Monetary Policy, Trend Inflation and Unemployment Volatility

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Abstract

The literature has long agreed that the canonical DMP model with search and matching frictions in the labor market can deliver large volatilities in labor market quantities, consistent with US data during the *Great Moderation* period (1985-2005), only if there is at least some wage stickiness. I show that the canonical model can deliver nontrivial volatility in unemployment without wage stickiness. By keeping average US inflation at a small but positive rate, monetary policy may be accountable for the standard deviations of labor market variables to have achieved those large empirical levels. Solving the Shimer (2005) puzzle, the role of long-run inflation holds even for an economy with flexible wages, as long as it has staggered price setting and search and matching frictions in the labor market.

Trend inflation, Staggered prices, Unemployment volatility puzzle.
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1 Introduction

The theoretical literature on search and matching frictions in the labor market has long recognized that the DMP model (after Diamond (1982), Mortensen (1982) and Pissarides (1985)) exhibits what came to be known as the *Shimmer Puzzle*: the model predicts that standard deviations of key labor market variables are of the same order of magnitude as the standard deviation of labor productivity, i.e. output per hours worked. However, empirical evidence from US data strongly suggests that standard deviations of the labor market variables are about ten times or more as large as what the model implies (see Shimer (2005), Hall (2005) and Costain and Reiter (2008)). Hence, a puzzle arises in reconciling these two perspectives.

Most of the existing literature agrees that the unemployment volatility puzzle is best resolved by incorporating wage rigidities.[1] As Pissarides (2009, pg. 1340) states, the "canonical model can deliver nontrivial volatility in unemployment only if there is at least some wage stickiness."

In this paper, I show that the canonical model can deliver nontrivial volatility in unemployment without wage stickiness. My main result is that by keeping average US inflation at a small but positive rate during the *Great Moderation* period 1985:Q1-2005:Q4, monetary policy may be accountable for the standard deviations of labor market quantities to achieve the levels observed in the US. That is, keeping average inflation at positive rates are just as relevant to explain the puzzle as wage rigidities are. The role of long-run inflation holds even for an economy with flexible wages, as long as it has staggered price setting and search and matching frictions in the labor market. Even though empirical results in the US strongly suggest that wage rigidities are not negligible, the evidence also points to relevant price staggering (see e.g. Kehoe and Midrigan (2015) and Nakamura and Steinsson (2008)) and non negligible positive inflation rate ($\bar{\pi} \approx 3.05\%$) during the Great Moderation period.[2]

To formalize my argument, I augment the DMP model with staggering price setting, as described in section 2. Each differentiated firm is subject to the aggregate technology shock, holds firm-specific employment stock, and simultaneously decides on price setting and vacancy postings.[3] Hours per worker and salaries are decided every period by Nash bargaining between firms and unions. The representative household is subject to preference shocks, and monetary policy decisions are subject to policy shocks.

I detach from the standard literature on search and matching frictions in the labor market by allowing not only for technology shocks, but also by considering the effects of preference and monetary policy shocks. Since labor productivity, i.e. aggregate output per total hours worked, endogenously responds to all types of shocks, I assess how labor

[1] For papers with wage rigidities in all jobs, see e.g. Hall (2005), Gertler et al. (2008), Gertler and Trigari (2009), Thomas (2008), Blanchard and Gali (2010). For wage rigidities in ongoing jobs only, see e.g. Haefke et al. (2008) and Pissarides (2009). Alternative approaches avoiding wage rigidities include: adding heterogeneous worker productivity (e.g. Pries (2008)); adding time-varying bargaining power (e.g. Ravenna and Walsh (2011)); and using non-standard calibration (e.g. Hagedorn and Manovskii (2008)).

[2] I skip the recent period, after the 2008 crisis, for considering that there is no consensus yet on how to properly incorporate more sophisticated modelling approaches to account for financial frictions, financial shocks, the zero lower bound constraint on the nominnal interest rates, and non-conventional monetary policy.

[3] Recent models in which firms simultaneously decide on vacancy postings and price setting are used in e.g. Foerster and Mustre-del Rio (2015), Kuester (2010), Sveen and Weinke (2009) and Thomas (2011). They assume, on the other hand, different structures on wage formation and do not focus on the consequences of trend inflation on labor market volatilities.

market volatilities change according to each one of them.

As for long-run positive inflation rates, I follow the literature on non-zero trend inflation.[4] I show that the reason why small levels of trend inflation are able to generate higher relative volatilities in market quantities is indeed simple. Part of the explanation parallels my findings in Alves (2014). In that paper, I show that trend inflation alone is able to generate what I called *endogenous trend inflation cost-push shocks* in the New Keynesian Phillips curve (NKPC). And the level of trend inflation acts as a shock amplifier in the NKPC: it is zero when the trend inflation is zero and increases as trend inflation rises.

Here, I also find that the level of trend inflation is an important fluctuations' amplifier into labor market variables. This important finding comes from combining established results from both the literature on search and matching frictions in the labor market and on staggered prices.[5] On the one hand, when wages are jointly decided by Nash bargaining between firms and workers, total surpluses from job matches increase with output and decrease with disutility to work. On the other hand, larger levels of inflation increases price dispersion, which in turn increases dispersion in production and hours worked. As a consequence, aggregate output falls and the average disutility to work rises.

Therefore, total surpluses from job matches fall as long-run inflation rises. This result is key in this paper when seen at the light of the findings by Hagedorn and Manovskii (2008) and Costain and Reiter (2008). They show that employment becomes more volatile as the total surplus decreases. If it is sufficiently small, then small movements in labor productivity have relevant impact on the total surplus and are able to cause proportionally larger fluctuations in the labor market.

Extending my results on the Divine Coincidence in Alves (2014), I also show that monetary policy has an important role in channeling fluctuations into variables from the goods and labor markets. That is, the monetary authority could reduce fluctuations in the labor market by choosing different ways to perform monetary policy. But that would likely increase fluctuations observed in the goods market.

For assessing how larger levels of trend inflation affect relative volatilities, i.e. standard deviations of endogenous variables divided by that of labor productivity, I first consider the static equilibrium version of the non-linear model and obtain closed form solutions for labor market variables as functions of structural parameters, trend inflation and structural shocks. Following, I perform simulations with the dynamic (log-linearized) version of the model to illustrate how theoretical relative volatilities achieve their empirical counterparts. For that, I use standard calibration for the US and set the level of trend inflation at the observed average of US CPI inflation during the Great Moderation period.

An important finding is that the Shimmer Puzzle is somehow minimized when the economy is hit only by preference shocks, instead of technology ones. When hit by policy shocks, even though they have no effect in equilibria with flexible prices, theoretical relative volatilities are already consistent with empirical evidence at zero trend inflation.

The remainder of the paper is organized as follows. The model is described in Section 2. Data and stylized facts are presented in Section 3. Analytical results for a static model on the role of trend inflation and monetary policy on relative volatilities are derived in Sections 4 and 5. Dynamic amplification effects are discussed in Section 6. Section 7 shows

[4] Good references on the trend inflation literature are found in e.g. Alves (2014), Amano et al. (2007), Ascari (2004), Ascari and Ropele (2007, 2009), Ascari and Sbordone (2014), Cogley and Sbordone (2008), Coibion and Gorodnichenko (2011), Coibion et al. (2012), Kichian and Kryvtsov (2007), Kiley (2007) and Sahuc (2006).

[5] See e.g. Woodford (2003) and Walsh (2010).

the calibration strategy, while Section 8 shows simulation results and assesses how trend inflation performs in amplifying fluctuations towards labor market quantities. Section 9 summarizes the paper's conclusions.

2 The model

The economy consists of a central bank that implements monetary policy, a representative household with a continuum of workers, and a unit mass of differentiated firms $z \in (0,1)$. Each firm produces using labor in both the extensive and the intensive margins, posts job vacancies at a cost and makes price decisions, subject to Calvo (1983) price stickiness.[6] Workers can be hired or lose their jobs. The labor market is subject to search frictions captured by a matching function. Salaries and hours are decided in a flexible Nash bargaining framework.

2.1 Labor flows

At the end of period t, a fraction n_t of household members is employed in existing jobs, where $\mathsf{n}_t = \int_0^1 \mathsf{n}_t(z)\,dz$ aggregates all end-of-period specific labor force $\mathsf{n}_t(z) \in (0,1)$ at firm z. At the beginning of each period, employed members separate from their jobs at the exogenous rate $\rho \in (0,1)$. Therefore, the beginning-of-period unemployment u_t accounts for the unemployed members at the end of last period u_{t-1}^e and the recently separated workers. During the period, $\mathsf{m}_t = \int_0^1 \mathsf{m}_t(z)\,dz$ workers are matched into new jobs, where $\mathsf{m}_t(z)$ is the number of matches into firm z. Firm z posts $\mathsf{v}_t^e(z)$ job vacancies at the end of each period. Therefore, $\mathsf{v}_t^e = \int_0^1 \mathsf{v}_t^e(z)\,dz$ is the total number of vacancy postings. Let $\mathsf{v}_t(z) \equiv \mathsf{v}_{t-1}^e(z)$ denote the number of job openings at firm z available at the beginning of period t, and $\mathsf{v}_t \equiv \mathsf{v}_{t-1}^e$ the total number of job openings available at the same time. The laws of motion are described by

$$\begin{array}{ll} \mathsf{n}_t(z) = (1-\rho)\,\mathsf{n}_{t-1}(z) + \mathsf{m}_t(z) & ; \ \mathsf{u}_t = 1 - (1-\rho)\,\mathsf{n}_{t-1} \\ \mathsf{n}_t = (1-\rho)\,\mathsf{n}_{t-1} + \mathsf{m}_t & ; \ \mathsf{u}_t^e = 1 - \mathsf{n}_t \end{array} \tag{1}$$

In this context, $\mathsf{p}_t \equiv \mathsf{m}_t/\mathsf{u}_t$ is the job-finding rate within the period, $\mathsf{q}_t \equiv \mathsf{m}_t/\mathsf{v}_t$ is the matching rate for vacancies and $\theta_t \equiv \mathsf{v}_t/\mathsf{u}_t$ is the labor market tightness. As in Pissarides (2000), m_t is given by the Cobb-Douglas matching function $\mathsf{m}_t = \eta \mathsf{v}_t^{1-a}\mathsf{u}_t^a$. Assuming that the aggregate matching rate is given, the number of matches into firm z is $\mathsf{m}_t(z) = \mathsf{q}_t \mathsf{v}_t(z)$.

Empirical evidence on the labor market is commonly inferred in terms of end-of-period variables. In this context, I define the end-of-period market tightness $\theta_t^e \equiv \mathsf{v}_t^e/\mathsf{u}_t^e$. This variable complements the set formed by the end-of-period unemployment u_t^e and end-of-period vacancy postings v_t^e. As in Shimer (2005), it is also important to consider the end-of-period mass of unemployed members that have remained so for less than one period without being matched, i.e. short-term unemployment is defined as $\mathsf{u}_t^s \equiv \rho \mathsf{n}_{t-1}(1-\mathsf{p}_t)$.

[6] I depart from the alternative approach that considers two production sectors for considering that individual firms indeed face the simultaneous problem of setting prices and posting vacancies. For the literature on two production sectors, see e.g. Blanchard and Gali (2010), Christoffel and Linzert (2006), Faia (2008), Gali (2010), Ravenna and Walsh (2010, 2011), Thomas (2008) and Walsh (2005).

2.2 Households

The representative household has unions specialized in negotiating wage and hours with each firm. Union z represents all $\mathsf{n}_t(z)$ workers when bargaining with firm z on hours per worker $h_t(z)$ and the total nominal salary $W_t(z) = P_t w_t(z)$ to be paid over the period, where P_t is the aggregate price and $w_t(z)$ is the real salary, while $\varpi_t(z) = w_t(z)/h_t(z)$ is the real wage. The union's disutility to its members' hours worked $H_t(z) \equiv \mathsf{n}_t(z) h_t(z)$ is $\upsilon_t(z) \equiv \chi H_t(z)^{1+\nu}/(1+\nu)$, where ν^{-1} is generally view as the Frisch elasticity of labor supply. Since the unions belong to the representative household, its aggregate disutility function is $\upsilon_t \equiv \int_0^1 \upsilon_t(z)\,dz$.[7]

As in Merz (1995), I assume full risk sharing of consumption among household members, employed and unemployed.[8] All members pool their income and evenly consume $c_t(z)$ units of good z. Unemployed workers earn monetary transfers from the government until they are matched into a firm. That generates $P_t w^u(1-\mathsf{n}_t)$ in nominal income for the household, where w^u is the fixed real unemployment compensation. Consumption $c_t(z)$ over all differentiated goods is aggregated into a bundle C_t, as in Dixit and Stiglitz (1977), and provides an external habit formation utility[9] $u_t \equiv \epsilon_{\mathrm{u},t}\left(C_t - \iota_{\mathrm{u}}\widetilde{C}_{t-1}\right)^{1-\sigma}/(1-\sigma)$, where $\tilde{C}_t$ is the average consumption level which equals C_t in equilibrium, σ^{-1} is the intertemporal elasticity of substitution, $\iota_{\mathrm{u}} \in (0,1)$ is the habit formation parameter, and $\epsilon_{\mathrm{u},t}$ is a preference shock. Aggregation and expenditure minimization relations are described by:

$$\begin{array}{ll} (C_t)^{\frac{\phi-1}{\phi}} = \int_0^1 c_t(z)^{\frac{\phi-1}{\phi}}\,dz & ;\ P_t^{1-\phi} = \int_0^1 p_t(z)^{1-\phi}\,dz \\ c_t(z) = C_t\left(\frac{p_t(z)}{P_t}\right)^{-\phi} & ;\ P_t C_t = \int_0^1 p_t(z)\, c_t(z)\,dz \end{array}$$

where $\phi > 1$ is the elasticity of substitution between goods.

As usual, the household chooses the sequence of C_t, government-issued bonds B_t and state-contingent securities portfolio A_{t+1} to maximize its welfare measure $\mathcal{W}_t \equiv \max(u_t - \upsilon_t) + \beta E_t \mathcal{W}_{t+1}$, subject to the budget constraint and a standard no-Ponzi condition, where β denotes the subject discount factor. The household internalizes the fact that its optimal intertemporal consumption allocation contemporaneously affects firms' optimal decisions on job openings,[10] which in turn are based on firms' demand functions. Since job openings affect future employment, the household also chooses the sequence future firm-specific employment $\mathsf{n}_{t+1}(z)$ to maximize $\mathcal{W}_t$, subject to each firm-specific employment law-of-motion:

$$\begin{aligned} \mathcal{W}_t = \max\ & (u_t - \upsilon_t) + \int_0^1 \lambda_t^{cn}(z)\left[(1-\rho)\,\mathsf{n}_t(z) + \mathsf{m}_{t+1}(z) - \mathsf{n}_{t+1}(z)\right] dz + \beta E_t \mathcal{W}_{t+1} \\ & + \lambda_t\left[A_t + I_{t-1}B_{t-1} + \int_0^1 \mathsf{n}_t(z)\, W_t(z)\,dz + P_t w^u(1-\mathsf{n}_t) - P_t C_t - \Xi_t - Q_{t+1}A_{t+1} - B_t\right] \end{aligned}$$

where B_t is the stock of government issued bonds held at the beginning of period t, A_t

[7] Using a unions-based aggregate disutility function instead of a workers-based one allows me to derive closed form equations describing the dynamics of the aggregate disutility to work in Section 2.4, which is important under trend inflation. The dynamics implied by Beveridge equations and Calvo price setting convolute in such a way that the derivation is not possible otherwise.

[8] Some authors have been making efforts to model imperfect consumption insurance and fully capture the distortions caused by unemployment. See e.g. Christiano et al. (2010).

[9] See e.g. Abel (1990) and Gali (1994).

[10] See Section 2.3.

is the state-contingent value of the portfolio of financial securities held at the beginning of period t, under complete financial markets, Ξ_t denotes nominal profits net of lump-sum taxes that finance the unemployment transfers, $I_t = 1 + i_t$ is the (gross) nominal interest rate set at period t, and Q_{t+1} is the stochastic discount factor from $(t+1)$ to t. In equilibrium, optimal intertemporal plans and arbitrage conditions are described as follows:

$$\begin{array}{llll} C_t: & 1 = \beta E_t \left(\frac{u'_{t+1}}{u'_t} \frac{I_t}{\Pi_{t+1}} \right) & A_{t+1}: & Q_t = \beta \frac{\lambda_t}{\lambda_{t-1}} \\ \mathsf{n}_{t+1}(z): & \lambda_t^{cn}(z) = \lambda_t P_t E_t Q_{t+1}^{\pi} \mathsf{u}_{t+1}(z) & B_t: & 1 = I_t E_t Q_{t+1} \end{array} \tag{2}$$

where the first order condition for C_t is the standard transformation of $u'_t = \lambda_t P_t$, $\Pi_t = 1 + \pi_t$ is the (gross) inflation rate at period t, $Q_t^{\pi} \equiv Q_t \Pi_t$ is the real stochastic discount factor, $u'_t \equiv \partial u_t / \partial C_t = \epsilon_{\mathrm{u},t} \left(C_t - \iota_{\mathrm{u}} C_{t-1} \right)^{-\sigma}$ is the marginal utility to consumption, and $\mathsf{u}_t(z) \equiv (1/u'_t)\, \partial \mathcal{W}_t / \partial \mathsf{n}_t(z)$ is the household real surplus enjoyed as the marginal worker is matched into firm z. This variable is relevant during Nash bargaining on salaries.

Since $\mathsf{n}_t = \int_0^1 \mathsf{n}_t(z)\, dz$, the Envelope Theorem gives us the evolution dynamics of $\mathsf{u}_t(z)$:

$$\mathsf{u}_t(z) = w_t(z) - w^u - (1+\nu) \frac{1}{\lambda_t P_t} \frac{\upsilon_t(z)}{\mathsf{n}_t(z)} + \frac{\lambda_t^{cn}(z)}{\lambda_t P_t}(1-\rho) + \int_0^1 \frac{\lambda_t^{cn}(\bar{z})}{\lambda_t P_t} \frac{\partial \mathsf{m}_{t+1}(\bar{z})}{\partial \mathsf{n}_t(z)} d\bar{z}$$

where $\bar{z}$ is a firm other than z. The term $\partial \mathsf{m}_{t+1}(\bar{z}) / \partial \mathsf{n}_t(z)$ captures the effect of a specific match into firm z on the matching functions of all firms $\bar{z}$. In order to pin down this term, I need more elaboration.

