively.

In this paper I will concentrate on symmetric stationary equilibria, where sellers face a stationary set of submarkets $(p, \theta(p), \mu, j)$ and buyers form an expectation about the conditional distribution of sellers that they attract in any submarket. Since this is an equilibrium with adverse selection, informational asymmetries will determine the types of sellers a particular submarket attracts which will in turn determine the expectation buyers form on the quality of the asset that will be available in each submarket. The free entry condition then determines the tightness of the submarket $\theta(p)$. Also note that the aggregate distribution of assets in the market does not affect equilibrium market tightness as in Shi (2009). This *block recursive* property of the equilibrium implies that submarket outcomes depend only on the distribution of the quality of the asset within a submarket rather than the aggregate.

As mentioned earlier, all buyers have a common prior about new sellers, but they form beliefs about sellers that have already sold an asset. As will become clearer in the analysis below, these beliefs are not only dependent on the quality of the asset a seller sold, but also on which submarket a seller chooses to search. Therefore, in order to make the analysis tractable, I will use backward induction and first concentrate on the outcome for sellers that have already sold an asset and have an individual reputation (market belief about their type).

2.3 Experienced Sellers

Consider the problem of a seller who has already sold an asset and holds a reputation μ. A buyer that posts the price p in a submarket open to experienced sellers with a particular reputation μ will in equilibrium post a price that satisfies the free entry constraint. Denote by y a seller's net valuation of the asset $s-c$. Then the seller's IC constraint requires:

$$V_2^*(y,\mu) = \max\{\frac{y}{r}, \max_{\hat{y}} \frac{y + p(\hat{y})m(\theta(\hat{y}))}{r + m(\theta(\hat{y}))}\} \tag{2.5}$$

A seller's net valuation of the asset, $s_i - c_j$ has four possible combinations. Suppose that $s_2 - s_1 > c_h - c_l$ so that the net valuations are ranked as follows: $y_1 = s_1 - c_h < y_2 = s_1 - c_l < y_3 = s_2 - c_h < y_4 = s_2 - c_l$. Then a buyers valuation is non-decreasing over y, which implies that there exists no pooling equilibrium.[4]

If however, $s_2 - s_1 < c_h - c_l$, [5] the ranking changes to: $y_1 = s_1 - c_h < y_2 = s_2 - c_h < y_3 = s_1 - c_l < y_4 = s_2 - c_l$ and the buyer's valuation is non-monotone over y. Denote $y_p \equiv s_p - c_h$ and $s_p = (1-\phi)s_1 + \phi s_2$. In this case, an equilibrium in which sellers of valuation y_2 and y_3 are pooled and y_1 and y_4 separate can be constructed as follows:

1) let $\theta^*(y_1) = \theta_2^{FB}(y_1)$

2) in the pooling equilibrium let $\theta^*(y_2) = \theta^*(y_3) = \theta^*(y_p)$, where $\theta^*(y_p)$ satisfies the IC constraint in 2.5. Note that since the buyers' valuation over y_p equals s_p, the free entry condition is automatically satisfied.

3) for y_4 set $\theta^*(y_4)$ to satisfy 2.5.

By construction, $y_p < s_2 - c_h < s_1 - c_l$, and since $\theta^*(y_p)$ satisfies the IC constraint

$$\frac{s_p - c_h + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p)}{r + m(\theta^*(y_p))} \geq \frac{s_p - c_h + \frac{s_2 m(\theta^*(y_4))}{r} - k\theta^*(y_4)}{r + m(\theta^*(y_4))}$$

it does so for both $y_2 = s_2 - c_h$ and $y_3 = s_1 - c_l$. Furthermore, any equilibrium that satisfies the free entry condition and the IC condition for sellers must induce $\theta_2^{FB}(y_1) \geq \theta^*(y_p) \geq \theta^*(y_4)$. To see that this holds in the semi- pooling equilibrium above, consider the first inequality, $\theta_2^{FB}(y_1) \geq \theta^*(y_p)$, where the latter must satisfy:

$$\max_{\theta} \frac{s_p - c_h + \frac{s_p m(\theta)}{r} - k\theta}{r + m(\theta)}$$

[4] See GSW and Chang.

[5] In this case sellers that hold the high value asset are not necessarily the ones willing to hold on to it the longest. One can think of this as a sudden need for funds or a correction of expectations about the future dividends of an asset.