The number of workers matched into firm z is $\mathsf{m}_t(z) = \mathsf{q}_t \mathsf{v}_t(z) = \mathsf{p}_t \mathsf{u}_t \mathsf{v}_t(z) / \mathsf{v}_t$. Likewise, $\mathsf{p}_t(z) \equiv \mathsf{m}_t(z) / \mathsf{u}_t$ is the job-finding rate for being matched at firm z and satisfies $\mathsf{p}_t = \int_0^1 \mathsf{p}_t(z)\, dz$. Let $\mathsf{s}_t(z) \equiv \mathsf{v}_t(z) / \mathsf{v}_t$ denote the firm's vacancy share. Conditioned on obtaining a new job, the probability that the worker is matched into firm z is $\mathsf{p}_t(z) / \mathsf{p}_t$, which also equals $\mathsf{v}_t(z) / \mathsf{v}_t = \mathsf{s}_t(z)$.

Being part of the household, unions internalize the effect of the specific match into firm z on the aggregate employment and on its consequences on the aggregate matching rate q_t, and on the matching functions of all firms $\bar{z}$. For that, unions conclude that:

$$\mathsf{m}_{t+1}(\bar{z}) = \mathsf{q}_{t+1} \mathsf{v}_{t+1}(\bar{z}) = \frac{\mathsf{p}_{t+1} \mathsf{u}_{t+1} \mathsf{v}_{t+1}(\bar{z})}{\mathsf{v}_{t+1}} = \mathsf{s}_{t+1}(\bar{z})\, \mathsf{p}_{t+1} \left[1 - (1-\rho) \int_0^1 \mathsf{n}_t(z)\, dz \right]$$

which implies that $\partial \mathsf{m}_{t+1}(\bar{z}) / \partial \mathsf{n}_t(z) = -(1-\rho)\, \mathsf{s}_{t+1}(\bar{z})\, \mathsf{p}_{t+1}$.

Therefore, the evolution dynamics of $\mathsf{u}_t(z)$ can be rewritten as follows:

$$\mathsf{u}_t(z) = w_t(z) - w^u - (1+\nu) \frac{1}{\lambda_t P_t} \frac{\upsilon_t(z)}{\mathsf{n}_t(z)} + (1-\rho) \left[\frac{\lambda_t^{cn}(z)}{\lambda_t P_t} - \mathsf{p}_{t+1} \int_0^1 \mathsf{s}_{t+1}(\bar{z}) \frac{\lambda_t^{cn}(\bar{z})}{\lambda_t P_t} d\bar{z} \right]$$

2.3 Firms and Bargaining

Firm z uses $H_t(z)$ hours to produce its differentiated good with technology $y_t(z) = \mathcal{A}_t H_t(z)^{\varepsilon}$, where $\varepsilon \in (0,1)$ and$\mathcal{A}_t$ is the aggregate technology shock, which evolves according to $\left(\mathcal{A}_t / \overline{\mathcal{A}} \right) = \epsilon_{\mathfrak{a},t} \left(\mathcal{A}_{t-1} / \overline{\mathcal{A}} \right)^{\phi_{\mathfrak{a}}}$, where $\epsilon_{\mathfrak{a},t}$ is the aggregate technology innovation and $\phi_{\mathfrak{a}} \in (0,1)$.

As in Calvo (1983), firm z optimally readjusts its price to $p_t(z) = p_t^*$ with probability $(1-\alpha)$. With probability α, its price is fixed at $p_t(z) = p_{t-1}(z)\Pi_t^{ind}$, where $\Pi_t^{ind} \equiv \Pi_{t-1}^{\gamma_\pi}$ and $\gamma_\pi \in (0,1)$.[11] Simultaneously, the firm optimally chooses the amount $\mathsf{v}_t^e(z)$ of job vacancies to be posted by the end of the period, in order to optimize its specific stock of employed workers $\mathsf{n}_{t+1}(z)$ in the next period. As in Ravenna and Walsh (2012), posting $\mathsf{v}_t^e(z)$ end-of-period vacancies requires $k\mathsf{v}_t^e(z)$ units of the final aggregate good. Optimal decisions are made by maximizing the firm's expected discounted flow of nominal profits subject to its demand curve and to the law of motion of $\mathsf{n}_t(z)$. Once the firm's price is set, production must meet demand. For that, the firm only has the intensive margin $h_t(z)$ to adjust, for the specific stock of employment $\mathsf{n}_t(z)$ is already fixed.

For the sake of presentation, I split the simultaneous optimization problem into two parts. In the first one, the firm chooses $\mathsf{v}_t^e(z)$ and $\mathsf{n}_{t+1}(z)$ to maximize its expected present discounted sum of nominal profits:

$$\begin{aligned}\mathcal{J}_t(z) = \max\ & \mathcal{R}_t(z) - P_t w_t(z)\,\mathsf{n}_t(z) - P_t k \mathsf{v}_t^e(z) + E_t Q_{t+1}\mathcal{J}_{t+1}(z_{\mathfrak{c}}) \\ & + P_t\lambda_t^n(z)\,E_t\left[(1-\rho)\,\mathsf{n}_t(z) + \mathsf{q}_t^f \mathsf{v}_t^e(z) - \mathsf{n}_{t+1}(z)\right]\end{aligned}$$

where $\mathcal{R}_t(z) \equiv p_t(z)\,y_t(z)$ is the revenue function. Considering the production and demand functions, it may be written as

$$\mathcal{R}_t(z) = P_t(Y_t)^{\frac{1}{\phi}}(y_t(z))^{1-\frac{1}{\phi}} = P_t(Y_t)^{\frac{1}{\phi}}\left[\mathcal{A}_t(h_t(z)\,\mathsf{n}_t(z))^{\varepsilon}\right]^{1-\frac{1}{\phi}}$$

The first order conditions are:

$$\mathsf{v}_t^e(z): \quad \lambda_t^n \equiv \lambda_t^n(z) = \frac{k}{\mathsf{q}_t^f} \qquad\qquad \mathsf{n}_{t+1}(z): \quad \lambda_t^n(z) = E_t Q_{t+1}^{\pi}\mathfrak{j}_{t+1}(z) \tag{3}$$

where $\mathsf{q}_t^f \equiv \mathsf{q}_{t+1}$ and $\mathfrak{j}_t(z) \equiv \frac{1}{P_t}\frac{\partial \mathcal{J}_t(z)}{\partial \mathsf{n}_t(z)}$ is the real value of the marginal worker to the firm, i.e. the firm's real match surplus, which is computed by means of the Envelope Theorem. The real value of the marginal worker to the firm can be written as follows:

$$\mathfrak{j}_t(z) \equiv \frac{\varepsilon}{\mu}\left(\frac{y_t(z)}{Y_t}\right)^{-\frac{1}{\phi}}\frac{y_t(z)}{\mathsf{n}_t(z)} - w_t(z) + (1-\rho)\,\lambda_t^n(z)$$

where $\mu \equiv \phi/(\phi-1)$ is the steady state price markup, as better characterized by the end of this section.

Real wages $\varpi_t(z)$ and hours per worker $h_t(z)$ are decided by Nash bargaining and maximize $b\log(\mathfrak{u}_t(z)) + (1-b)\log(\mathfrak{j}_t(z))$, where b is the workers' bargaining power, and $\mathfrak{u}_t(z)$ and $\mathfrak{j}_t(z)$ are the worker's and firm's real match surpluses when the marginal worker is matched into firm z. The solution is $\frac{\mathfrak{u}_t(z)}{b} = \mathfrak{s}_t(z) = \frac{\mathfrak{j}_t(z)}{1-b}$, where $\mathfrak{s}_t(z) \equiv \mathfrak{u}_t(z) + \mathfrak{j}_t(z)$ is the total surplus of each match. Plugging $\mathfrak{u}_t(z) = \frac{b}{1-b}\mathfrak{j}_t(z)$ and the firm's first order conditions into the household's first order conditions allows me to pin down $\lambda_t^{cn}(z)$ as a

[11] Even though I allow for price indexation, the model is consistent with small or null price indexation, as supported by empirical evidence (e.g. Cogley and Sbordone (2005, 2008), Klenow and Malin (2010), and Levin et al. (2005)).

function of $\lambda_t^n(z)$:

$$\frac{\lambda_t^{cn}(z)}{\lambda_t P_t} = E_t Q_{t+1}^{\pi} \mathfrak{u}_{t+1}(z) = \frac{b}{1-b} E_t Q_{t+1}^{\pi} \mathfrak{j}_{t+1}(z) = \frac{b}{1-b} \lambda_t^n$$

Therefore, I rewrite $\mathfrak{u}_t(z)$ as follows:

$$\mathfrak{u}_t(z) = w_t(z) - w^u - (1+\nu) \frac{v_t(z)/\mathsf{n}_t(z)}{\lambda_t P_t} + \frac{b}{(1-b)} (1-\rho)(1-\mathsf{p}_{t+1}) \lambda_t^n$$

Since $\mathfrak{j}_t(z) = \frac{(1-b)}{b} \mathfrak{u}_t(z)$, I obtain the firm's salary curve:

$$w_t(z) = b \frac{\varepsilon}{\mu} \frac{p_t(z)}{P_t} \frac{y_t(z)}{\mathsf{n}_t(z)} + (1-b) w^u + (1-b)(1+\nu) \frac{1}{u_t'} \frac{v_t(z)}{\mathsf{n}_t(z)} + b(1-\rho) k \theta_t^f$$

where $\theta_t^f \equiv \theta_{t+1}$ and $v_t(z)/\mathsf{n}_t(z)$ is the disutility per worker in firm z.

From the first order condition for vacancies, I obtain the firm's job creation curve:

$$\frac{k}{\mathsf{q}_t^f} = E_t Q_{t+1}^{\pi} \left[\frac{\varepsilon}{\mu} \frac{p_{t+1}(z)}{P_{t+1}} \frac{y_{t+1}(z)}{\mathsf{n}_{t+1}(z)} - w_{t+1}(z) + (1-\rho) \frac{k}{\mathsf{q}_{t+1}^f} \right]$$

Let $w_t \equiv (1/\mathsf{n}_t) \int_0^1 w_t(z) \mathsf{n}_t(z) dz$ denote the aggregate salary. With this definition, I obtain the aggregate salary and job creation curves:

$$\begin{aligned} w_t &= b \tfrac{\varepsilon}{\mu} \tfrac{Y_t}{\mathsf{n}_t} + (1-b)(1+\nu) \tfrac{1}{u_t'} \tfrac{v_t}{\mathsf{n}_t} + b(1-\rho) k \theta_t^f + (1-b) w^u \\ \tfrac{k}{\mathsf{q}_t^f} &= E_t Q_{t+1}^{\pi} \left[\tfrac{\varepsilon}{\mu} \tfrac{Y_{t+1}}{\mathsf{n}_{t+1}} - w_{t+1} + (1-\rho) \tfrac{k}{\mathsf{q}_{t+1}^f} \right] \end{aligned} \tag{4}$$

where v_t/n_t is the average disutility per worker, and again $Q_t^{\pi} \equiv Q_t \Pi_t$ is the real stochastic discount factor.

As for price setting, I need first to compute the firm's real marginal cost $mc_t(z) \equiv \partial (w_t(z) \mathsf{n}_t(z)) / \partial y_t(z)$. Using the demand function and considering that the firm-specific employment stock $\mathsf{n}_t(z)$ is already set in period t, $mc_t(z)$ satisfies:

$$mc_t(z) = b \frac{\varepsilon}{\mu^2} \left(\frac{p_t(z)}{P_t} \right) + (1-b) \chi (1+\omega) \left(\frac{1}{\epsilon_{\mathrm{u},t}} \right) \left(\frac{1}{\mathcal{A}_t} \right)^{(1+\omega)} (Y_t)^{\omega} \left(C_t^{ad} \right)^{\sigma} \left(\frac{p_t(z)}{P_t} \right)^{-\phi\omega}$$

where $C_t^{ad} \equiv (C_t - \iota_{\mathrm{u}} C_{t-1})$ is the habit-adjusted consumption level, and $\omega \equiv \frac{1+\nu}{\varepsilon} - 1$ captures the curvatures in the production and disutility functions.

When the firm is optimally resetting its price $p_t(z)$, we can work with a simplified version of the discounted flow of nominal profits:

$$\begin{aligned} \mathcal{J}_t(z) = & \max_{\{p_t(z)\}} \sum_{\tau \geq t} Q_{t,\tau} \alpha^{\tau-t} \left[p_t(z) \Pi_{t,\tau}^{ind} Y_\tau \left(\tfrac{p_t(z) \Pi_{t,\tau}^{ind}}{P_\tau} \right)^{-\phi} - P_\tau w_\tau(z) \mathsf{n}_\tau(z) - P_\tau k \mathsf{v}_\tau^e(z) \right] \\ & + tip \end{aligned}$$

where tip stands for terms independent of price $p_t(z)$.

Following e.g. Alves (2014), Ascari and Sbordone (2014, Section 3) and Ascari (2004, online Appendix), the firm's first order condition for the optimal resetting price p_t^* can be conveniently written as $(p_t^*/P_t)^{1+\phi\omega} = N_t/D_t$. The numerator N_t and the denominator D_t functions can be written in recursive forms, avoiding infinite sums:

$$\begin{array}{ll} N_t = \left(1 - \frac{b\varepsilon}{\mu}\right)(X_t)^{\omega}(X_t^c)^{\sigma} + E_t n_{t+1} N_{t+1} & ; n_t = \alpha Q_t \mathcal{G}_t \Pi_t \left(\frac{\Pi_t}{\Pi_t^{ind}}\right)^{\phi(1+\omega)} \\ D_t = \left(1 - \frac{b\varepsilon}{\mu}\right) + E_t d_{t+1} D_{t+1} & ; d_t = \alpha Q_t \mathcal{G}_t \Pi_t \left(\frac{\Pi_t}{\Pi_t^{ind}}\right)^{(\phi-1)} \end{array} \tag{5}$$

where $\mathcal{G}_t \equiv Y_t/Y_{t-1}$ denotes the gross output growth rate, $X_t \equiv Y_t/Y_t^n$ is the gross output gap, $X_t^c \equiv (C_t - \iota_{\mathrm{u}} C_{t-1}) / \left(C_t^n - \iota_{\mathrm{u}} C_{t-1}^n\right)$ is the gross habit-adjusted consumption gap, and C_t^n and Y_t^n are the aggregate consumption and output levels that would prevail under an equilibrium with flexible prices, i.e. when $\alpha = 0$. The Calvo pricing structure also implies the following dynamics:

$$1 = (1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{-(\phi-1)} + \alpha\left(\frac{\Pi_t}{\Pi_t^{ind}}\right)^{(\phi-1)} \tag{6}$$

In the (natural) equilibrium with flexible prices, system (5) to (6) simplifies to the following equation, describing the dynamics of (natural) consumption C_t^n and output Y_t^n:

$$(Y_t^n)^{\omega}\left(C_t^n - \iota_{\mathrm{u}} C_{t-1}^n\right)^{\sigma} = \frac{\left(1 - \frac{b\varepsilon}{\mu}\right)}{(1-b)\,\mu\chi\,(1+\omega)} \epsilon_{\mathrm{u},t} \mathcal{A}_t^{(1+\omega)} \tag{7}$$

In this context, I define the gross unemployment gap $X_t^u \equiv \mathsf{u}_t^e/\mathsf{u}_t^{en}$, the gross employment gap $X_t^n \equiv \mathsf{n}_t/\mathsf{n}_t^n$ and the gross job openings gap $X_t^v \equiv \mathsf{v}_t^e/\mathsf{v}_t^{en}$, where u_t^{en}, n_t^n and v_t^{en} are the end-of-period aggregate unemployment, employment and job openings levels that would prevail under the equilibrium with flexible prices.

The aggregate market clearing condition is $Y_t = C_t + k\mathsf{v}_t^e$, where $Y_t^{\frac{\phi-1}{\phi}} = \int_0^1 y_t(z)^{\frac{\phi-1}{\phi}} dz$. Since the term $k\mathsf{v}_t^e$ represents aggregate intermediate consumption for production, I purge it out and define $\mathcal{Y}_t = C_t$ as the model's GDP measure.

2.4 Aggregates and productivity

Following, I present a set of equations describing the evolution of the aggregate disutility $\upsilon_t \equiv \int_0^1 \upsilon_t(z)\,dz$ to labor and the aggregate hours worked $H_t \equiv \int_0^1 H_t(z)\,dz$. For that, let $\mathcal{P}_t^{-\phi(1+\omega)} \equiv \int_0^1 (p_t(z)/P_t)^{-\phi(1+\omega)} dz$ and $\mathcal{P}_{Ht}^{-\phi(1+\tilde{\omega})} \equiv \int_0^1 (p_t(z)/P_t)^{-\phi(1+\tilde{\omega})} dz$ denote two distinct measures of aggregate relative prices, where $\tilde{\omega} \equiv \frac{1}{\varepsilon} - 1$.

Using the Calvo (1983) price setting structure, I am able to derive the laws of motion of $\mathcal{P}_t$ and $\mathcal{P}_{Ht}$.[12] The result is general and independent of any level of trend inflation. The following system describes the evolution of υ_t, H_t, $\mathcal{P}_t$ and $\mathcal{P}_{Ht}$:

$$\begin{array}{l} \upsilon_t = \frac{\chi}{1+\nu}\left(\frac{Y_t}{\mathcal{A}_t}\right)^{(1+\omega)} \mathcal{P}_t^{-\phi(1+\omega)} \quad ; \quad H_t = \left(\frac{Y_t}{\mathcal{A}_t}\right)^{(1+\tilde{\omega})} \mathcal{P}_{Ht}^{-\phi(1+\tilde{\omega})} \\ \mathcal{P}_t^{-\phi(1+\omega)} = (1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{-\phi(1+\omega)} + \alpha\left(\frac{\Pi_t}{\Pi_t^{ind}}\right)^{\phi(1+\omega)} \mathcal{P}_{t-1}^{-\phi(1+\omega)} \\ \mathcal{P}_{Ht}^{-\phi(1+\tilde{\omega})} = (1-\alpha)\left(\frac{p_t^*}{P_t}\right)^{-\phi(1+\tilde{\omega})} + \alpha\left(\frac{\Pi_t}{\Pi_t^{ind}}\right)^{\phi(1+\tilde{\omega})} \mathcal{P}_{Ht-1}^{-\phi(1+\tilde{\omega})} \end{array} \tag{8}$$

[12] The way I derive the law of motion of $\mathcal{P}_t$ and $\mathcal{P}_{Ht}$ is very similar to how e.g. Alves (2014), Ascari (2004), Schmitt-Grohe and Uribe (2007) and Yun (2005) derive relevant price dispersion variables for aggregate output, employment, resource constraints and aggregate disutility in their models.

Note that $\mathcal{P}_t$ and $\mathcal{P}_{Ht}$ are reciprocals of price dispersion measures. Therefore, υ_t and H_t increase with price dispersion. I highlight that the inflation rate has first order effects on the two measures of price dispersion under positive trend inflation, as better explained in Section 6. This effect, which is completely absent under zero trend inflation, is very important to explain most of the fluctuations that are transmitted into the labor market.

Since the model has extensive and intensive labor margins, the appropriate measure of labor productivity is the GDP per total hours ratio $\wp_t \equiv \mathcal{Y}_t/H_t$. Aggregate hours per worker are computed as $h_t \equiv H_t/\mathsf{n}_t$.

2.5 Monetary policy

The central bank is assigned a inflation target $\bar{\pi} \geq 0$ to pursuit and follows a Taylor-type rule, whose specification is in line with the one estimated by Coibion and Gorodnichenko (2011) for a trend inflation economy from 1983 to 2002:

$$\left(\frac{I_t}{\bar{I}}\right) = \epsilon_{\mathrm{i},t}\left(\frac{I_{t-1}}{\bar{I}}\right)^{\phi_{\mathrm{i}_1}}\left(\frac{I_{t-2}}{\bar{I}}\right)^{\phi_{\mathrm{i}_2}}\left[\left(E_t\frac{\Pi^{av}_{t+1}}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{\mathcal{Y}_t}{\bar{\mathcal{Y}}}\right)^{\phi_y}\left(\frac{\mathcal{Y}_t}{\mathcal{Y}_{t-1}}\right)^{\phi_{gy}}\right]^{1-\phi_{\mathrm{i}_1}-\phi_{\mathrm{i}_2}} \tag{9}$$

where $\epsilon_{\mathrm{i},t}$ is the monetary policy shock, $(\phi_{\mathrm{i}_1}, \phi_{\mathrm{i}_2}) \in (0, 1)$ are policy smoothing parameters, and the response parameters ϕ_π, ϕ_y and ϕ_{gy} are consistent with stability and determinacy in equilibria with rational expectations. As described in Coibion and Gorodnichenko (2011), Π^{av}_{t+1} is the expected inflation average over the next two quarters, i.e. $\Pi^{av}_{t+1} \equiv (\Pi_{t+1} \cdot \Pi_{t+2})^{1/2}$.

The authors find that reacting to the observed output growth has two major advantages: (*i*) it has more stabilizing properties when the trend inflation is not zero; and (*ii*) it is empirically more relevant.[13] Their result reinforce the suggestions by Orphanides and Williams (2007) and Walsh (2003).

3 Data and stylized facts

In this section, I describe the way I obtain labor and goods market original quantities from data sets, and construct relevant empirical measures using the dynamics of the underlying model described in Section 2. When original data is available at monthly frequency, I obtain quarterly stocks (e.g. the number of employed and unemployed workers) by considering observations from the last month of each quarter. As for quarterly flows (e.g. the number of job openings) I obtain them by adding up all monthly observations of each quarter.

I obtain quarterly (seasonally adjusted) gross inflation rates Π_t from the (all urban consumers) CPI,[14] released by the Bureau of Labor Statistics (BLS). As for quarterly (seasonally adjusted) GDP and hours per worker, I obtain them from the BLS Major Sector Productivity and Costs program. Before applying any detrending method, I obtain

[13] The authors estimate the generalized Taylor rule using Greenbook forecasts prepared for each meeting of the Federal Open Market Committee (FOMC) as real-time measures of expected inflation, output growth, and the output gap. This approach is advantageous because it avoids any extra assumption on how the FED's expectations are formed.

[14] US city average, all items. Quarterly inflation rates are obtained the usual way, i.e. $\Pi^{quart}_t = \prod_{j=1}^{3}\Pi^{mon}_{t,j}$.

$\mathcal{Y}_t$ from the (nonfarm business) GDP, and h_t from the (nonfarm business) average weekly hours worked.

I obtain measures of (seasonally adjusted) labor force statistics[15] from the Current Population Survey (CPS). Using quarterly observations, before applying any detrending method, I obtain n_t from the employment level, u_t^e from the unemployment level, and $(\mathsf{u}_t^e - \mathsf{u}_t^s)$ from the unemployment level for 15 weeks & over. From those measures, I construct time series of empirical separation rates[16] (ρ_t), job-finding rates (p_t), hires (m_t) and beginning-of-period unemployment (u_t) using dynamic relations described in Section 2.1:[17]

$$\mathsf{p}_t = 1 - \frac{(\mathsf{u}_t^e - \mathsf{u}_t^s)}{\mathsf{u}_{t-1}^e} \quad ; \ \rho_t = \frac{\mathsf{u}_{t+1}^s}{\mathsf{n}_t(1-\mathsf{p}_{t+1})} \quad ; \ \mathsf{m}_t = \frac{\mathsf{u}_t^e}{\left(\frac{1}{\mathsf{p}_t} - 1\right)} \quad ; \ \mathsf{u}_t = \mathsf{u}_t^e + \mathsf{m}_t$$

I construct quarterly (seasonally adjusted) end-of-period vacancy postings (v_t^e) by merging the (total nonfarm) number of job openings, from the BLS Job Openings and Labor Turnover Survey (JOLTS), to the composite Help-Wanted Index, provided by Barnichon (2010).[18] Since JOLTS job openings are only available from Dec. 2000 on, I retrieve what this series would look like before Dec. 2000 by level-adjusting the Help-Wanted Index in order to conform it to the (JOLTS) units of measurement.[19]

Since CPS and JOLTS statistics have the same units of measurement, I directly construct time series of beginning-of-period vacancy postings (v_t), matching rates (q_t), beginning-of-period tightness ratio (θ_t), and end-of-period tightness ratio (θ_t^e):

$$\mathsf{v}_t = \mathsf{v}_{t-1}^e \quad ; \ \mathsf{q}_t = \frac{\mathsf{m}_t}{\mathsf{v}_t} \quad ; \ \theta_t = \frac{\mathsf{v}_t}{\mathsf{u}_t} \quad ; \ \theta_t^e = \frac{\mathsf{v}_t^e}{\mathsf{u}_t^e}$$

I construct the quarterly (seasonally adjusted) measure of hours per worker and labor productivity by directly computing[20] $H_t = h_t \mathsf{n}_t$ and $\wp_t \equiv \mathcal{Y}_t / H_t$. Since GDP per total hours worked $(\wp_t)$ is a wide-spread statistics to measure labor productivity, it plays an important role in my results.

During the Great Moderation period (1985Q1 to 2005Q4), the following are sample averages I have computed using quarterly observed and retrieved data: $\bar{\pi} = 3.05\%$, $\bar{\rho} = 14.7\%$, $\bar{\mathsf{n}} = 94.3\%$, $\bar{\mathsf{u}}^e = 5.7\%$, $\bar{\mathsf{u}} = 19.7\%$, $\bar{\mathsf{m}} = 13.9\%$, $\bar{\mathsf{v}} = 10.1\%$, $\bar{\theta}^e = 1.852$, $\bar{\theta} = 0.526$, $\bar{\mathsf{p}} = 0.704$ and $\bar{\mathsf{q}} = 1.389$. Reported sample averages $\bar{\mathsf{n}}$, $\bar{\mathsf{u}}^e$, $\bar{\mathsf{u}}$, $\bar{\mathsf{m}}$ and $\bar{\mathsf{v}}$ are the rates computed by normalizing original data by the empirical working age population, i.e. the total number of employed and unemployed workers.

As for detrending (log) empirical quantities from the labor and goods markets, in order

[15] Numbers in thousands, 16 years and over.

[16] By empirical separation rates, I am implicitely loosing the assumption of a constant rate, i.e. I assume that the separation rate is time-varying and is set by the end of each period. In this case, $\rho_{t-1}\mathsf{n}_{t-1}$ employed workers would separate at the begining of each period.

[17] Note that my approach slightly differs from that used by Shimer (2005) when constructing quarterly job-finding (p_t) and separation (ρ_t) rates. The difference comes from the specific timing I use to describe Beveridge equations. Nonetheless, monthly probabilities obtained by both methods are very similar. When comparing quarterly rates I show here with monthly rates shown in Shimer (2005), have in mind that frequency conversion should roughly hold, i.e. $rate_t^{quart} \approx 1 - (1 - rate_t^{mon})^3$.

[18] The composite Help-Wanted Index, currently available from Jan. 1951 to Dec. 2014, combines the information on the old newspaper and current online (since 2005) job advertisements provided by the Conference Board. Prior to 1995, the series is the newspaper Help-Wanted Index.

[19] Pissarides (2009) and Gervais et al. (2011) use similar approaches to rescale the number of vacancy postings into JOLTS units of measurement.

[20] Even though the BLS Major Sector Productivity and Costs program also releases measures of total hours worked and ouput per hour, I preferred gathering original series to compute those statiscs. The differences from my measures and BLS ones are minimal, but not negligible.

to infer their sample standard deviations, I consider three approaches: (i) hp - standard Hodrick Prescott detrending with quarterly smoothing parameter 1600; (ii) sh - Hodrick Prescott detrending with smoothing parameter 10^5, as in Shimer (2005);[21] and (iii) lin - linear detrending.[22]

In what follows, relative volatilities stand for ratios of standard deviations of endogenous variables over the standard deviation of labor productivity $\wp_t$. The first three rows of Table 1 show empirical relative volatilities (1985Q1 to 2005Q4), using different detrending approaches. The last two rows show theoretical relative volatilities[23] implied by the model for the case of flexible prices, using the benchmark calibration described in Section 7, when the only shock hitting the economy is either technology or preference (demand) shocks. Whereas monetary policy shocks have no influence over the equilibrium with flexible prices, technology and demand shocks do not affect inflation in the equilibrium with flexible prices.

In line with what is highlighted in Shimer (2005), Hall (2005) and Costain and Reiter (2008), the empirical relative volatilities for labor market quantities are about one order of magnitude larger than what the model predicts, under flexible prices, when the economy is only hit by technology shocks. This inability to attain empirical volatilities is commonly known as the Shimmer puzzle. Even though this puzzle is somehow attenuated when using the linear detrending approach, the differences in relative volatilities (empirical against theoretical) are still large.

When only hit by preference shocks, empirical relative volatilities for labor market quantities are still larger than what the model predicts under flexible prices. The differences, however, are not as large as in the case where the economy is only hit by technology shocks.

Table 1: Empirical and Theoretical Relative Volatilities

Rel StDev	m_t	u_t	u_t^e	v_t	θ_t	θ_t^e	p_t	q_t	n_t	h_t	$\mathcal{Y}_t$	π_t
Empirical (1985*Q*1 *to* 2005*Q*4)												
hp	12.3	7.3	13.5	15.4	16.9	28.4	6.9	14.5	1.1	0.6	1.8	0.6
sh	7.9	4.7	9.2	11.7	12.1	20.3	4.5	10.1	0.8	0.4	1.6	0.3
lin	5.9	4.1	7.5	8.9	9.8	15.9	3.4	8.1	0.6	0.4	1.3	0.2
Theoretical												
$flex_{\mathfrak{a}}$	0.6	1.1	1.3	1.7	2.6	2.8	1.3	1.3	0.2	0.4	0.9	-
$flex_{\mathfrak{u}}$	3.4	2.5	3.0	6.2	6.7	6.8	3.3	3.3	0.5	3.3	2.1	-

Note: (*Relative volatility*) StDev(EndogVariable)/StDev(LaborProductivity); ($\text{flex}_{\mathfrak{a}}$) theoretical relative volativity, under flexible prices equilibrium and technology shocks; ($\text{flex}_{\mathfrak{u}}$) theoretical relative volatility, under flexible prices equilibrium and preference shocks; (hp) empirical relative volatility, HP-detrending with smoothing parameter 1600; (sh) empirical relative volatility, HP-detrending with smoothing parameter 10^5, as in Shimer (2005); (lin) empirical relative volatility, linear detrending.

[21] Due to Shimer (2005) results, this specific detrending approach has become very common in the labor literature.

[22] Recall the the linear filtering is the limit as the Hodrick Prescott parameter goes to infinity.

[23] Since the model's variables have no trends, I do not use detrending methods for computing theoretical relative volatilities.

4 The role of trend inflation

I start this section by summarizing the distortions caused by inflation. As it rises, the dispersion of relative prices $p_t(z)/P_t$ increases. Due to monopolistic competition, this dispersion translates into dispersion in production $y_t(z)$. By means of the production function, this dispersion is passed through hours worked $H_t(z)$, which also become more dispersed. Because disutility to work $v_t(z)$ is a convex function of $H_t(z)$, the aggregate disutility v_t increases as inflation rises, deteriorating welfare and profits.

An important feature of staggered prices under non-zero trend inflation is that it induces a stationary dispersion of relative prices, whose deterioration effects hold even in the steady state.[24] Figure 1 depicts how the steady state levels of important quantities behave as trend inflation rises, using the Great Moderation calibration described in Section 7.

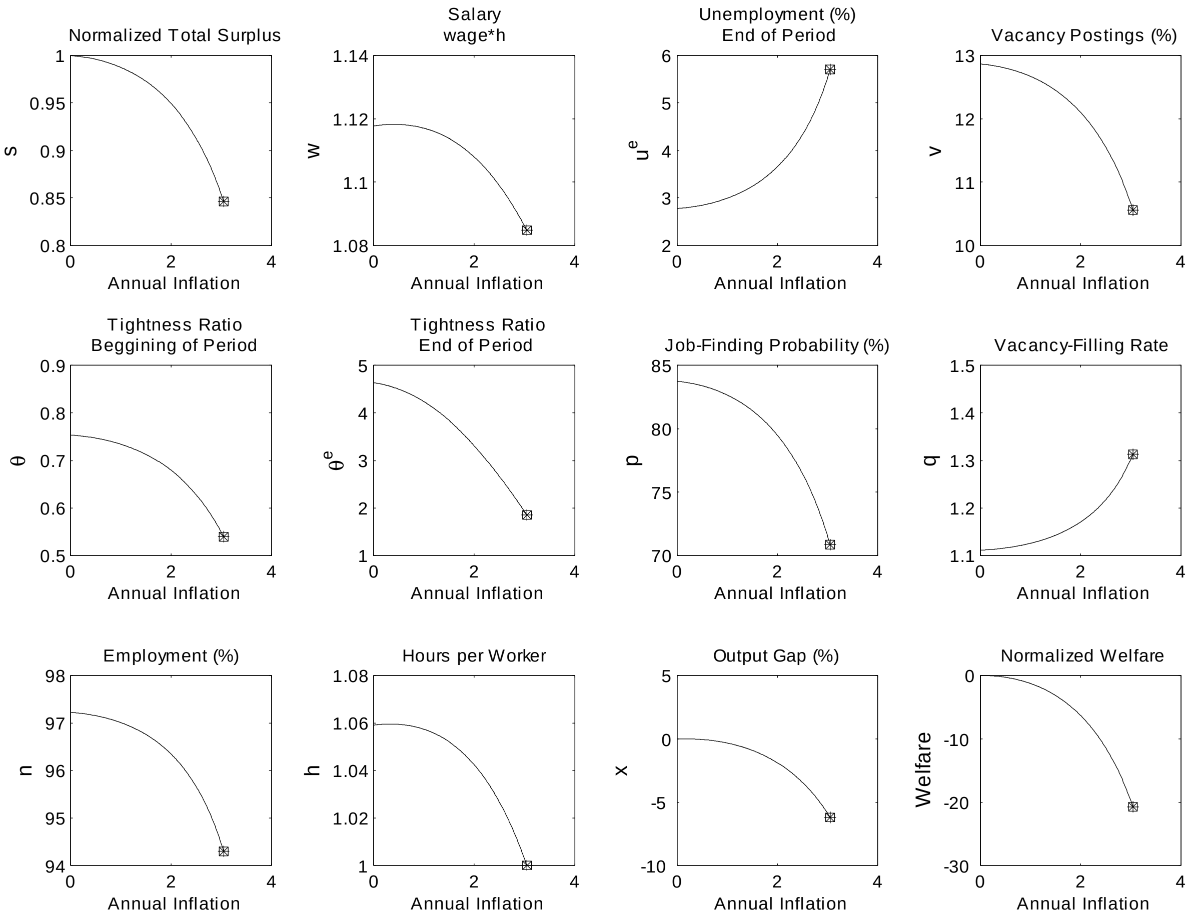

Figure 1: Steady state levels, as trend inflation rises

Note: *Steady states consistent with parametrization described in Section 7.*
Stars: points at the Great Moderation inflation rate.

As previously noticed in the literature, very small levels of trend inflation tends to benefit the economy by correcting some of the distortionary effects created by monopolistic

[24] Even if the shocks remains at their means, positive trend inflation implies that there is always a fraction of firms whose relative prices lag behind their optimal levels. Consequently, firms adjust above the aggregate price trend when resetting their prices. Interestingly, the aggregate variables converge to time invariant steady states – the individual level dispersion cancels out.

competition. When augmenting the model to account for distortions created by search and matching frictions in the labor market, I find that this phenomena changes accordingly. For instance, output gap achieves its maximum at $\bar{\pi} = 0.14\%$, while welfare peaks at $\bar{\pi} = -0.01\%$ and unemployment is minimized at $\bar{\pi} = -0.42\%$.