$s.t$

$$\frac{y_1 + \frac{s_1 m(\theta_2^{FB}(y_1))}{r} - k\theta_2^{FB}(y_1)}{r + m\left(\theta_2^{FB}(y_1)\right)} \geq \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p)}{r + m\left(\theta^*(y_p)\right)}$$

If the constraint above was slack, then it is easy to see that $\theta^*(y_p) = \theta_2^{FB}(y_1)$ since the solution to $\max_\theta \dfrac{s_p - c_h + \frac{s_p m(\theta)}{r} - k\theta}{r + m(\theta)}$ is the same as the one for $\max_\theta \dfrac{s_1 - c_h + \frac{s_1 m(\theta)}{r} - k\theta}{r + m(\theta)}$. However, $\theta^*(y_p) = \theta_2^{FB}(y_1)$ would imply that $\frac{y_1 + \frac{s_1 m\left(\theta_2^{FB}(y_1)\right)}{r} - k\theta_2^{FB}(y_1)}{r + m\left(\theta_2^{FB}(y_1)\right)} < \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p)}{r + m(\theta^*(y_p))}$ since $s_p > s_1$, which would violate the constraint. Therefore the constraint must hold with equality in equilibrium. This fact together with the first order conditions of the above problem implies the desired result. The reasoning behind $\theta^*(y_p) \geq \theta^*(y_4)$ is similar.[6] The behaviour of $\theta^*(y_p)$ with respect to beliefs can be easily deduced from the IC constraint above. Given that the LHS is constant, any increase in s_p will result in a decrease in equilibrium tightness of the semi-pooling submarket since the RHS of the equation must be increasing in θ at the point of intersection. Since s_p depends positively on μ, we have that $\dfrac{\partial \theta^*(y_p)}{\partial \mu} < 0$. Therefore, in semi-pooling submarkets that admit sellers with lower reputation (lower μ), it is easier to find buyers for a seller. Note that this effect of reputation on market tightness affects the upper separating market for y_2 as well through the IC constraint. More specifically we can also write that $\dfrac{\partial \theta^*(y_2)}{\partial \mu} < 0$. In this manner, even in submarkets where reputation plays no role in determining expectations as to the quality of the asset, it plays a role in liquidity. In fact this characteristic would suggest that markets with very reputable sellers are less liquid and more expensive.

Note that in submarkets that attract sellers of type y_1 and y_4, expectations about a seller's type do not determine the price of the exchange because buyers know the type of asset they are about to receive. In those markets the value to the seller is simply given by $\dfrac{y_i + \frac{s_j m}{r} - k\theta^*(y_i)}{r + m\left(\theta^*(y_i)\right)}$. In the pooling market however, the expected value to the buyer depends on the expectations about a seller's type through ϕ. More specifically we can write $V_2^*(y, \mu) = \dfrac{y_i + \frac{s_p m}{r} - k\theta^*(y_i)}{r + m\left(\theta^*(y_i)\right)}$ and by the envelope theorem we have that $\frac{\partial V_2^*(y,\mu)}{\partial \mu} > 0$. Therefore, if $s_2 - s_1 > c_h - c_l$ and a semi-pooling equilibrium cannot be sustained, there is no value to reputation given that the quality of a seller's asset will eventually be revealed. If however $s_2 - s_1 < c_h - c_l$ there exists a semi-pooling equilibrium as described above that induces reputational concerns. As a consequence, the manner in which beliefs are formed

[6] Here I have chosen to concentrate on semi-pooling equilibria with downward distortion in tightness.

determines the expected future payoff of a new seller.

It is important to note that the condition $s_2 - s_1 < c_h - c_l$ allows us to construct a semi-pooling equilibrium but does not exclude the possibility of a fully separating equilibrium in the market for experienced sellers. It is worth noting here that in a fully separating equilibrium, the value function of a seller does not depend on the reputation parameter μ. This is due to the fact that in such an equilibrium, buyers expect to attract sellers holding a particular type of asset, which implies that prices are determined with certainty. More specifically, a buyer's expectations about a seller's type are irrelevant given that they know precisely the type of asset a seller is holding in any submarket.