Under positive levels of trend inflation, labor market variables deteriorate as trend inflation rises. Employment, hours per worker, vacancy postings and salary are reduced. Matches and surpluses fall and unemployment increases. Empirically, my theoretical predictions are backed by Berentsen et al. (2011), who find a strong and positive long-run relationship between inflation and unemployment in quarterly US data from 1955–2005.

In order to develop an analytical assessment of the role of trend inflation on labor market volatilities, I need to characterize the *dynamic* and *static* equilibria in this model economy. The dynamic equilibria with rational expectations, as usual, is defined by the equations describing all first order conditions (household, firms and bargaining), a transversality condition $\lim_{T\to\infty} E_T q_{t,T} B_T = 0$, where $q_{t,T} \equiv \Pi_{\tau=t+1}^{T} q_\tau$, market clearing conditions, and a policy rule describing the way monetary policy is implemented in an economy where all agents have complete and perfect information on the whole structure of the model.

The static equilibrium, on the other hand, is obtained when we consider simplified forms of all dynamic equations, as all lagged and expected future variables are replaced by their current-time peers. For instance, a static simplification of a hypothetical term $\varkappa_t + \ell_1 E_t \varkappa_{t+1} - \ell_2 \varkappa_{t-1}$ is $(1 + \ell_1 - \ell_2)\, \varkappa_t$. The roles of expectations and inertia might be strongly dampened in the static equilibrium, as volatility inducing dynamic terms such as $(\varkappa_t - E_t \varkappa_{t+1})$ and $(\varkappa_t - \varkappa_{t-1})$ disappear in static specifications. Notwithstanding, the static equilibrium is very useful for providing us with intuitions.

Even though it is unfeasible to derive closed-form solutions under the dynamic equilibrium, I am able to describe how inflation affects volatilities in the static general equilibrium, under particular parametrizations. Of course, this analytical tractability comes at the cost of loosing important sources of volatilities arising from the system dynamics. Therefore, the results I present below are only meant for illustrative purposes.

Consider that monetary policy is described by the static version of the Taylor rule (9) for the case of $\phi_y = 0$, i.e. $I_t/\bar{I} = (\epsilon_{\mathrm{i},t})^{1/(1-\phi_{\mathrm{i}})} \left(\Pi_t/\bar{\Pi}\right)^{\phi_\pi}$, where $\phi_{\mathrm{i}} \equiv \phi_{\mathrm{i}_1} + \phi_{\mathrm{i}_2}$, $\bar{I} = \bar{\Pi}/\beta$ is the steady state level of the (gross) nominal interest rate and $\bar{\Pi}$ is the constant inflation target (trend inflation). Consider first the case in which there is no monetary policy shock, i.e. $\epsilon_{\mathrm{i},t} = 1\ \forall t$. Using the static simplification $1 = \beta\left(I_t/\Pi_t\right)$ of the consumption Euler equation (2), we conclude that adopting this rule in the static equilibrium is sufficient for keeping the (gross) inflation rate constant at target $\bar{\Pi}$ as long as ϕ_π is different from unity. If monetary policy shocks have a role in the economy, then inflation responds to policy shocks $\epsilon_{\mathrm{i},t}$ in the static equilibrium, i.e. $\Pi_t = \bar{\Pi}\left(\epsilon_{\mathrm{i},t}\right)^{-1/[(1-\phi_{\mathrm{i}})(\phi_\pi - 1)]}$.

The static equivalent of the Beveridge equations (1) implies that n_t and all remaining labor market variables are monotonic functions of v_t. Therefore, I choose v_t as the representative labor market variable when showing analytical results in this section.

Under very weak assumptions, it is not hard to obtain quasi-closed-form solutions[25] to the non-linear static equilibrium as functions of the preference $\epsilon_{\mathrm{u},t}$, technology $\mathcal{A}_t$ and policy $\epsilon_{\mathrm{i},t}$ shocks. Consider first the particular case in which policy shocks are absent, i.e. $\epsilon_{\mathrm{i},t} = 1\ \forall t$. The solutions for GDP $\mathcal{Y}_t$, total hours H_t, labor productivity $\wp_t$ and vacancy

[25] Closed form solutions are impossible to derive. However, I obtain quasi-closed-form solution with the help of only two steady state levels: the replacement ratio and the aggregate total surplus.

postings v_t are:

$$\mathcal{Y}_t = (\mathfrak{S}_1)^{\frac{1}{(\sigma+\omega)}} \frac{(1-\mathfrak{M}_2)^{\frac{\omega}{(\sigma+\omega)}} (\mathfrak{C}_3)^{\frac{1}{(\sigma+\omega)}\frac{(1+\phi\omega)}{(\phi-1)}}}{(\mathfrak{M}_1)^{\frac{1}{(\sigma+\omega)}} (\mathfrak{C}_2)^{\frac{1}{(\sigma+\omega)}}} (\epsilon_{\mathfrak{u},t})^{\frac{1}{(\sigma+\omega)}} (\mathcal{A}_t)^{\frac{(1+\omega)}{(\sigma+\omega)}}$$

$$\mathsf{v}_t = \frac{1}{k}\frac{\mathfrak{M}_2}{(1-\mathfrak{M}_2)}\mathcal{Y}_t \quad ; \; H_t = \frac{\mathfrak{C}_0(\mathfrak{C}_3)^{\frac{-(1+\phi\tilde{\omega})}{(\phi-1)}}}{(1-\mathfrak{M}_2)^{(1+\tilde{\omega})}} \left(\frac{\mathcal{Y}_t}{\mathcal{A}_t}\right)^{(1+\tilde{\omega})} \quad ; \; \wp_t = \frac{\mathcal{Y}_t}{H_t}$$

The composite parameters are defined as follows:

$$\mu \equiv \frac{\theta}{\theta-1} \quad ; \; \varrho \equiv \frac{1-\beta}{\beta} + \rho \quad ; \; \omega \equiv \frac{1+\nu}{\varepsilon} - 1 \quad ; \; \tilde{\omega} \equiv \frac{1}{\varepsilon} - 1 \quad ; \; b^u \equiv 1 - (1-b)\,\varrho_{wu}$$

$$\bar{\alpha} \equiv \alpha \left(\bar{\Pi}\right)^{(\phi-1)(1-\gamma_\pi)} \quad ; \; \bar{\vartheta} \equiv \left(\bar{\Pi}\right)^{(1+\phi\omega)(1-\gamma_\pi)} \quad ; \; \tilde{\vartheta} \equiv \left(\bar{\Pi}\right)^{(1+\phi\tilde{\omega})(1-\gamma_\pi)} \quad ; \; \mathfrak{C}_0 \equiv \frac{1-\bar{\alpha}}{1-\bar{\alpha}\tilde{\vartheta}}$$

$$\mathfrak{C}_1 \equiv \frac{1-\bar{\alpha}}{1-\bar{\alpha}\bar{\vartheta}} \quad ; \; \mathfrak{C}_2 \equiv \frac{1-\bar{\alpha}\beta}{1-\bar{\alpha}\bar{\vartheta}\beta} \quad ; \; \mathfrak{C}_3 \equiv \frac{1-\alpha}{1-\bar{\alpha}} \quad ; \quad \mathfrak{C}_4 \equiv \frac{\mathfrak{C}_1}{\mathfrak{C}_2}$$

$$\mathfrak{S}_1 \equiv \frac{\varepsilon}{\chi\mu(1-\iota_{\mathfrak{u}})^{\sigma}} \quad ; \; \mathfrak{M}_1 \equiv \frac{(1+\nu)(1-b)}{\left(1-\frac{b\varepsilon}{\mu}\right)} \quad ; \; \mathfrak{M}_2 \equiv \frac{\frac{(1-b)}{b^u}\frac{\varepsilon}{\mu}\left((1-\varrho_{wu})-\frac{\mathfrak{C}_4}{\mathfrak{M}_1}\right)}{\left(\frac{\varrho}{\rho}+(1-\rho)\frac{b}{b^u}\frac{\eta}{\rho}\left(\frac{\beta(1-b)\eta}{k}\right)^{\frac{1-a}{a}}(\bar{\mathfrak{s}})^{\frac{1-a}{a}}\right)}$$

in which ϱ_{wu} is the steady state replacement ratio, i.e. unemployment compensation over aggregate salary, and $\bar{\mathfrak{s}}$ is the steady state level of the aggregate total surplus of matches.

Now I am able to pin down the relative volatility of v_t, with respect to that of the labor productivity $\wp_t$:

$$\frac{StDv\left(\mathsf{v}_t\right)}{StDv\left(\wp_t\right)} = \frac{1}{k}\frac{(\mathfrak{S}_1)^{\frac{(1+\tilde{\omega})}{(\sigma+\omega)}}(\mathfrak{M}_2)(\mathfrak{C}_0)(\mathfrak{C}_3)^{\frac{1}{(\phi-1)}\left[\frac{(1+\phi\omega)-\tilde{\omega}(\phi\sigma-1)}{(\sigma+\omega)}-1\right]}}{(\mathfrak{M}_1)^{\frac{(1+\tilde{\omega})}{(\sigma+\omega)}}(1-\mathfrak{M}_2)^{\left[1+\frac{\sigma(1+\tilde{\omega})}{(\sigma+\omega)}\right]}(\mathfrak{C}_2)^{\frac{(1+\tilde{\omega})}{(\sigma+\omega)}}}\frac{S^{\mathsf{v}}_{\mathfrak{u},\mathfrak{a}}}{S^{\wp}_{\mathfrak{u},\mathfrak{a}}} \tag{10}$$

where $S^{\mathsf{v}}_{\mathfrak{u},\mathfrak{a}} = \sqrt{Var\left((\epsilon_{\mathfrak{u},t})^{\frac{1}{(\sigma+\omega)}}(\mathcal{A}_t)^{\frac{(1+\omega)}{(\sigma+\omega)}}\right)}$ and $S^{\wp}_{\mathfrak{u},\mathfrak{a}} = \sqrt{Var\left((\epsilon_{\mathfrak{u},t})^{\frac{-\tilde{\omega}}{(\sigma+\omega)}}(\mathcal{A}_t)^{\left[1-\frac{(1-\sigma)\tilde{\omega}}{(\sigma+\omega)}\right]}\right)}$.

I define $\mathfrak{S}_1$ as the basic *structural multiplier*, for it is a composite of structural parameters related to preferences and production, captured by χ, ω, σ and $\iota_{\mathfrak{u}}$. For instance, it tells us that relative volatilities tend to increase as risk aversion σ and habit persistence $\iota_{\mathfrak{u}}$ rise, or when the curvature parameter ω falls. However, as I show below, there are stronger multipliers arising when trend inflation has a role and workers have a sizeable bargaining power. And the basic structural parameters interact with them. Indeed, the decreasing role of ω is very small when compared to its increasing role in pricing multipliers, defined below.

Positive trend inflation is a fluctuation amplifier, which is particularly important in the labor market. In this context, I define *pricing multipliers* (i.e. $\mathfrak{C}_0$, $\mathfrak{C}_1$, $\mathfrak{C}_2$, $\mathfrak{C}_3$ and $\mathfrak{C}_4$) as the ones which do not depend on labor market parameters and are direct functions of trend inflation, by means of the composite parameters $\bar{\alpha} \geq \alpha$, $\bar{\vartheta} \geq 1$ and $\tilde{\vartheta} \geq 1$. In common, all pricing multipliers are greater than unity and monotonically increase with trend inflation.

Composite parameters $\bar{\alpha}$, $\bar{\vartheta}$ and $\tilde{\vartheta}$ are key in describing the dynamics under trend inflation: $\bar{\vartheta}$ and $\tilde{\vartheta}$ are positive transformations of trend inflation $\bar{\pi}$, and $\bar{\alpha}$ is the effective degree of price stickiness.[26] All increase as trend inflation rises.

Note that $\mathfrak{C}_2$ is the only pricing multiplier that individually pushes down the relative volatility of v_t as trend inflation rises. However, since empirical evidence suggests that

[26]The composite parameters $\bar{\alpha}$ and $\bar{\vartheta}$ are bounded by $\max\left(\bar{\alpha}, \bar{\alpha}\bar{\vartheta}\right) < 1$ to guarantee the existence of an equilibrium with trend inflation.

the reciprocal σ of the intertemporal elasticity of substitution is typically close to unity, and micro-evidence on labor supply suggests that the reciprocal ν of the Frisch elasticity is typically a large parameter ($\nu \approx 2$), it happens that that $\omega >> \tilde{\omega}$ and so the fall in $(\mathfrak{C}_2)^{-(1+\tilde{\omega})/(\sigma+\omega)}$ tends to be much more than compensated by the rise in the remaining pricing multipliers. Since there is a monotonic function linking the remaining labor market to vacancy openings, relative volatilities in the labor market increase as trend inflation rises in the static equilibrium.

I define $\mathfrak{M}_1$ and $\mathfrak{M}_2$ as *labor multipliers*, for it varies with parameters related to the labor market dynamics and salary bargaining. Note that $\mathfrak{M}_2$ is an hybrid labor multiplier, for it interacts with pricing multiplier $\mathfrak{C}_4$. As workers' bargaining power b rises, $\mathfrak{M}_1$ falls and $\mathfrak{M}_2$ increases. Multiplier $\mathfrak{M}_2$ also increases as the separation rate ρ rises. This volatility rise reflects the increase in employment turnover. Pinning down the net effect of a change in the elasticity a of unemployment in the matching function needs more elaboration. If a falls, unemployment $\bar{\mathfrak{u}}$ tends to rise, and so $\mathfrak{M}_2$ increases. As for the net effect of an increase of the replacement ratio ϱ_{wu}, its effect is not clear only by studying result (10). Note that using ϱ_{wu} for solving the system is equivalent to a model reparametrization, i.e. ancillary parameter k must conform to ϱ_{wu}, as better detailed in Section 7 and Appendix B. Nevertheless, performing comparative statics simulations in Section 8.2 allows me to conclude that labor market relative volatility increases as ϱ_{wu} rises.

Note that multiplier $\mathfrak{M}_2$ increases, and so do relative volatilities, as total surplus $\bar{\mathfrak{s}}$ falls. This result parallels the findings first identified by Hagedorn and Manovskii (2008) and Costain and Reiter (2008). They show that employment becomes more volatile as the total surplus decreases. Here, we must also take into account that all factors changing pricing multipliers also change steady state total surplus $\bar{\mathfrak{s}}$. For instance, Figure 1 confirms that total surplus decreases as trend inflation rises. In my comparative statics analyses in Section 8.2, I confirm that total surpluses have a negative correlation with relative volatilities obtained from simulations with the dynamic model.

Finally, note that the relative volatility depends on the shocks' standard deviations ratio $S^{\mathfrak{v}}_{\mathfrak{u},\mathfrak{a}}/S^{\wp}_{\mathfrak{u},\mathfrak{a}}$. It means that the relative volatilities prevailing under either pure technology or preference shocks increase in parallel paths as trend inflation rises. Indeed, the ratio only changes with parameters σ, ν and ε, and is invariant with respect to the level of trend inflation. I find that, under reasonable parametrization described in Section 7, relative volatilities generated by pure preference shocks are larger than those generated by pure technology shocks. See Section 8 for more results.

Even though the results I obtained for the static equilibrium do not depend on monetary policy, it is not a general rule. For instance, when monetary policy shocks $\epsilon_{\mathfrak{i},t}$ are in, the solution depends on a more entangled way on policy response parameters. In Section 5, I elaborate more on the role of monetary policy.

5 The role of monetary policy

In an economy with nominal rigidities, monetary policy has an important role in the way equilibrium volatilities are achieved. In the static equilibrium I considered in the last section, static volatilities are only independent of monetary policy parameters because I have assumed absence of monetary policy shocks $\epsilon_{\mathfrak{i},t}$. This assumption were only meant for illustration purposes, for I only focused on the role of trend inflation in amplifying volatilities.

Should monetary policy shocks were allowed to exist, results from the last section would be modified as follows. Let $\bar{\alpha}_t \equiv \bar{\alpha}\left(\epsilon_{\mathrm{i},t}\right)^{-\mathfrak{P}_1}$ denote a policy-adjusted effective degree of price stickiness, where $\mathfrak{P}_1 \equiv (\phi-1)(1-\gamma_\pi) / [(1-\phi_{\mathrm{i}})(\phi_\pi-1)]$ is a *policy multiplier.* Substitute $\bar{\alpha}_t$ for $\bar{\alpha}$ in all pricing and labor multipliers $\mathfrak{C}_{0,t}$, $\mathfrak{C}_{1,t}$, $\mathfrak{C}_{2,t}$, $\mathfrak{C}_{3,t}$, $\mathfrak{C}_{4,t}$ and $\mathfrak{M}_{2,t}$, which now are time-varying as they respond to monetary policy shocks. Note that simply assuming that $\epsilon_{\mathrm{i},t}$ varies is enough to allow policy parameters ϕ_{i} and ϕ_π to affect all volatility measures, even in the static equilibrium.

Pricing and labor multipliers have now a stronger influence on the way relative volatilities increase as trend inflation rises, as they also amplify policy shock fluctuations. Under reasonable parametrization described in Section 7, I find that relative volatilities generated by pure monetary policy shocks are much larger than those generated by pure preference or technology shocks.

In the dynamic equilibrium, timing is also just as important. If monetary policy respond to e.g. lagged inflation rates or expected output growth, volatilities will be affected. What about leaving the family of Taylor-type rules? In general, there are infinite ways monetary policy can be implemented. And so formally assessing the general case is impossible.

Therefore, how can we obtain a general result for the role of monetary policy as an extra component in determining labor market volatilities? I answer this question by showing, for economies with non-zero long-run inflation rates, that central banks face a serious trade-off in stabilizing volatilities. For that, I consider the simplest scenario in which monetary policy is set to keep the inflation rate constant at a pre-determined target. As I show below, under general conditions, monetary policy is always unable stabilize labor market gaps when inflation is chosen to be stabilized at non-zero rates.

I start by showing that the dynamics of all labor market variables are completely determined when the dynamics of job openings v_t^e are set.