2.4 New Sellers

Now consider the problem of a new seller that is in possession of an asset of quality s_i. Given the non-monotonicity of a buyer's matching value, a pooling equilibrium can potentially be constructed as above. However, belief updating determines the nature of this equilibrium, so at this point it is necessary to clarify how beliefs depend on the ratios of seller types in each of the submarkets. The belief parameter μ^k is the ex-post probability that a particular seller is of type H given the type of asset he sold as a new seller s_i and the submarket he participated in x_k, where x_1 denotes the submarket $(p^{FB}, \theta_1^{FB}, \pi, 1)$, x_p the pooling submarket and by x_2 the upper separating submarket.[7] More formally this can be written as:

$$\mu^k = \Pr[H|s_i, x_k] = \frac{\Pr[s_i, x_k|H]\Pr[H]}{\Pr[s_i, x_k|H]\Pr[H] + \Pr[s_i, x_k|L]\Pr[L]}$$

by Bayesian updating. Rewriting $\Pr[s_i, x_k|j]$ as $\Pr[s_i, x_k, j]/\Pr[j] = \Pr[s_i|x_k, j]\Pr[s, j]/\Pr[j]$ we can rearrange the above expression.

$$\mu^k = \Pr[H|s_i, x_k] = \frac{\Pr[s_i|x_k, H]}{\Pr[s_i|x_k, H] + \Pr[s_i|x_k, L]\frac{\Pr[x_k, L]}{\Pr[x_k.H]}} = \frac{1}{1 + \frac{\Pr[s_i|x_k, L]}{\Pr[s_i|x_k, H]}\frac{\Pr[x_k, L]}{\Pr[x_k.H]}} \qquad (\mu)$$

Note that for any type of seller, as far as actions are concerned, ex-post beliefs depend on the ratio of low types to high types in the submarket they decide to enter. That is to say that when buyers post a price p, their expectations about the tightness of the market that will result in equilibrium will take into account the extra incentive that sellers have

[7]Here I am abusing notation a bit to clarify the source of market beliefs. Inherently and ^{k} are the same object.

to pretend to be higher types. In the market for experienced sellers for example, the trade-off was between the tightness of the market and the price paid, while in the current scenario, low quality asset sellers have incentives beyond this trade-off given that, as shown in the subsection above, future payoffs depend positively on reputation, which is negatively affected by the ratio of low to high types in a given market.

In a pooling equilibrium, the expression $\frac{\Pr[x_k,L]}{\Pr[x_k.H]}$ for each submarket is:

$$\begin{aligned}
x_1 &: \frac{\pi}{(1-\pi)}\frac{(1-\lambda^L)}{(1-\lambda^H)} \\
x_p &: \frac{\pi}{(1-\pi)}\frac{(1-\lambda^L)(1-\gamma)+\lambda^L\gamma}{(1-\lambda^H)(1-\gamma)+\lambda^H\gamma} \\
x_2 &: \frac{\pi}{(1-\pi)}\frac{\lambda^L}{\lambda^H}
\end{aligned}$$

It is interesting to note that since $\lambda^L < \lambda^H$ the ratio of low to high types is so that $\frac{\Pr[x_1.L]}{\Pr[x_1,H]} > \frac{\Pr[x_p.L]}{\Pr[x_p,H]} > \frac{\Pr[x_2.L]}{\Pr[x_2,H]}$. As a way of an illustration consider $\Pr[H|s_1,x_1]$ and $\Pr[H|s_1,x_p]$. Since $\Pr[s_1|x_1,j] = \Pr[s_1|x_1] = 1$ we have that

$$\begin{aligned}
\Pr[H|s_1,x_1] &= \frac{1}{1+\frac{\Pr[x_1.L]}{\Pr[x_1,H]}} = \frac{1}{1+\frac{\pi}{(1-\pi)}\frac{(1-\lambda^L)}{(1-\lambda^H)}} \\
\Pr[H|s_1,x_p] &= \frac{1}{1+\frac{\pi}{(1-\pi)}\frac{(1-\lambda^L)}{(1-\lambda^H)}\frac{(1-\lambda^L)(1-\gamma)+\lambda^L\gamma}{(1-\lambda^H)(1-\gamma)+\lambda^H\gamma}}
\end{aligned}$$

which implies that in a case with no reputational concerns ($\lambda^L = \lambda^H$), updating does not change the initial prior, as expected. When, however, reputational concerns are present, $\Pr[H|s_1,x_p] > \Pr[H|s_1,x_1]$ so that μ is larger if a seller enters the pooling submarket, which illustrates the extra incentive a seller holding an asset of value s_1 has to enter it. A similar comparison can be made between $\Pr[H|s_2,x_p]$ and $\Pr[H|s_2,x_2]$.

Having described belief updating, we now consider the pooling equilibrium in submarkets with only new sellers. A pooling submarket with new sellers can be constructed in a similar manner to the pooling equilibrium above, according to the following steps:

1) let $\theta^*(y_1) = \theta_1^{FB}(y_1)$

2) in the pooling equilibrium let

$$\theta^*(y_p) = \arg\max_{\theta} \frac{s_p - c_h + \frac{s_p m(\theta)}{r} - k\theta + m(\theta)E[V_2(s^{'},p^{'},\theta^{'}(p^{'}))|\mu^p]}{r+m(\theta)}$$

s.t

$$\frac{y_1 + \frac{s_1 m(\theta_1^{FB}(y_1))}{r} - k\theta_1^{FB}(y_1) + m(\theta)E[V_2(s^{'}, p^{'}, \theta^{'}(p^{'}))|\mu^1]}{r + m\left(\theta_1^{FB}(y_1)\right)}$$

$$\geq \quad \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p) + m(\theta)E[V_2(s^{'}, p^{'}, \theta^{'}(p^{'}))|\mu^p]}{r + m\left(\theta^*(y_p)\right)}$$

where μ^i refers to the ex-post belief on the type of a seller currently participating in market i. Since $\Pr[H|s_1, x_p] > \Pr[H|s_1, x_1]$ as argued above, a seller holding s_1 has a higher future value if he participates in the pooling market given that $\mu^1 < \mu^p$. In short, participating in the pooling market gives a low quality asset holding seller more "cover", which may lead buyers to overestimate the likelihood that a seller is of high type. It is also important to emphasize here that both types of sellers have this incentive to try to induce buyers into a higher belief about their type. But, since high type sellers are less likely to be holding on to a low quality asset, on average, it is mostly low quality sellers that are trying to confound buyers.

For ease of notation, denote $E[V_2(s^{'}, p^{'}, \theta^{'}(p^{'}))|\mu^i]$ by $T(\mu^i)$. Because in expectation an experienced seller's value is constant save for the decision as to which market the seller will participate, this notation is particularly convenient.

3) for y_4 set $\theta^*(y_4)$ to satisfy a new seller's version of 2.5:

$$V_1^*(y, \pi) = \max\{\frac{y}{r}, \max_{y_i} \frac{y + p(y_i)m(\theta(y_i)) + m(\theta(y_i)T(\mu^i)}{r + m(\theta(y_i))}\}$$

The argument as to why such an equilibrium exists are very similar to those made for constructing the pooling equilibrium above. The same argument applies to showing that the downward IC constraint binds and that in equilibrium $\theta^*(y_1) > \theta^*(y_p) > \theta^*(y_4)$, i.e. that there is downward distortion in the market tightness. However, as argued above, the IC constraint in the case of new sellers contains an extra term that is action dependent, which implies that if there is downward distortion in a pooling equilibrium, the degree of the distortion will depend on reputational concerns. In fact, since sellers' future value depends on the type of submarket they will be allowed to enter (as denoted by μ), and this reputation is determined through their interactions as new sellers, the role of reputation here is crucial. This can be seen from the parameter ϕ, which is defined as $\Pr[s_2]$ for an individual with reputation μ. We know that $\phi = (1-\mu)\lambda^L + \mu\lambda^H$, and that $s_p = (1-\phi)s_1 + \phi s_2$, which implies that $\frac{\partial s_p}{\partial \mu} = (\lambda^H - \lambda^L)(s_2 - s_1)$. If high types are much more likely to receive s_2 and/or the differences in the buyer's valuation increases, the value of having a high reputation for a new seller increases, which induces more distortion in the market tightness. Therefore it is important to formalize the effect that reputational concerns have on market tightness (θ).

Proposition 1. *Denote by $\tilde{\theta}(y_i)$ the equilibrium tightness in a semi-pooling equilibrium where $\lambda^L = \lambda^H$. Then $\tilde{\theta}(y_i) > \theta(y_i)$ for all y_i, where $\theta(y_i)$ represents equilibrium tightness in the case of $\lambda^L < \lambda^H$.*

Proof. If $\lambda^L = \lambda^H$, as illustrated above, ex-post beliefs that a seller holding s_1 is of type H are the same as the common prior $1 - \pi$. Therefore, the future value to a seller in this case is $T(\pi)$ regardless of which market he enters. Rewriting the condition for $\theta_1^{FB}(y_1)$ we have:

$$\tilde{\theta}_1^{FB} : \frac{c_j + rT(\pi)}{k} = \frac{r + m(\theta) - \theta m'}{m'}$$

while in the case of $\lambda^L < \lambda^H$, this is given by:

$$\theta_1^{FB} : \frac{c_j + rT(\mu)}{k} = \frac{r + m(\theta) - \theta m'}{m'}$$

where $\mu < 1 - \pi$ and therefore $T(\mu) < T(\pi)$. Given that the RHS of the above expressions is increasing in θ due to the concavity of m, it must be that $\tilde{\theta}_1^{FB} > \theta_1^{FB}$. Now consider the tightness of the pooling market. With no reputational concerns, the IC constraint implies:

$$\frac{y_1 + \frac{s_1 m\left(\tilde{\theta}_1^{FB}\right)}{r} - k\tilde{\theta}_1^{FB} + m\left(\tilde{\theta}_1^{FB}\right)T(\pi)}{r + m\left(\tilde{\theta}_1^{FB}\right)} = \frac{y_1 + \frac{s_p m\left(\tilde{\theta}^*(y_p)\right)}{r} - k\tilde{\theta}^*(y_p) + m\left(\tilde{\theta}^*(y_p)\right)T(\pi)}{r + m\left(\tilde{\theta}^*(y_p)\right)}$$