Lemma 1 *Provided that the initial state $(\mathsf{n}_0, \mathsf{v}_0^e)$ at $t = 0$ is known, once the dynamics of end-of period job openings v_t^e is set, the path of all remaining labor market quantities are completely determined.*

The proof is shown in Appendix E.

Now, I generalize Alves (2014) main result on the lack of divine coincidence[27]. Under general circumstances, monetary policy faces a trade-off in simultaneously stabilizing the (gross) inflation rate $\Pi_t \equiv 1+\pi_t$, (gross) output gap $X_t \equiv 1+x_t$, (gross) consumption gap $X_t^c \equiv 1 + x_t^c$, and all remaining labor market gaps, summarized by $X_t^v \equiv 1 + x_t^v$. Indeed, stabilizing all of those variables requires that monetary authority stabilizes the inflation rate at exactly zero percent or firms follow an exact full indexation mechanism ($\gamma_\pi = 1$) when not optimally resetting their prices. Otherwise, monetary policy is able to stabilize at most only one of them.

The following proposition states the policy trade-off results for the nonlinear model. Therefore, they are robust to any log-linearized approximation and do not depend on assuming that distortions are sufficiently close to zero.

Proposition 2 *With staggered price setting ($\alpha > 0$) and partial indexation ($\gamma_\pi \neq 1$), there exists a trade-off in stabilizing the inflation rate π_t, output gap x_t, consumption gap x_t^c, and labor market gap x_t^v whenever the monetary policy chooses a non-zero rate*

[27] See Blanchard and Gali (2007) for details on the divine coincidence property.

for stabilizing π_t: it is impossible for π_t and anyone of the three gaps (i.e. x_t, x_t^c or x_t^v) to simultaneously have zero variance if π_t is stabilized at $\bar{\pi} \neq 0$. If $\bar{\pi} = 0$ or $\gamma_\pi = 1$, there is no stabilization trade-off and the divine coincidence holds.

The proof is shown in Appendix E.

As highlighted in Alves (2014), empirical evidence strongly supports the conditions for a policy trade-off: inflation has been systematically positive since the World War II until the 2008 Great Recession,[28] and empirical evidence from macro and micro data suggests that there is very small indexation on individual prices.[29]

6 Dynamic Volatility Amplification

In this section, I assess the way the fluctuations are transmitted into the variables in the dynamic equilibrium, by means of log-linearized equations. For any variable $\mathcal{X}_t$, the hatted representation $\widehat{\mathcal{X}}_t$ is its log-deviation from its steady state level $\bar{\mathcal{X}}$. See appendices A and D for all steady state levels, a complete description of the composite parameters, and the list of the log-linearized equations.

Total fluctuation created in general equilibrium depends on the variances and covariances of all variables, and on the way shocks' fluctuations are amplified throughout the dynamic system. For instance, the law of motion of employment is highly inertial and this fact alone amplifies the volatility of shocks. Since the general equilibrium solution with rational expectations cannot be derived algebraically, I focus my analysis on specific log-linearized equations containing relevant fluctuation multipliers.

Fluctuations are brought into $\widehat{\mathsf{q}}_t^f$, which is then transmitted into the remaining labor variables by means of Beveridge equations. For instance, log-linearized Beveridge equations allows me to pin down the law of motion of posted vacancies as a function of $\widehat{\mathsf{q}}_t^f$:

$$\widehat{\mathsf{v}}_t^e = \mathfrak{m}_v \widehat{\mathsf{v}}_{t-1}^e - \frac{1}{a}\widehat{\mathsf{q}}_t^f + \frac{1}{a}(1-\rho)(1-a\bar{\mathsf{p}})\widehat{\mathsf{q}}_{t-1}^f \tag{11}$$

where $\mathfrak{m}_v \equiv (1-\rho)(1-\bar{\mathsf{p}})$. In order to get the intuition why this law of motion amplifies fluctuations brought by $\widehat{\mathsf{q}}_t^f$ into $\widehat{\mathsf{v}}_t^e$, consider the long-run (static) relation $\widehat{\mathsf{v}}_t^e = \bar{\mathfrak{m}}_v \widehat{\mathsf{q}}_t^f$, where $\bar{\mathfrak{m}}_v \equiv 1 + (1-a)\rho / [a(\bar{\mathsf{p}}(1-\rho)+\rho)]$. Since $\bar{\mathfrak{m}}_v > 1$, we expect the standard deviation of vacancy postings to be larger than that of the job filling rate. In this context, we may think of $\mathfrak{m}_v$ and $\bar{\mathfrak{m}}_v$ as short-run and long-run labor multipliers. So, we need to understand how fluctuations brought by technology $\widehat{\mathcal{A}}_t$, preference $\hat{\epsilon}_{\mathrm{u},t}$ and monetary policy $\hat{\epsilon}_{\mathrm{i},t}$ shocks are passed and possibly amplified into $\widehat{\mathsf{q}}_t^f$.

Using the aggregate salary and job creation curves, described in system (4), I obtain:

$$\begin{aligned} \hat{\mathsf{q}}_t^f &= (1-\rho)\left(1-\tfrac{b}{a}\bar{\mathsf{p}}\right)\beta E_t \hat{\mathsf{q}}_{t+1}^f + E_t(\hat{\imath}_t - \hat{\pi}_{t+1}) \\ &+ \tfrac{1}{\beta\bar{\mathsf{s}}}\tfrac{(1+\nu)\bar{\upsilon}}{\bar{u}'\bar{\mathsf{n}}}\beta E_t\left(\hat{\upsilon}_{t+1} - \hat{u}'_{t+1} - \hat{\mathsf{n}}_{t+1}\right) - \tfrac{1}{\beta\bar{\mathsf{s}}}\tfrac{\varepsilon\bar{Y}}{\mu\bar{\mathsf{n}}}\beta E_t\left(\hat{Y}_{t+1} - \hat{\mathsf{n}}_{t+1}\right) \end{aligned} \tag{12}$$

where $\hat{u}'_t = \hat{\epsilon}_{\mathrm{u},t} - \sigma\hat{C}_t^{ad}$, $\hat{C}_t^{ad} \equiv \hat{C}_t - \iota_{\mathrm{u}}\hat{C}_{t-1}$.

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[28] Indeed, average annual inflation ranged between 4% to 10% in European countries, about 3-5% in the US. As for Japan, even though the CPI annual inflation rate averaged 3.14% from 1971 to 2008, the average from 1999 to 2005 was -0.46%. Source: Japan's Statistics Bureau.

[29] See e.g. Bils and Klenow (2004), Cogley and Sbordone (2005, 2008), Klenow and Kryvtsov (2008), Klenow and Malin (2010) and Levin et al. (2006).

Since $\bar{\mathsf{p}} = \eta \left(\beta \left(1-b\right) \frac{\eta}{k} \bar{\mathfrak{s}}\right)^{\frac{(1-a)}{a}}$, dynamic multipliers $\left(1 - \frac{b}{a}\bar{\mathsf{p}}\right)$ and $1/\left(\beta \bar{\mathfrak{s}}\right)$ increase whenever the aggregate total surplus $\bar{\mathfrak{s}}$ falls. The former is a dynamic multiplier that enhances the ergodic standard deviation of $\hat{\mathsf{q}}_t^f$ once expectations channels are taken into account. The latter is a dynamic multiplier that amplifies how fluctuations brought from output per worker $\hat{Y}_t - \hat{\mathsf{n}}_t$, aggregate disutility $\hat{v}_t$, aggregate consumption $\hat{C}_t$ and preference shocks $\hat{\epsilon}_{\mathrm{u},t}$ are brought into the labor market. Among all of those variables, I find that it is the aggregate disutility which has the major role in amplifying fluctuations created by shocks. I dedicate the last part of this section elaborating more on this role.

Monetary policy, captured by the real interest rate $\hat{r}_t = \hat{\imath}_t - E_t \hat{\pi}$, also has a role in amplifying fluctuations into labor market variables. There are two channels by which the volatility of $\hat{r}_t$ increases: (i) an indirect one caused by monetary policy, when responding to inflation $\hat{\pi}_t$ and output growth $\left(\widehat{\mathcal{Y}}_t - \widehat{\mathcal{Y}}_{t-1}\right)$ in the Taylor rule (9), and subject to monetary policy shocks $\hat{\epsilon}_{\mathrm{i},t}$; and (ii) a direct one caused by the effect of the expected inflation rate. Of course, changes in the nominal interest rate also directly influence intertemporal consumption $\hat{C}_t$ allocation by means of the Euler equation (2).

As for channel (i), the Taylor rule defines the monetary authority's preferences in the trade-off between dampening inflation fluctuations vis-a-vis fluctuations in output, consumption and labor market quantities. The trade-off will always occur, according to the premises of Proposition 2.

Regarding channel (ii), the following system describes the labor-augmented Generalized New Keynesian Phillips Curve (LaGNKPC), under trend inflation:[30]

$$\begin{aligned}
\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) = {} & \beta E_t \left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \bar{\kappa} \left(\omega \hat{x}_t + \sigma \hat{x}_t^c\right) + \left(\bar{\vartheta} - 1\right) \bar{\kappa}_{\varpi} \beta E_t \hat{\varpi}_{t+1} + \hat{\varsigma}_t \\
\hat{\varpi}_t = {} & \bar{\alpha}\bar{\vartheta}\beta E_t \hat{\varpi}_{t+1} + \phi \left(1+\omega\right) \left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) + \left(1 - \bar{\alpha}\bar{\vartheta}\beta\right) \left(\omega \hat{x}_t + \sigma \hat{x}_t^c\right) \\
+ {} & \left(\hat{x}_t - \hat{x}_{t-1}\right) - \sigma \left(\hat{x}_t^c - \hat{x}_{t-1}^c\right) \\
\hat{\varsigma}_t = {} & \bar{\alpha}\bar{\vartheta}\beta E_t \hat{\varsigma}_{t+1} + \left(\bar{\vartheta} - 1\right) \beta E_t \hat{\xi}_{t+1} \\
\hat{\xi}_t = {} & \bar{\kappa}_{\varpi} \left(1+\omega\right) \left[\left(\hat{Y}_t^n - \hat{Y}_{t-1}^n\right) - \left(\widehat{\mathcal{A}}_t - \widehat{\mathcal{A}}_{t-1}\right)\right]
\end{aligned} \tag{13}$$

where $\hat{\pi}_t^{ind} = \gamma_\pi \hat{\pi}_{t-1}$ is the indexation term, $\hat{\varpi}_t$ is an ancillary variable with no obvious interpretation,[31] $\hat{\xi}_t$ is an aggregate shock term that collects fluctuations created by the technology $\widehat{\mathcal{A}}_t$ and preference $\hat{\epsilon}_{\mathrm{u},t}$ shocks, and $\hat{\varsigma}_t$ is the endogenous *trend inflation cost-push shock*, which ultimately depends only on technology and preference shocks. Since $\bar{\alpha}$ and $\bar{\vartheta}$ increase as trend inflation rises, the trend inflation cost-push shock $\hat{\varsigma}_t$ amplifies, by means of multipliers $\left(\bar{\vartheta} - 1\right)$ and $\bar{\alpha}\bar{\vartheta}\beta$ (on $E_t \hat{\varsigma}_{t+1}$), the effect of the aggregate shock $\hat{\xi}_t$ and transmits it through the inflation dynamics. In line with Alves (2014) findings, cost-push shock $\hat{\varsigma}_t$ is innocuous when trend inflation is zero ($\bar{\pi} = 0$). The remaining composite parameters are $\bar{\kappa} \equiv \frac{\left(1-\bar{\alpha}\bar{\vartheta}\beta\right)\left(1-\bar{\alpha}\right)}{\bar{\alpha}\left(1+\phi\omega\right)}$ and $\bar{\kappa}_{\varpi} \equiv \frac{\left(1-\bar{\alpha}\right)}{\left(1+\phi\omega\right)}$.

In line with well documented results in the trend inflation literature, the LaGNKPC becomes flatter ($\bar{\kappa}$ decreases) and more forward looking ($\left(\bar{\vartheta} - 1\right) \bar{\kappa}_{\varpi}\beta$ and $\bar{\alpha}\bar{\vartheta}\beta$ increases)

[30] Ascari and Sbordone (2014) coined the term Generalized New Keynesian Phillips Curve (GNKPC), to describe Phillips curves under trend inflation.

[31] In the literature on trend inflation, there are two usual ways to describe trend inflation Phillips curves: (i) with ancillary variables (e.g. Ascari and Ropele (2007) and Alves (2014)); and (ii) with infite sums (e.g. Cogley and Sbordone (2008) and Coibion and Gorodnichenko (2011)).

with trend inflation. The effect of $\hat{\varpi}_t$ on the inflation dynamics is to make it even more forward looking. This is due to the fact that the coefficients $\left(\bar{\vartheta}-1\right)\bar{\kappa}_{\varpi}$ on $E_t\hat{\varpi}_{t+1}$, in the first equation, and $\bar{\alpha}\bar{\vartheta}\beta$ on $E_t\hat{\varpi}_{t+1}$, in the second equation, increase as trend inflation rises.

Now I asses the role of aggregate disutility. For comparison, I simultaneously study the dynamics of labor productivity $\hat{\wp}_t = \hat{Y}_t - \hat{H}_t$. Note that aggregate total hours is an important component. Therefore, consider the following log-linearized equations describing the dynamics of $\hat{\upsilon}_t$ and $\hat{H}_t$:

$$\hat{\upsilon}_t = (1+\omega)\left(\hat{Y}_t - \widehat{\mathcal{A}}_t - \phi\hat{\mathcal{P}}_t\right) \quad ; \; \hat{H}_t = (1+\tilde{\omega})\left(\hat{Y}_t - \widehat{\mathcal{A}}_t - \phi\hat{\mathcal{P}}_{Ht}\right)$$

Those two equations share remarkable similarity. I find that the larger part of action comes into play by means of aggregate relative prices $\hat{\mathcal{P}}_t$ and $\hat{\mathcal{P}}_{Ht}$. That been said, note that the multipliers passing fluctuations from $\hat{\mathcal{P}}_t$ and $\hat{\mathcal{P}}_{Ht}$ into $\hat{\upsilon}_t$ and $\hat{H}_t$ are different. Indeed, we always have that $((1+\nu)/\varepsilon - 1) = \omega > \tilde{\omega} = (1/\varepsilon - 1)$. We also find strong, but distinct, short-run pricing multipliers in the log-linearized laws of motions of $\hat{\mathcal{P}}_t$ and $\hat{\mathcal{P}}_{Ht}$:

$$\hat{\mathcal{P}}_t = \bar{\alpha}\bar{\vartheta}\hat{\mathcal{P}}_{t-1} - \tfrac{\left(\bar{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) \quad ; \; \hat{\mathcal{P}}_{Ht} = \bar{\alpha}\tilde{\vartheta}\hat{\mathcal{P}}_{Ht-1} - \tfrac{\left(\tilde{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) \tag{14}$$

Note first that inertia parameters ($\bar{\alpha}\bar{\vartheta}$ and $\bar{\alpha}\tilde{\vartheta}$) and short-run pricing multipliers ($\frac{\left(\bar{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})}$ and $\frac{\left(\tilde{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})}$) all grow as trend inflation rises. Those short-run pricing multipliers amplify inflation fluctuations into $\hat{\mathcal{P}}_t$ and $\hat{\mathcal{P}}_{Ht}$, while the inertia parameters amplify short-run volatilities when assessing long-run (ergodic) standard deviations. Secondly, since $\bar{\vartheta}$ and $\tilde{\vartheta}$ are only zero when trend inflation is exactly zero, price dispersion has first effects even when trend inflation slightly deviates from zero.

It is easy to verify that the aggregate disutility absorbs greater volatility amplification than total hours worked as trend inflation rises. Since $\omega > \tilde{\omega}$, it is always the case that $\bar{\vartheta} > \tilde{\vartheta}$, $\bar{\alpha}\bar{\vartheta} > \bar{\alpha}\tilde{\vartheta}$ and $\frac{\left(\bar{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})} > \frac{\left(\tilde{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})}$.

This result implies, in turn, that labor market volatilities grow faster than productivity volatility as trend inflation rises. The amplified fluctuations of $\hat{\mathcal{P}}_t$ are transferred to aggregate disutility $\hat{\upsilon}_t$, which in turn are weakly amplified by labor multipliers and transmitted to the labor market by means of the the aggregate wage and job creation curves, as summarized by equation (12). The weaker volatility of $\hat{\mathcal{P}}_{Ht}$, on the other hand, is transmitted to the labor productivity variable $\hat{\wp}_t$ (output per hours) by means of the aggregate hours worked $\hat{H}_t$.

7 Calibration

The calibration, summarized in Appendix C, is described as follows. As in Cooley and Prescott (1995),[32] the technology shock follows an AR(1) process $\widehat{\mathcal{A}}_t = \phi_{\mathfrak{a}}\widehat{\mathcal{A}}_{t-1} + \hat{\epsilon}_{\mathfrak{a},t}$, where $\hat{\epsilon}_{\mathfrak{a},t}$ is a white-noise disturbance. I set the subject discount factor at $\beta = 0.99$, the elasticity to hours at the production function at $\varepsilon = 0.64$ and the autoregressive coefficient

[32] More specifically, I refer to their model with no government consumption.

of the technology shock at $\phi_{\mathfrak{a}} = 0.95$.[33] The variance of $\hat{\epsilon}_{\mathfrak{a},t}$ is irrelevant for computing relative volatilities.

Since the literature estimate the matching elasticity to unemployment in the range $[0.4\ ,\ 0.6]$,[34] I fix it roughly in the midpoint $a = 0.50$. I also impose Hosios (1990) optimal condition in the steady state by setting the workers' bargaining power at $b = a = 0.50$.[35]

I set the elasticity of substitution at $\phi = 7$, which implies a steady state price markup of $\mu = 1.17$.[36] I calibrate the following parameters according to central estimates obtained by Smets and Wouters (2007),[37] estimated for sample 1984Q1 to 2004Q4. For the reciprocal of the intertemporal elasticity of substitution, I set $\sigma = 1.47$. Habit persistence parameter is set at $\iota_{\mathfrak{u}} = 0.68$. As for the elasticity ν of the disutility from hours worked $H_t(z)$, i.e. the reciprocal of the Frisch elasticity, I use $\nu = 2.30$. Note that this value is consistent with micro evidence, as reported by Chetty et al. (2011).[38] I set the degree of price stickiness at $\alpha = 0.73$, while the price indexation parameter is set at $\gamma_\pi = 0.21$.

On the generalized Taylor rule, I use central estimates from Coibion and Gorodnichenko (2011) for the 1983–2002 period. Their Mixed Taylor rule is characterized by $\phi_{i1} = 1.05$, $\phi_{i2} = -0.13$, $\phi_\pi = 2.20$ and $\phi_{gy} = 1.56$. Their estimate for the response parameter ϕ_y on the output gap from trend is $0.43/4 = 0.11$, when doing frequency conversion to quarterly data. However, I find that large response parameters brings instability when the trend inflation model is augmented with labor matching frictions. The model used Coibion and Gorodnichenko (2011), on the other hand, does not have any labor matching frictions. Thus they do not find any instability issues. To cope with this issue, I choose the nearest feasible parameter consistent with stability, i.e. $\phi_y = 0.02$.

As for the nuisance parameters $[\eta, w^u, k, \chi]$, I calibrate them based on steady state values, as described in Appendix B. I set the steady state aggregate hours per worker is at $\bar{h} = 1$. As in Ravenna and Walsh (2008), I fix the replacement ratio $\varrho_{wu} \equiv w^u/\bar{w}$ at 0.40. I fix the steady state levels end-of-period unemployment rate $\bar{\mathfrak{u}}^e$ and end-of-period tightness rate $\bar{\theta}^e$ according to the averages observed during the Great Moderation sample (1985Q1 to 2005Q4), as reported in Section 3., i.e. I set $\bar{\mathfrak{u}}^e = 0.057$ and $\bar{\theta}^e = 1.85$. The separation rate[39] $\rho = 0.147$ and trend inflation level[40] $\bar{\pi} = 3.05$ are also set according to the observed averages.

[33] Note that we find very similar coefficients when regressing $\Delta \log(Y_t) = c_y + \bar{\varepsilon}\Delta \log(n_t \cdot h_t) + \xi_t$ over the Great Moderation Sample (1985Q1 to 2005Q4). I use original data, before applying any detrending technique, of Y_t, n_t and h_t obtained according to what is described in Section 3. The relevant estimate is $\widehat{\bar{\varepsilon}} = 0.61$ ($se = 0.13$). Since the regression residual may be interpreted as $\Delta\widehat{\mathcal{A}}_t$, I obtain a series of $\widehat{\mathcal{A}}_t$ and regress it against its lagged value, i.e. $\widehat{\mathcal{A}}_t = c_a + \bar{\phi}_{\mathfrak{a}}\widehat{\mathcal{A}}_{t-1} + \hat{\xi}_{a,t}$. The relevant estimate is $\widehat{\bar{\phi}}_{\mathfrak{a}} = 0.95$ ($se = 0.03$).

[34] See e.g. Andolfatto (1996), Blanchard and Diamond (1989), Hagedorn and Manovskii (2008), Hall (2005), Merz (1995), Mortensen and Nagypal (2007), and Shimer (2005).

[35] Flinn (2006) estimates this parameter at $b = 0.40$.

[36] For instance, Ravenna and Walsh (2008, 2011) set the steady state price markup to 1.2.

[37] I use posterior distribution modes, as reported at Table 5 in Smets and Wouters (2007).

[38] The authors conduct meta analyses of existing micro evidence. Their point estimate of the Frisch elasticity of intensive margin is $(1/\nu) = 0.54$.

[39] Note that this value is roughly consistent with empirical evidence, as reported by Shimer (2005), that jobs last about 10 quarters in the US. Indeed, the average employment duration implied by the value ρ is calibrated is $\bar{\tau} = \frac{1}{\rho} \approx 7$ quarters.

[40] The restriction $\max\left(\bar{\alpha}, \bar{\alpha}\bar{\vartheta}\right) < 1$ implies that maximum level of trend inflation is $\bar{\pi}^{\max} = 4.51$, which is much greater than the sample inflation average $\bar{\pi} = 3.05$. Therefore, my results are not influenced by disortions arising in the near vicinity of $\bar{\pi}^{\max}$.

8 Simulations

In all simulation exercises, I assess dynamic properties as trend inflatyion rises from zero to $\bar{\pi} = 3.05$. In this regard,m there are two methods to choose: (i) as I did when plotting Figure 1, I may compute the nuisance parameters $[\eta, w^u, k, \chi]$ using $\bar{\pi} = 3.05$, and keep them constant for all simulations carried out at $\bar{\pi} < 3.05$; and (ii) I may recompute the nuisance parameters for each level of trend inflation. As shown in Appendix B, recomputing nuisance parameters based on fixed steady state levels for $\left[\bar{h}, \varrho_{wu}, \bar{\mathsf{u}}^e, \bar{\theta}^e\right]$ does not change steady state levels of any labor market variable. It does change the levels of $\bar{Y}$ and $\bar{\mathcal{Y}} = \bar{C}$. As for (log-deviation) dynamics, they are almost invariant to the method I choose. However, in order to strongly verify the role of trend inflation, I follow the second method.

In my first exercise, I show impulse responses to technology, preferences and monetary policy shocks considering different levels of trend inflation. I also show how labor productivity $\wp_t \equiv \mathcal{Y}_t/H_t$ responds to each shock. In this exercise, it gets crystal clear the important effects of rising trend inflation in both the amplitude and inertia of labor market responses.

Next, I compute relative volatilities $StDev\left(\hat{\varkappa}_t\right)/StDev\left(\wp_t\right)$ for each endogenous (log-linearized) variable $\hat{\varkappa}_t$. For that, I simulate the log-linearized model with rational expectations considering different levels of trend inflation, when the economy is hit either by pure technology, preferences or policy shocks, and compute unconditional standard deviations of every endogenous variable. In order to test how robust are the volatilities to changes in the parameter set, I also consider comparative statics analysis by redoing the simulations for different levels of structural parameters and steady state values used for calibration of nuisance parameters.

At last, I compare the empirical correlation matrix obtained when data is detrended as in Shimer (2005), i.e. Hodrick Prescott detrending with smoothing parameter 10^5,[41] with theoretical ones obtained when the economy is hit only by either technology, preferences or monetary policy shocks.

8.1 Impulse Responses

Figure 2 shows the responses to a normalized positive technology shock, for different levels of trend inflation. Blueish hues stand for low levels of trend inflation, starting at $\bar{\pi} = 0$, while reddish hues stand for larger levels, up to $\bar{\pi} = 3.05$. The benchmark response, i.e. at $\bar{\pi} = 3.05$, is marked with bold lines. As a normalization rule, I set to unity the maximum (in absolute terms) response amplitude of GDP when trend inflation is $\bar{\pi} = 3.05$. This normalization makes it easy when comparing the responses to technology shocks with those to preference and monetary shocks. The responses are measured in percentage change for all variables, except for annualized inflation and annualized interest rates, whose changes are measured in percentage points.

As expected, regardless the level of trend inflation, quarterly GDP rises (up to 1% in the benchmark level of trend inflation). Annualized inflation rate falls down by approximately 1.5 p.p., which in turn leads to a fall in annualized nominal interest rates by approximately 0.4 p.p.. Due to the fall in prices, it gets cheaper to post more vacancies, which leads to a 8% increase in employment when $\bar{\pi} = 3.5$. Hourly real wages rise by 4%

[41] Using the other two detrending methods, as described in Section 3, do not change much empirical correlations.

and so the increase in employment (extensive margin) is accompanied by a 12% reduction in hours (intensive margin). The net effect is a 8% reduction in real salaries. Since employment and hours per worker achieve their maximum amplitude by the same time, total hours $H_t = \mathsf{n}_t h_t$ then decreases by 4%. The combination of increased GDP with reduced total hours lead labor productivity $\wp_t \equiv \mathcal{Y}_t / H_t$ to increase by 4%.

After the shock, the job finding rate increases up by 50%, i.e. $\mathsf{p}_t \approx 1.5\bar{\mathsf{p}}$, while the vacancy filling rate decreases by 30%. As a consequence, the tightness ratio more than doubles after the shock. Note that the dynamics of hourly wages follow that of labor productivity after the technology shock $\wp_t \equiv \mathcal{Y}_t / H_t$. I find that this property roughly continues to hold even when the economy is hit shocks other than technology, i.e. pure preference or monetary policy shocks. As trend inflation rises, the maximum amplitude achieved by GDP decreases. The effect on inflation and labor productivity dynamics are barely noticed. However, the impact of larger levels of trend inflation in very strong on vacancy postings, i.e. v_t^e increases by 35% when trend inflation is zero, while it increases by 115% when trend inflation is set at $\bar{\pi} = 3.05$. This phenomenon is in line with the role of pricing multipliers described in Sections 4 and 6. The response amplitude of v_t^e increase as trend inflation rises, increasing even more the amplitude and persistence of employment responses and those of the remaining labor market variables. Responses to labor productivity seems robust to trend inflation, and so relative volatilities are expect to increase as trend inflation rises.

Figure 3 shows the responses to a normalized positive preference shock, for different levels of trend inflation. Again, I set to unity the maximum (in absolute terms) response amplitude of GDP when trend inflation is $\bar{\pi} = 3.05$. As expected for demand shocks, quarterly GDP rises (up to 1% in the benchmark level of trend inflation). Annualized inflation rate falls down by approximately 0.1 p.p., which in turn leads to a hike in annualized nominal interest rates by approximately 0.4 p.p.. Due to the increase in prices, it gets more expensive to post more vacancies, which leads to a 0.6% reduction in employment when $\bar{\pi} = 3.5$. Hourly real wages fall by 0.17% and so the reduction in employment (extensive margin) is accompanied by a 1.5% increase in hours (intensive margin). The net effect is a 1.3% increase in real salaries. Hours per worker hike on spot as the shock hits, while employment reduction takes some time to build up. Therefore, total hours $H_t = \mathsf{n}_t h_t$ then increases on spot by about 1.4%. Since total hours increases more than GDP, labor productivity $\wp_t \equiv \mathcal{Y}_t / H_t$ actually falls by 0.4%. After the shock, the job finding rate decreases down by 3.5%, i.e. $\mathsf{p}_t \approx 0.665\bar{\mathsf{p}}$, while the vacancy filling rate increases by about 3.6%. As a consequence, the tightness ratio falls after the shock.

Note that the dynamics of hourly wages roughly follow that of labor productivity $\wp_t \equiv \mathcal{Y}_t / H_t$, even the economy being hit by a preference shock. As before, increased levels of trend inflation have minimal effects on goods market variables after preference shocks. The main channel is to amplify the reduction in vacancy postings, i.e. v_t^e decreases by 3.3% when $\bar{\pi} = 0$, while the reduction achieves 7% when $\bar{\pi} = 3.05$. And thus amplitudes and inertia of labor market variables largely increase. Again, responses to labor productivity are only slightly changed with trend inflation and so relative volatilities are expect to achieve large levels as trend inflation rises.

Figure 4 shows the responses to a normalized positive monetary policy shock, for different levels of trend inflation. Again, I set to unity the maximum (in absolute terms) response amplitude of GDP when trend inflation is $\bar{\pi} = 3.05$. As expected for monetary policy shocks, quarterly GDP falls (down to 1% in the benchmark level of trend inflation). Annualized inflation rate falls down by approximately 1.5p.p.. Due to the fall in prices, it gets cheaper to post more vacancies, leading to a 6.5% increase in n_t when $\bar{\pi} = 3.5$.

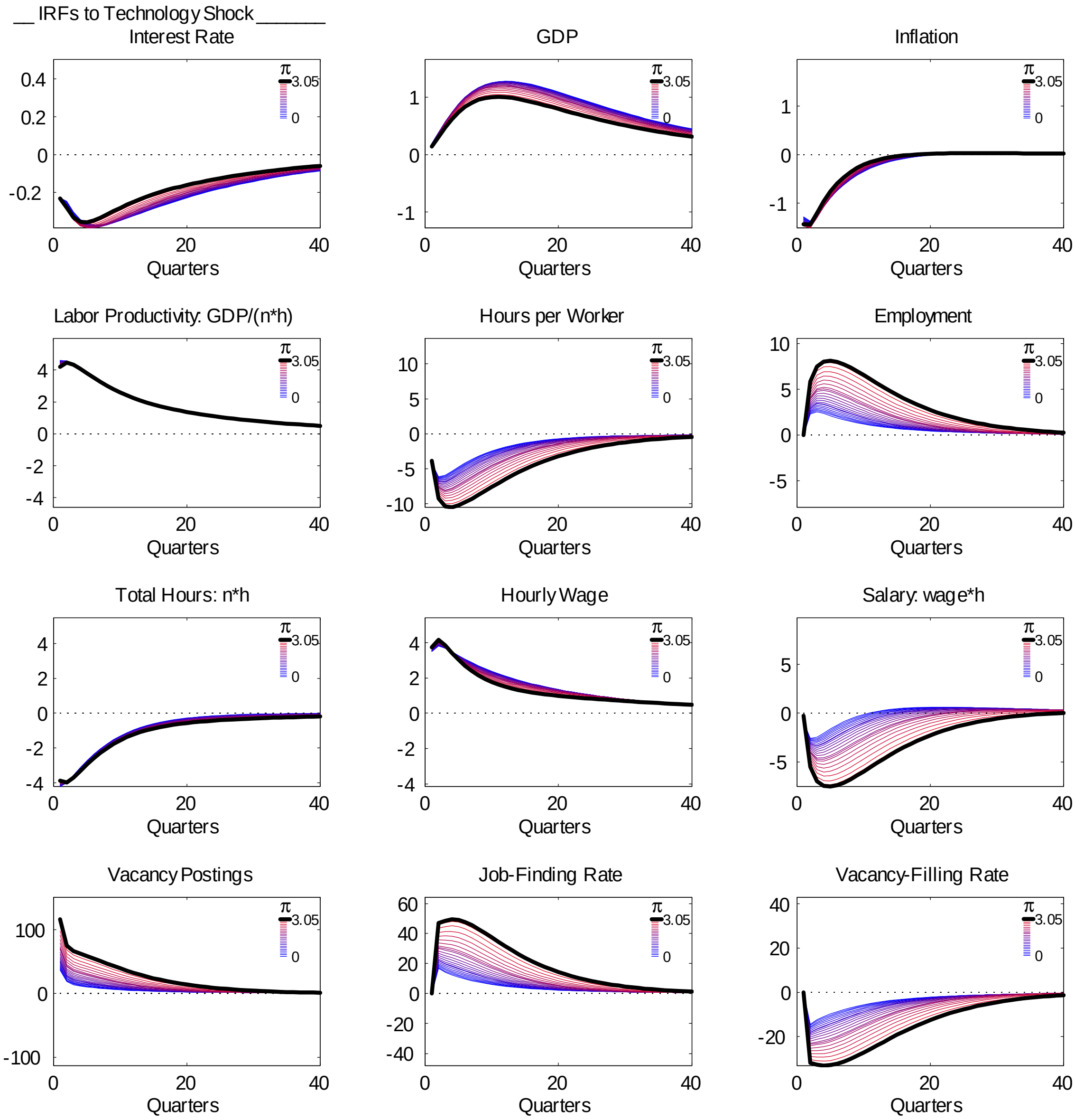

Figure 2: Impulse Responses to Technology Shock

Note: (Bold Line) impulse response at $\bar{\pi}$=3.05; (Color Gradient) Blue is impulse response at $\bar{\pi}$=0, Red is impulse response at $\bar{\pi}$=3.05. (Normalized Shocks) maximum absolute impact on GDP is 1% at $\bar{\pi}$=3.05. Responses measured in percentage variation (%) over steady state levels, except for annualized inflation rates and annualized nominal interest rates, whose responses are measured in percentage points.

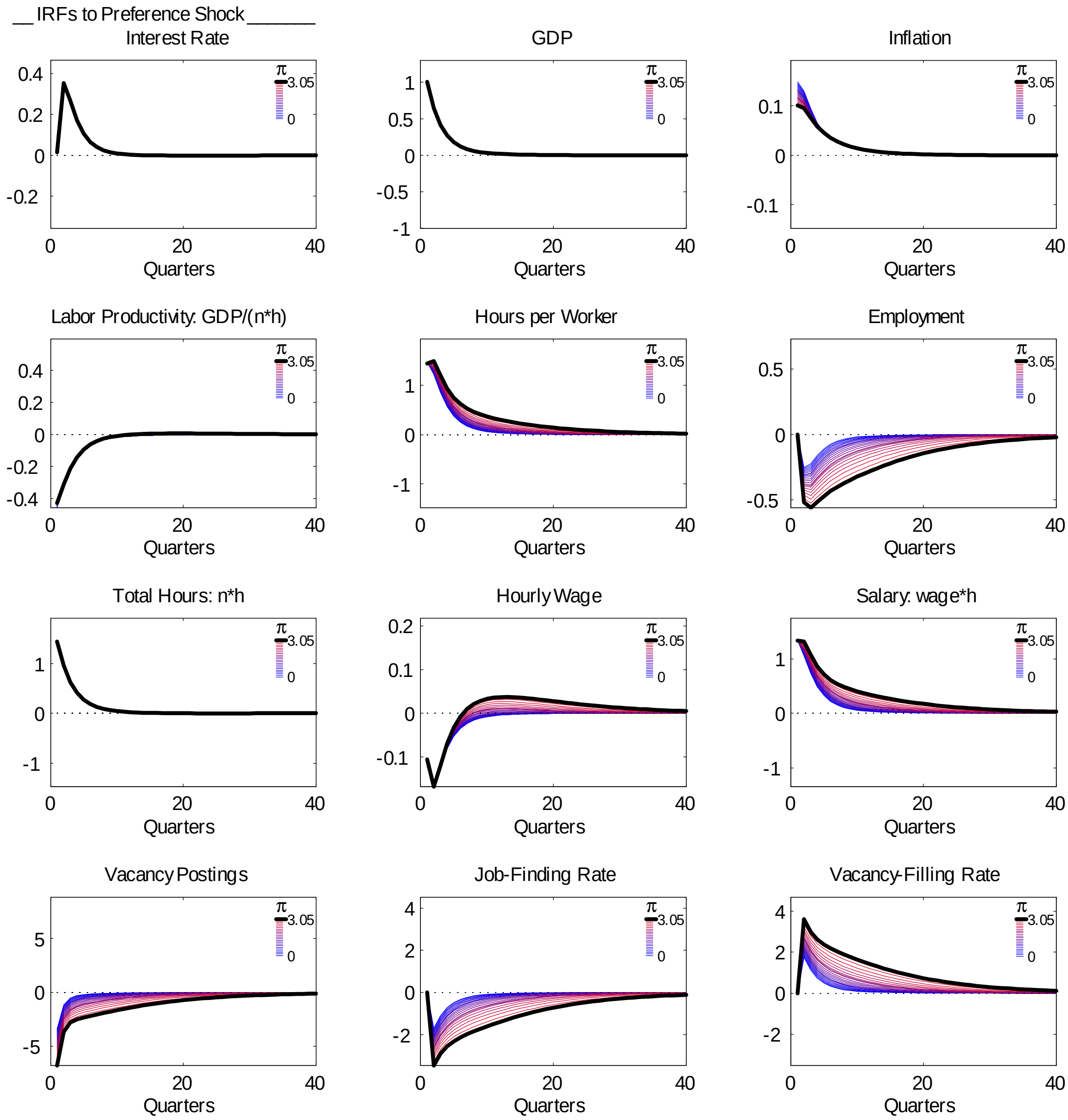

Figure 3: Impulse Responses to Preference Shock

Note: (Bold Line) impulse response at $\bar{\pi}$=3.05; (Color Gradient) Blue is impulse response at $\bar{\pi}$=0, Red is impulse response at $\bar{\pi}$=3.05. (Normalized Shocks) maximum absolute impact on GDP is 1% at $\bar{\pi}$=3.05. Responses measured in percentage variation (%) over steady state levels, except for annualized inflation rates and annualized nominal interest rates, whose responses are measured in percentage points.