(A)

while in the case of reputational concerns we have:

$$\frac{y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k\theta_1^{FB} + m\left(\theta_1^{FB}\right)T(\mu^1)}{r + m\left(\theta_1^{FB}\right)} = \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p) + m\left(\theta^*(y_p)\right)T(\mu^p)}{r + m\left(\theta^*(y_p)\right)}$$

(B)

where $T(\pi) > T(\mu^p) > T(\mu^1)$. Given this the RHS of A lies strictly above that of B for all θ as depicted in the figure below. Now consider $\Delta LHS|_{\theta_1^{FB} = \tilde{\theta}_1^{FB}} = \frac{m\left(\tilde{\theta}_1^{FB}\right)}{r + m\left(\tilde{\theta}_1^{FB}\right)}(T(\pi) - T(\mu^1))$, which, given the fact that $\tilde{\theta}_1^{FB} > \theta_1^{FB}$ and that both $\tilde{\theta}_1^{FB}$ and θ_1^{FB} are maximands, is strictly smaller than ΔLHS. If $\Delta RHS|_{\theta^*(y_p) = \tilde{\theta}^*(y_p)} < \Delta LHS|_{\theta_1^{FB} = \tilde{\theta}_1^{FB}} < \Delta LHS$ then $\theta^*(y_p)$ lies strictly to the left of $\tilde{\theta}^*(y_p)$ as per the figure below. Given that $\theta^*(y_p)$ and $\tilde{\theta}^*(y_p)$ are smaller

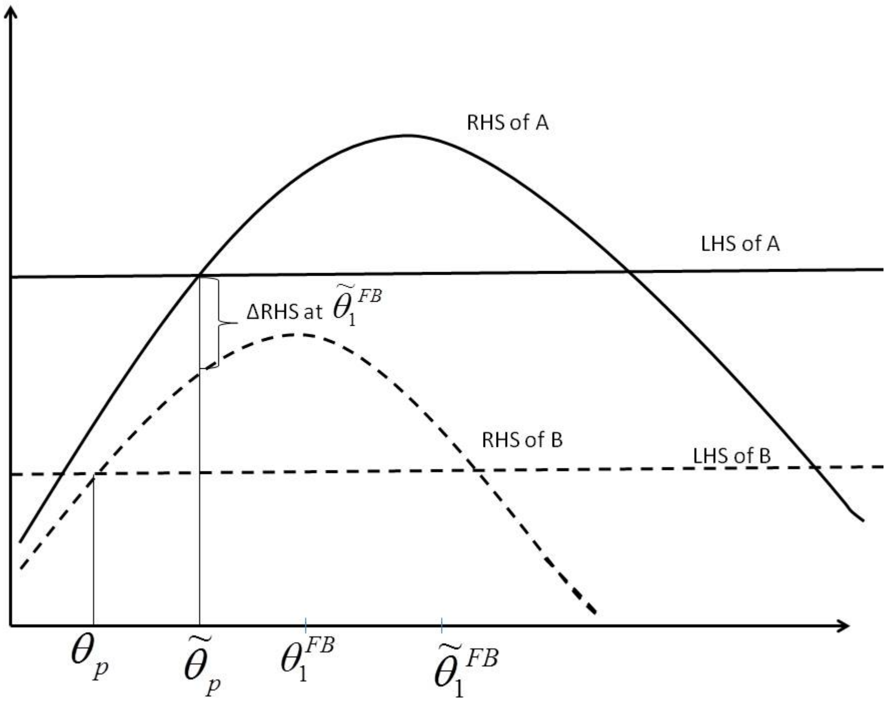

Figure 2: The IC constraint

then their respective first best market tightness, the RHS for both A and B is increasing in θ. Moreover, since $T(\pi) > T(\mu^p)$ the curves have the shape depicted in the figure below. Now $\Delta RHS|_{\theta^*(y_p)=\tilde{\theta}^*(y_p)} = \frac{m\left(\tilde{\theta}_1^{FB}\right)}{r+m\left(\tilde{\theta}_1^{FB}\right)}(T(\pi) - T(\mu^p))$, which, given that $T(\mu^p) > T(\mu^1)$ is strictly smaller than $\frac{m\left(\tilde{\theta}_1^{FB}\right)}{r+m\left(\tilde{\theta}_1^{FB}\right)}(T(\pi) - T(\mu^1)) = \Delta LHS|_{\theta_1^{FB}=\tilde{\theta}_1^{FB}}$. □