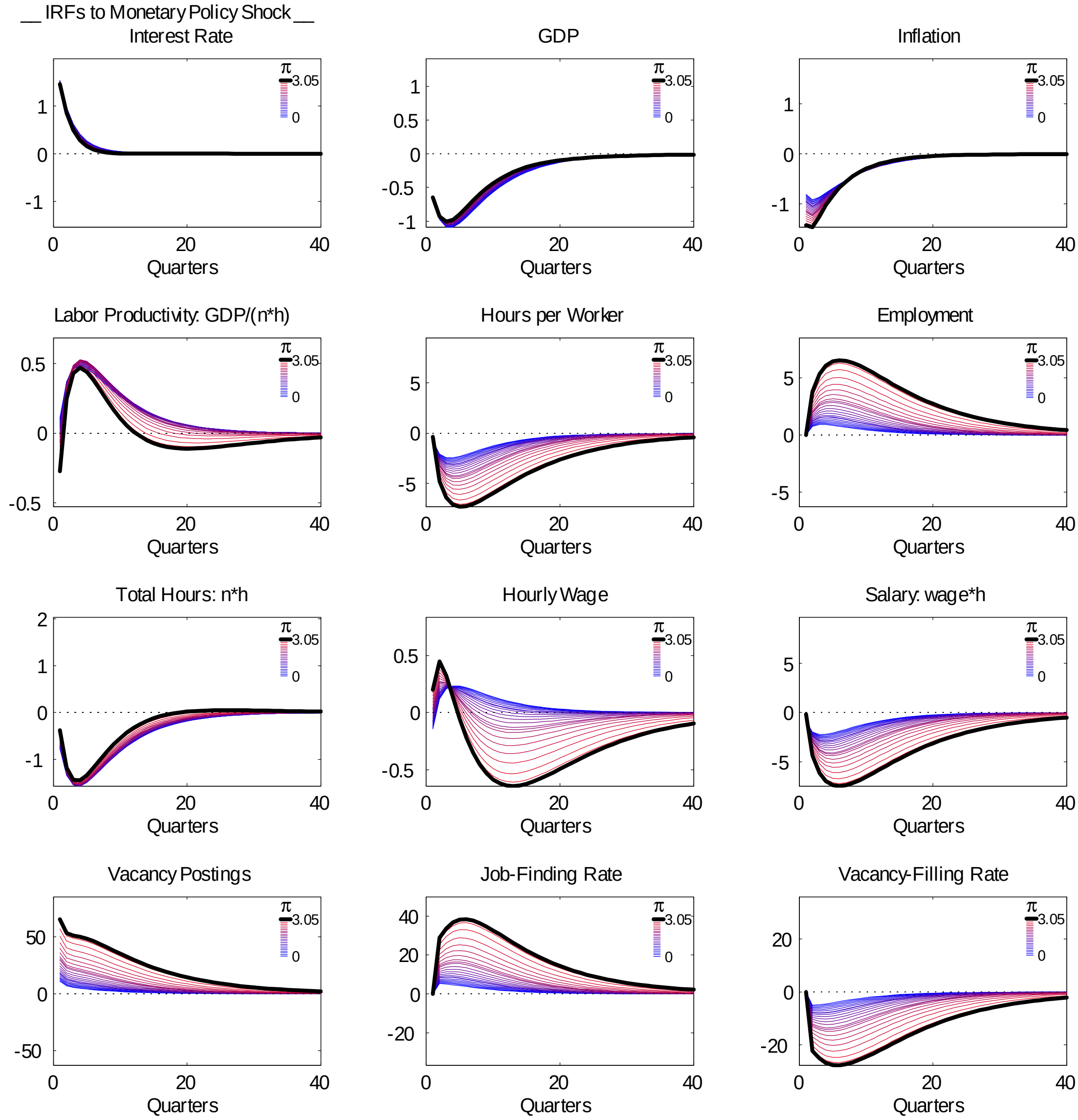

Figure 4: Impulse Responses to Monetary Policy Shock

Note: (Bold Line) impulse response at $\bar{\pi}$=3.05; (Color Gradient) Blue is impulse response at $\bar{\pi}$=0, Red is impulse response at $\bar{\pi}$=3.05. (Normalized Shocks) maximum absolute impact on GDP is 1% at $\bar{\pi}$=3.05. Responses measured in percentage variation (%) over steady state levels, except for annualized inflation rates and annualized nominal interest rates, whose responses are measured in percentage points.

Hourly real wages initially rise by 0.5% and then strongly falls by 0.65%, remaining below its steady state level for many quarters. Therefore, the increase in employment (extensive margin) is accompanied by a 7.5% fall in hours (intensive margin). The net effect is a 7.5% fall in real salaries. Total hours $H_t = \mathsf{n}_t h_t$ then falls by about 1.4%, which leads in turn to a 0.45% rise in labor productivity $\wp_t \equiv \mathcal{Y}_t/H_t$. After the shock, the job finding rate increases up by 40%, i.e. $\mathsf{p}_t \approx 1.40\bar{\mathsf{p}}$, while the vacancy filling rate falls by about 30%. As a consequence, the tightness ratio rises after the shock.

Again, the dynamics of hourly wages roughly follow that of labor productivity $\wp_t \equiv \mathcal{Y}_t/H_t$, even the economy being hit by a monetary policy shock.

As predicted by the analytical results shown in Section 5, the effect of rising trend inflation is a little bit entangled after monetary policy shocks. Just as policy shocks affect pricing multipliers in the non-linear static model, we expect the role of rising trend inflation to be more complicated when monetary shocks hit the dynamic model. Indeed, we see that the changes in hourly wages responses are not monotonic with trend inflation. Moreover, rising trend inflation has an important effect on the response of labor productivity $\wp_t \equiv \mathcal{Y}_t/H_t$. As trend inflation rises, its maximum amplitude and inertia decreases. And so we expect the standard deviation of $\wp_t$ to decrease as trend inflation rises.

8.2 Relative Volatilities

Figures 5 to 7 show theoretical and empirical relative volatility ratios for the labor and goods markets. I plot theoretical values when inflation trend is set at the sample average $\bar{\pi} = 3.05$ (square), theoretical values in the equilibrium with flexible prices (diamond), and empirical values (dots), obtained using different detrending methods.

Figures 8 to 10 show comparative statics analyses on theoretical relative volatilities, as the economy is hit by technology, preferences and monetary policy shocks. I only show the analyses for end-of-period unemployment $\hat{\mathsf{u}}_t^e$. Lessons for the remaining labor market variables would be similar. Blueish hues stand for the smallest parameters in each interval, while reddish hues stand the largest ones.

Figure 11 shows comparative statics analyses on steady state level of total surplus. The main lesson is that there is a negative correlation between changes in total surplus and changes in relative volatilities.

With pure technology shocks (Figure 5), the equilibrium with flexible prices generates very small theoretical relative volatilities when compared to empirical assessments. In the literature, Shimer (2005), Hall (2005) and Costain and Reiter (2008) have already found this property. Using technology shocks as the only source of fluctuations in their models, they have reported this property as a puzzle, or the Shimmer Puzzle as it came to be known. Note that the model does a pretty god job in explaining the relative volatilities of employment n_t, hours per worker h_t, output Y_t and inflation π_t, even under the equilibrium with flexible prices. That is, the puzzle is not there when assessing those variables.

If $\bar{\pi} = 0$, my results are similar to those reported by Thomas (2011), i.e. I find that allowing each firm to simultaneously make pricing and vacancy postings decisions does help increase relative volatilities when compared to the flexible price equilibrium. However, the volatility amplification is still too small to explain the puzzle. As trend inflation rises, on the other hand, the relative volatilities significantly increase and reach the region where the empirical values are located.

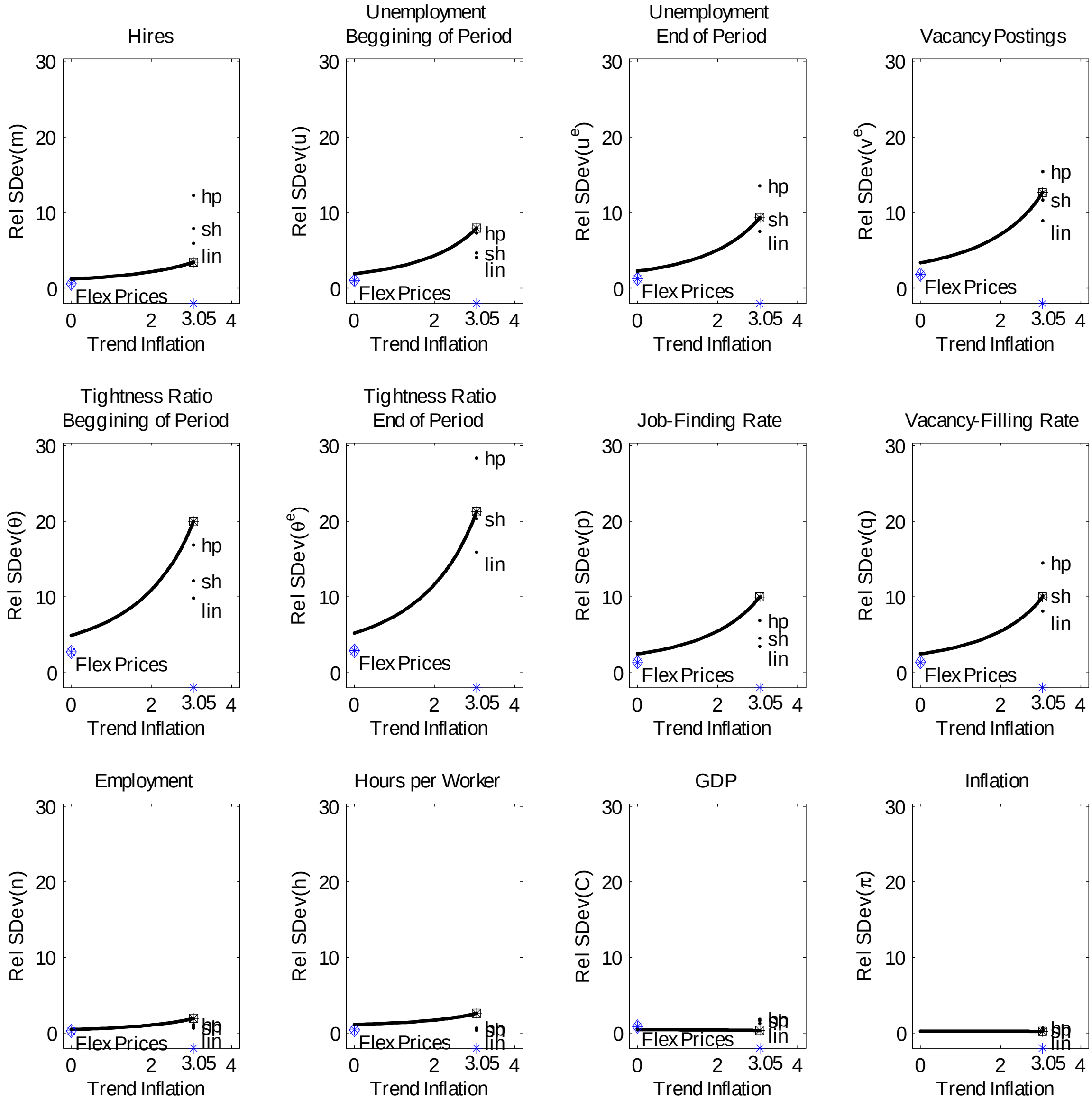

Figure 5: Relative volatilities (pure technology shocks)

Note: (Bold Line) theoretical relative volatility, i.e. StDev(EndogVariable)/StDev(LaborProductivity); (Square) theoretical relative volatility at sample average $\bar{\pi}$=3.05; (Dot hp) empirical relative volatility, HP-detrended with quarterly smoothing parameter 1600; (Dot sh) empirical relative volatility, HP-detrended with smoothing parameter 10^5, as in Shimer (2005); (Dot lin) empirical relative volatility, linear detrending; (Diamond) theoretical relative volatility with flexible prices.

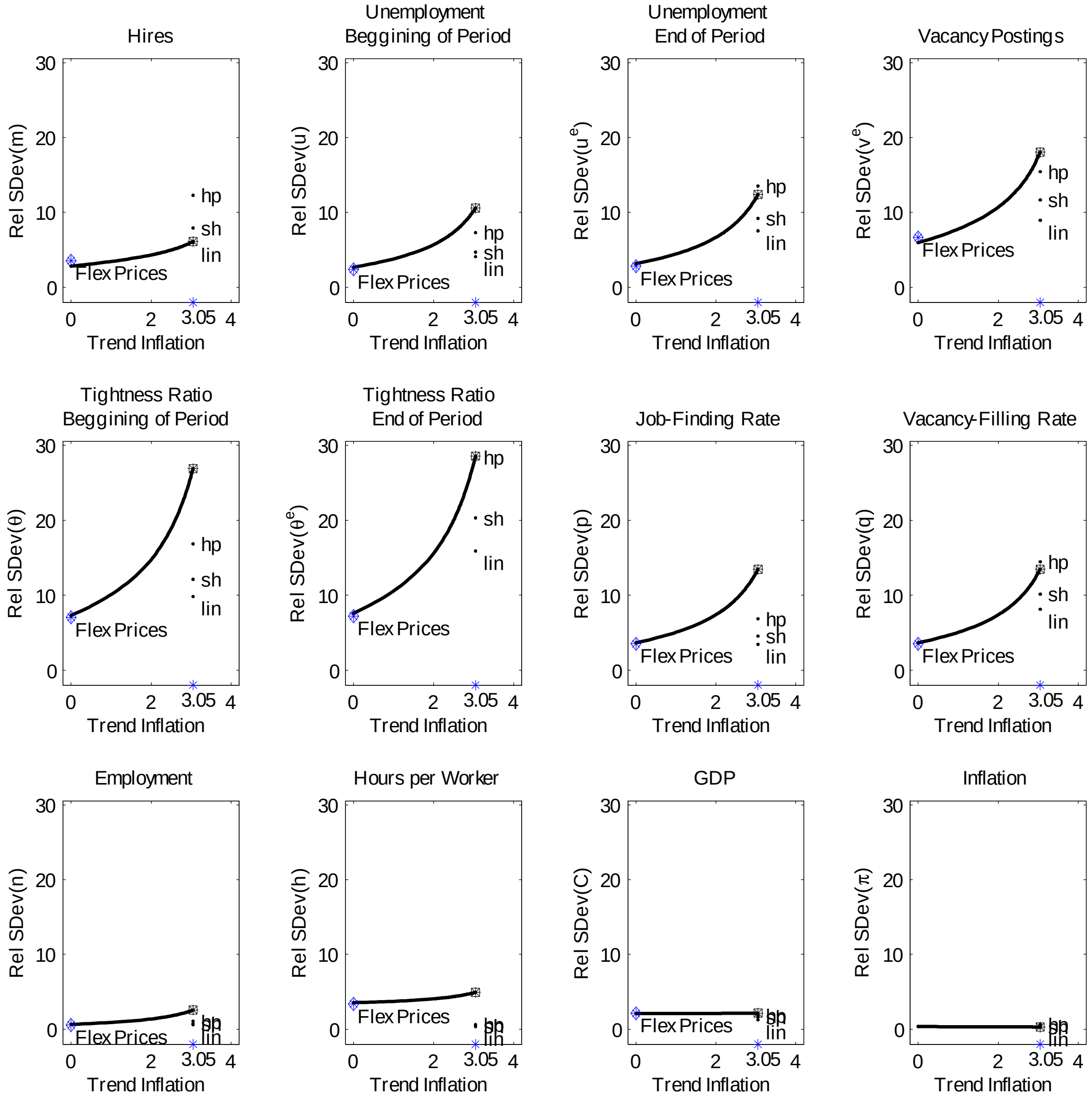

Figure 6: Relative volatilities (pure preference shocks)

Note: (Bold Line) theoretical relative volatility, i.e. StDev(EndogVariable)/StDev(LaborProductivity); (Square) theoretical relative volatility at sample average $\bar{\pi}$=3.05; (Dot hp) empirical relative volatility, HP-detrended with quarterly smoothing parameter 1600; (Dot sh) empirical relative volatility, HP-detrended with smoothing parameter 10^5, as in Shimer (2005); (Dot lin) empirical relative volatility, linear detrending; (Diamond) theoretical relative volatility with flexible prices.