Claim 1 above gets at a fundamental aspect of directed search markets where reputational concerns play a role; namely that the liquidity distortion in such markets is deeper. Although in this section we concentrate on semi-pooling markets with downward distortion, a similar argument can be made when an equilibrium with upward liquidity distortion (as in Chang) can be constructed.

However, the multiplicity of equilibria in markets for both new and experienced sellers raises questions about the relationship between timing and equilibrium selection through the reputation channel. As clarified above, if a fully separating equilibrium prevails in markets for experienced sellers, then there is no return to reputation, since payoffs are equal conditional on the type of asset a seller holds. If this is the case, it seems that the incentive sellers have to imitate up due to reputational concerns vanishes, which should in theory make the selection of a semi pooling equilibrium for new sellers less likely. It also seems intuitive that the converse should hold, so that in the case when a semi-pooling equilibrium prevails in the market for experienced sellers, then it is more likely that a semi-pooling equilibrium prevails in the market for new sellers too. In this way, a seller's continued market presence coupled with reputational concerns has deep implications for equilibrium selection. This intuition leads to the following result:

Proposition 2. *Denote by $\hat{\theta}(y_p)$ and $\hat{\theta}(y_2)$ the equilibrium tightness in markets for new sellers in the semi-pooling and fully separating cases respectively when there is full separation in markets for experienced sellers and by $\theta(y_p)$ and $\theta(y_2)$ when there is a semi-pooling equilibrium in the market for experienced sellers. Then the following must hold:*

1. $\hat{\theta}(y_p) - \hat{\theta}(y_2) < \theta(y_p) - \theta(y_2)$

2. in the absence of reputational concerns $\hat{\theta}(y_p) - \hat{\theta}(y_2) = \theta(y_p) - \theta(y_2)$.

Proof. Consider the IC constraints for both cases. When there is a semi-pooling equilibrium in markets for experienced sellers P describes the downward IC constraint when the semi-pooling equilibrium prevails in markets for new sellers while S describes the downward IC constraint when the separating equilibrium prevails in these markets:

$$\frac{y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k\theta_1^{FB} + m\left(\theta_1^{FB}\right)T(\mu^1)}{r + m\left(\theta_1^{FB}\right)} = \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p) + m\left(\theta^*(y_p)\right)T(\mu^p)}{r + m\left(\theta^*(y_p)\right)} \quad (P)$$

$$\frac{y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k\theta_1^{FB} + m\left(\theta_1^{FB}\right)T(\mu^1)}{r + m\left(\theta_1^{FB}\right)} = \frac{y_1 + \frac{s_2 m(\theta^*(y_2))}{r} - k\theta^*(y_2) + m\left(\theta^*(y_2)\right)T(\mu^2)}{r + m\left(\theta^*(y_2)\right)} \quad (S)$$

where for now let us suppose that $T(\mu^1) < T(\mu^p) < T(\mu^2)$. On the other hand, when there is a separating equilibrium in markets for experienced sellers $\hat{P}$ describes the downward IC constraint when the semi-pooling equilibrium prevails in markets for new sellers while $\hat{S}$ describes the downward IC constraint when the separating equilibrium prevails in these markets:

$$\frac{y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k\theta_1^{FB} + m\left(\theta_1^{FB}\right)T(\mu^1)}{r + m\left(\theta_1^{FB}\right)} = \frac{y_1 + \frac{s_p m\left(\hat{\theta}^*(y_p)\right)}{r} - k\hat{\theta}^*(y_p) + m\left(\hat{\theta}^*(y_p)\right)T(\mu^p)}{r + m\left(\hat{\theta}^*(y_p)\right)} \quad (\hat{P})$$

$$\frac{y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k\theta_1^{FB} + m\left(\theta_1^{FB}\right) T(\mu^1)}{r + m\left(\theta_1^{FB}\right)} = \frac{y_1 + \frac{s_2 m\left(\hat{\theta}^*(y_2)\right)}{r} - k\hat{\theta}^*(y_2) + m\left(\hat{\theta}^*(y_2)\right) T(\mu^2)}{r + m\left(\hat{\theta}^*(y_2)\right)}$$

$(\hat{S})$

where $T(\mu^1) = T(\mu^p) = T(\mu^2)$ since there is separation in markets for experienced sellers, and therefore the value of an experienced seller depends only on the quality of the asset he holds. Consider equations $\hat{P}$ and $\hat{S}$. The fact that, both $\hat{\theta}^*(y_p)$ and $\hat{\theta}^*(y_2)$ are smaller than θ_1^{FB} implies that $\frac{\partial RHS}{\partial \theta}|_{\theta=\hat{\theta}^*(y_2)} > 0$ and $\frac{\partial RHS}{\partial \theta}|_{\theta=\hat{\theta}^*(y_p)} > 0$ (see figure above). Since the RHS of P is equal to the RHS of S, and since $s_2 > s_p = (1-\phi)s_1 + \phi s_2$, it must be that $\hat{\theta}^*(y_2) < \hat{\theta}^*(y_p)$. The same argument can be used to show that $\theta^*(y_2) < \theta^*(y_p)$. Note however that in the case of P and S, $T(\mu^p) < T(\mu^2)$ while in the case of $\hat{P}$ and $\hat{S}$, $T(\mu^p) = T(\mu^2)$ which implies that $\hat{\theta}(y_p) - \hat{\theta}(y_2) < \theta(y_p) - \theta(y_2)$. All we need to show at this point is that $T(\mu^p) < T(\mu^2)$ when there is semi-pooling in the market for experienced sellers. This holds *iff* $\mu^2 > \mu^p$. Now μ^2 is given by

$$\frac{1}{1 + \frac{\pi}{1-\pi}\frac{\lambda^L}{\lambda^H}}$$

while μ^p is given by $\Pr[H|s_1, x_p]$ above. In order to show the desired result it is sufficient to show that

$$\frac{\lambda^H}{\lambda^L} > \frac{(1-\lambda^H)(1-\gamma) + \lambda^H\gamma}{(1-\lambda^L)(1-\gamma) + \lambda^L\gamma}$$

which holds *iff* $\lambda^H > \lambda^L$. Part 2 simply follows from the fact that in the absence of reputational concerns $T(\mu^p) = T(\mu^2)$.

□

Given the multiplicity of equilibria in markets for both new and experienced sellers, equilibrium selection depends on the expectations of buyers as to what market tightness will result in equilibrium when a certain price is posted. Part 1 of the claim above highlights the fact that when a fully separating equilibrium occurs in the market for experienced sellers the distance between the semi-pooling and fully separating equilibria in markets for new sellers gets smaller as measured by the difference between the respective equilibrium tightness. This is due to the fact that a fully separating equilibrium for experienced sellers makes the action of choosing a submarket to participate in for new sellers irrelevant to future value. This is because the outcome for experienced sellers depends *only* on the quality of the asset a seller holds. On the other hand, when a semi-pooling equilibrium prevails in the market for experienced sellers, there is added incentive for new sellers to gain in reputation, which increases their incentive to pool, thus making the selection of a pooling equilibrium in the market for new sellers more likely. In fact, the claim above tells us that the deviation needed to jump from a semi-pooling to a separating equilibrium in the market for new sellers is smaller when there is full separation in markets for experienced sellers. More specifically, if there is a semi-pooling equilibrium in the market for experienced sellers, any shock that reduces the semi-pooling equilibrium tightness $\theta(y_p)$ in the market for new sellers has to be larger to induce an equilibrium switch. In this way, equilibrium selection in one submarket (experienced sellers) determines equilibrium selection for another submarket (new sellers). Part 2 of the claim simply establishes the fact that when there are no reputational concerns, any incentive that sellers have to pool vanishes, which makes equilibrium selection in the market for new sellers independent of the outcome in the market for experienced sellers.

It is important to reiterate here that the presence of reputational concerns exacerbates the incentive that sellers holding s_1 have to enter the pooling submarket. Therefore, intuitively, reputational concerns can amplify the effects of shocks as measured by the response of market tightness. To be more specific, consider an increase in the entry cost k. This reduces the probability that a seller will find a buyer in any submarket. However, because with reputational concerns sellers holding s_1 have an extra incentive to enter the pooling submarket, the fall in the tightness of the latter must compensate for this. This intuition is represented by the claim below:

Proposition 3. *Denote by $\theta(y_p)$ and $\bar{\theta}(y_p)$ the equilibrium market tightness in the pooling submarket in the case of $\lambda^L < \lambda^H$ and $\lambda^L = \lambda^H$ respectively. Suppose there is a one time increase in k. Then $\Delta\theta(y_p) < \Delta\bar{\theta}(y_p) < 0$.*

Proof. First, it is useful to show that the response of θ_1^{FB} is larger in magnitude than that of $\hat{\theta}_1^{FB}$, where $\hat{\theta}_1^{FB}$ represents the first best tightness for the case where $\lambda^L = \lambda^H$. Note that we have already shown that $\theta_1^{FB} < \hat{\theta}_1^{FB}$. Consider the expression for θ_1^{FB}.

$$\begin{aligned} \theta_1^{FB} &: \quad \frac{c_j + rT(\mu^1)}{k} = \frac{r + m(\theta) - \theta m'}{m'} \\ \hat{\theta}_1^{FB} &: \quad \frac{c_j + rT(\pi)}{k} = \frac{r + m(\theta) - \theta m'}{m'} \end{aligned}$$

Taking derivatives with respect to k we have:

$$\begin{aligned}
\theta_1^{FB} &: \quad \frac{c_j + rT(\mu^1)}{k^2} = \frac{\partial \theta}{\partial k}\left(\frac{(r+m(\theta))\,m''}{(m')^2}\right) \\
\hat{\theta}_1^{FB} &: \quad \frac{c_j + rT(\pi)}{k^2} = \frac{\partial \theta}{\partial k}\left(\frac{(r+m(\theta))\,m''}{(m')^2}\right)
\end{aligned}$$

Given that $T(\pi) > T(\mu^1)$ as shown above and the concavity of m we have that $0 > \frac{\partial \hat{\theta}_1^{FB}}{\partial k} > \frac{\partial \theta_1^{FB}}{\partial k}$. Now consider the IC constraints for both cases for the submarket for new sellers:

$$f(\theta_1^{FB}, \mu^1) = \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p) + m\left(\theta^*(y_p)\right)T(\mu^p)}{r + m\left(\theta^*(y_p)\right)} \tag{C}$$

$$f(\hat{\theta}_1^{FB}, \pi) = \frac{y_1 + \frac{s_p m\left(\hat{\theta}^*(y_p)\right)}{r} - k\hat{\theta}^*(y_p) + m\left(\hat{\theta}^*(y_p)\right)T(\pi)}{r + m\left(\hat{\theta}^*(y_p)\right)} \tag{D}$$

where

$$f(x,y) \equiv \frac{y_1 + \frac{s_1 m(x)}{r} - kx + m\left(x\right)T(y)}{r + m\left(x\right)}$$

Due to the above result, we know that the $\Delta f(\theta_1^{FB}, \mu^1) < \Delta f(\hat{\theta}_1^{FB}, \mu^1) < 0$. We also know that at any θ, the derivative with respect to θ for the RHS of C is strictly smaller than that of D (see figure above). Given that the change in the left hand side is larger in magnitude for C then implies the desired result. $\square$

The intuition for a larger fall in liquidity in the pooling submarket in the case of reputational concerns was given above. What is as interesting is the fact that the effect is not isolated to the semi-pooling market tightness. The first best tightness falls more in the case with reputational concerns because the costs to waiting are lower for those sellers whose reputation is relatively low ($\mu < \pi \Rightarrow T(\pi) > T(\mu)$). In that case, individuals are willing to wait longer to meet a buyer, causing the larger drop in liquidity. In this way, reputational incentives result in higher sensitivity of liquidity to cost shocks.

3 Conclusion

In this paper I have constructed a model with adverse selection in markets with directed search. I find that reputational concerns deepen the impact that adverse selection has on market liquidity as measured by the ratio of buyers to sellers. Furthermore, the results suggests that equilibrium selection is significantly affected by the reputational mechanism, with the type of equilibrium that prevails in the second period of a seller's life determining the type of equilibrium that occurs when the seller is a new entrant. More specifically, if a fully separating equilibrium is the outcome in the later stages of a seller's career, then a semi-pooling equilibrium is harder to maintain when the seller is new. The converse seems to also be true. Moreover, shocks that reduce market participation seem to have larger effects when sellers have reputational concerns.

The modeling choices of the paper in terms of the types of sellers and quality of assets available in the market are intentionally limited to highlight relevant features of the results. However, any extension of the model that utilized multiple types on both dimensions could be achieved without substantial change to the main results of the paper. Where the difficulty lies however, is in the extension of a seller's life to more than two periods. This approach is more difficult to undertake due to the large set of informational possibilities that would result in such a scenario. Nonetheless, further work in this direction could prove fruitful and helpful in understanding market participation in this environment.

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