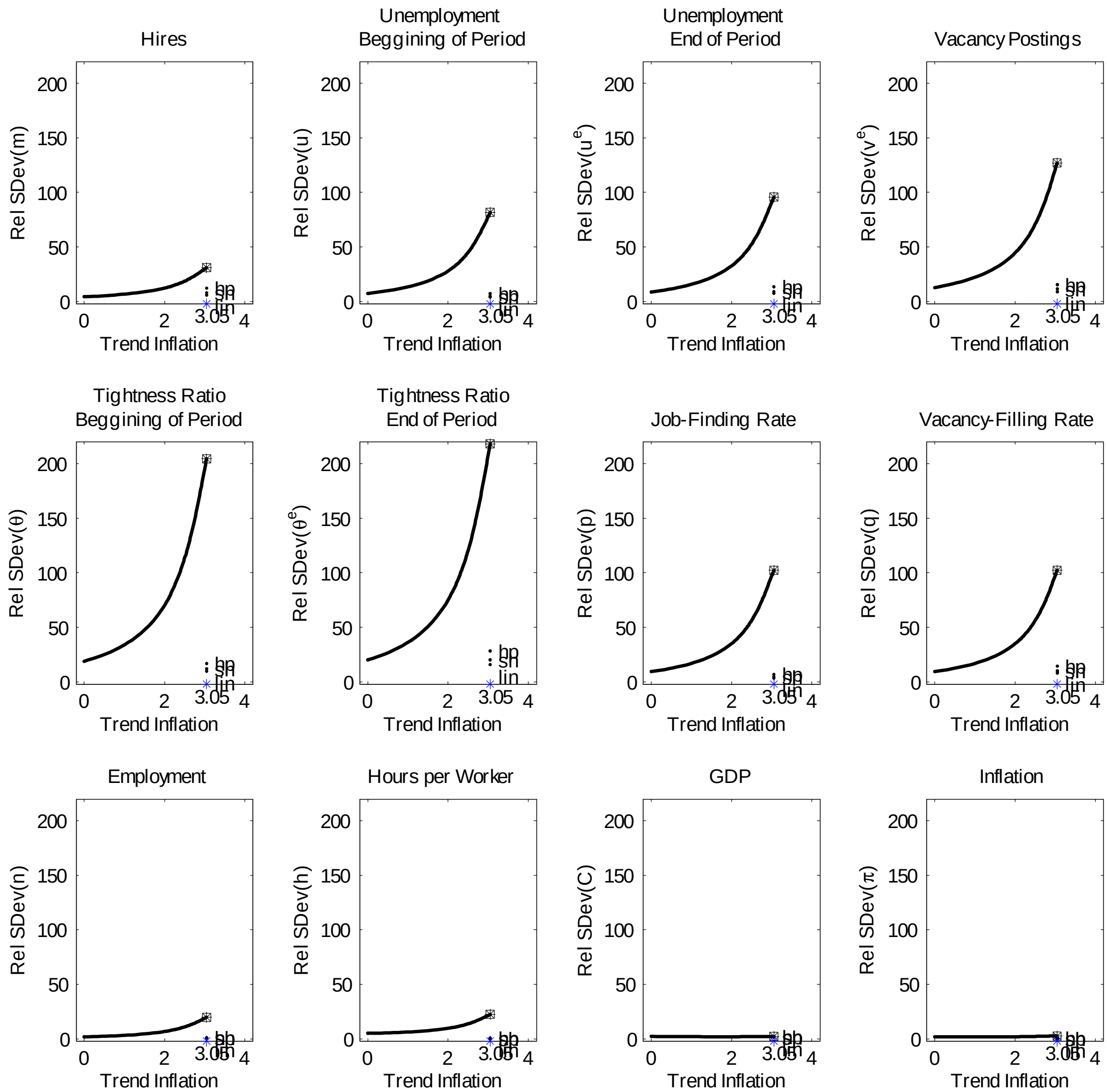

Figure 7: Relative volatilities (pure monetary policy shocks)

Note: (Bold Line) theoretical relative volatility, i.e. StDev(EndogVariable)/StDev(LaborProductivity); (Square) theoretical relative volatility at sample average $\bar{\pi}$=3.05; (Dot hp) empirical relative volatility, HP-detrended with quarterly smoothing parameter 1600; (Dot sh) empirical relative volatility, HP-detrended with smoothing parameter 10^5, as in Shimer (2005); (Dot lin) empirical relative volatility, linear detrending; (Diamond) theoretical relative volatility with flexible prices.

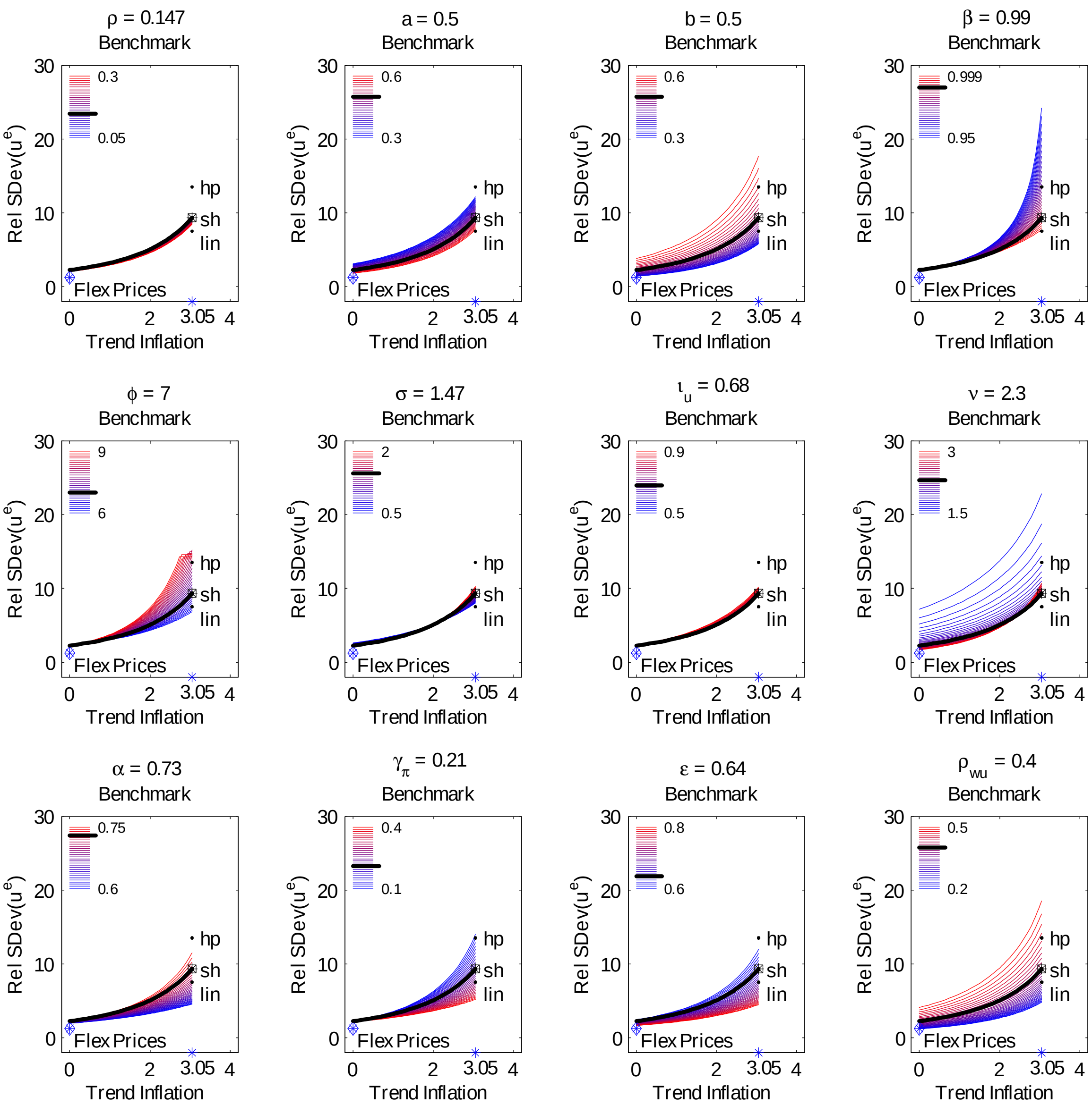

Figure 8: Comparative Statics - Relative Volatilities (Pure Technology Shocks)

Note: (Bold Line) theoretical relative volatility, i.e. StDev(EndogVariable)/StDev(LaborProductivity) of end-of-period unemployment u_t^e at benchmark calibration; (Color Gradient) Blue is relative volatility at smallest parameter in interval, Red is relative volatility at largest parameter; (Square) theoretical relative volatility at sample average $\bar{\pi}$=3.05; (Dot hp) empirical relative volatility, HP-detrended with quarterly smoothing parameter 1600; (Dot sh) empirical relative volatility, HP-detrended with smoothing parameter 10^5, as in Shimer (2005); (Dot lin) empirical relative volatility, linear detrending; (Diamond) theoretical relative volatility with flexible prices.

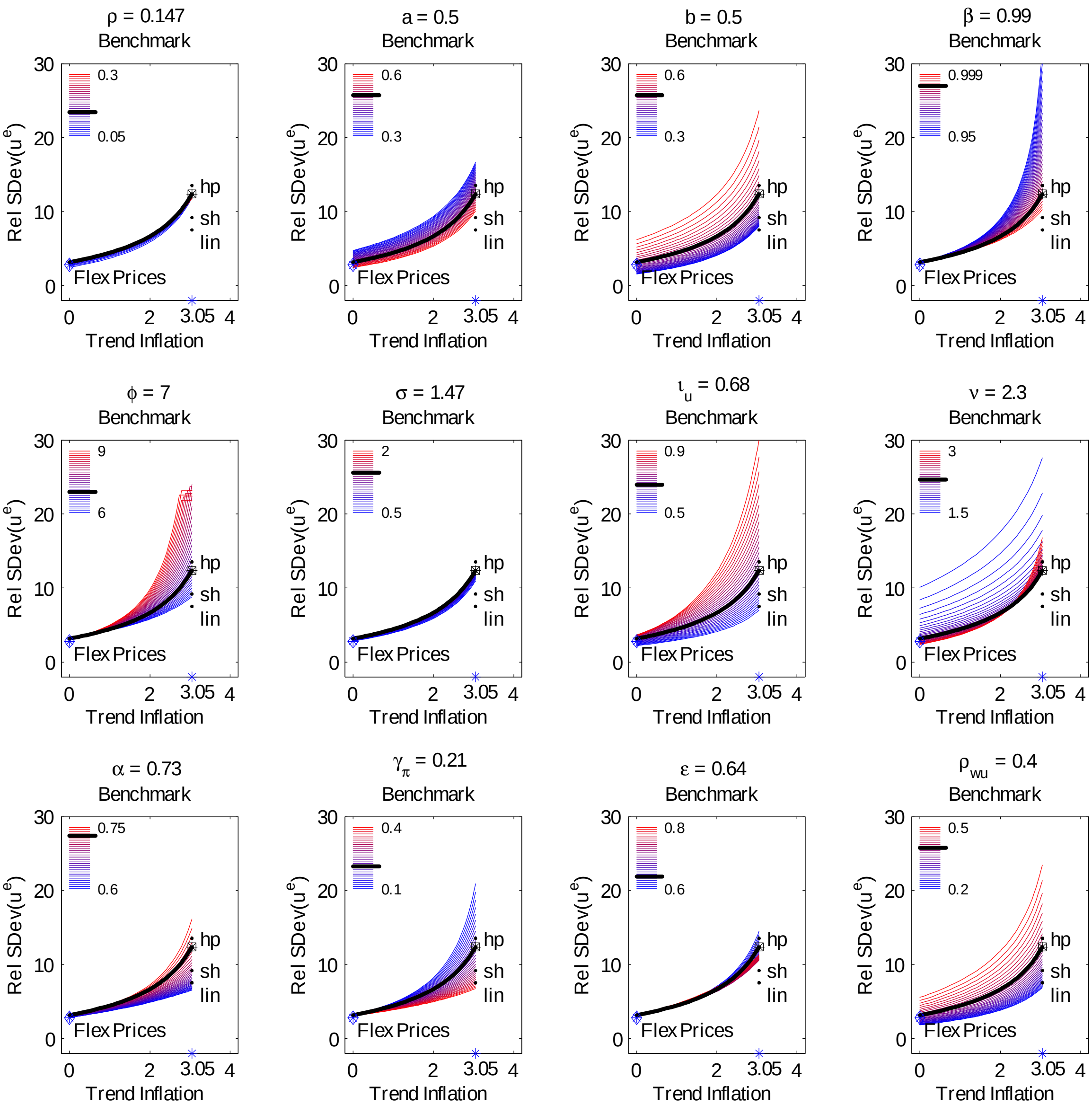

Figure 9: Comparative Statics - Relative Volatilities (Pure Preference Shocks)

Note: (Bold Line) theoretical relative volatility, i.e. StDev(EndogVariable)/StDev(LaborProductivity) of end-of-period unemployment u_t^e at benchmark calibration; (Color Gradient) Blue is relative volatility at smallest parameter in interval, Red is relative volatility at largest parameter; (Square) theoretical relative volatility at sample average $\bar{\pi}$=3.05; (Dot hp) empirical relative volatility, HP-detrended with quarterly smoothing parameter 1600; (Dot sh) empirical relative volatility, HP-detrended with smoothing parameter 10^5, as in Shimer (2005); (Dot lin) empirical relative volatility, linear detrending; (Diamond) theoretical relative volatility with flexible prices.

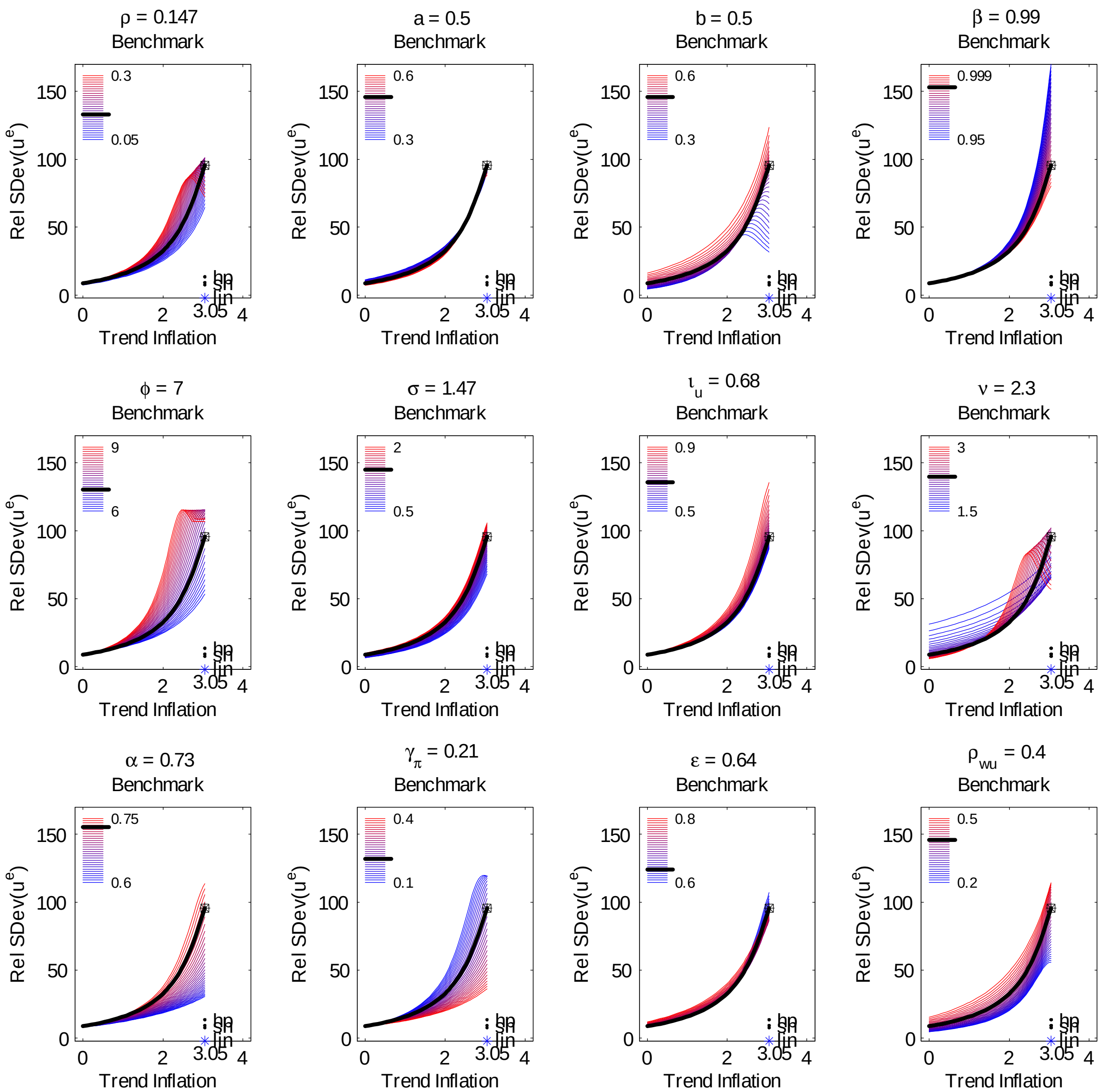

Figure 10: Comparative Statics - Relative Volatilities (Pure Monetary Policy Shocks)

Note: (Bold Line) theoretical relative volatility, i.e. StDev(EndogVariable)/StDev(LaborProductivity) of end-of-period unemployment u_t^e at benchmark calibration; (Color Gradient) Blue is relative volatility at smallest parameter in interval, Red is relative volatility at largest parameter; (Square) theoretical relative volatility at sample average $\bar{\pi}$=3.05; (Dot hp) empirical relative volatility, HP-detrended with quarterly smoothing parameter 1600; (Dot sh) empirical relative volatility, HP-detrended with smoothing parameter 10^5, as in Shimer (2005); (Dot lin) empirical relative volatility, linear detrending; (Diamond) theoretical relative volatility with flexible prices.

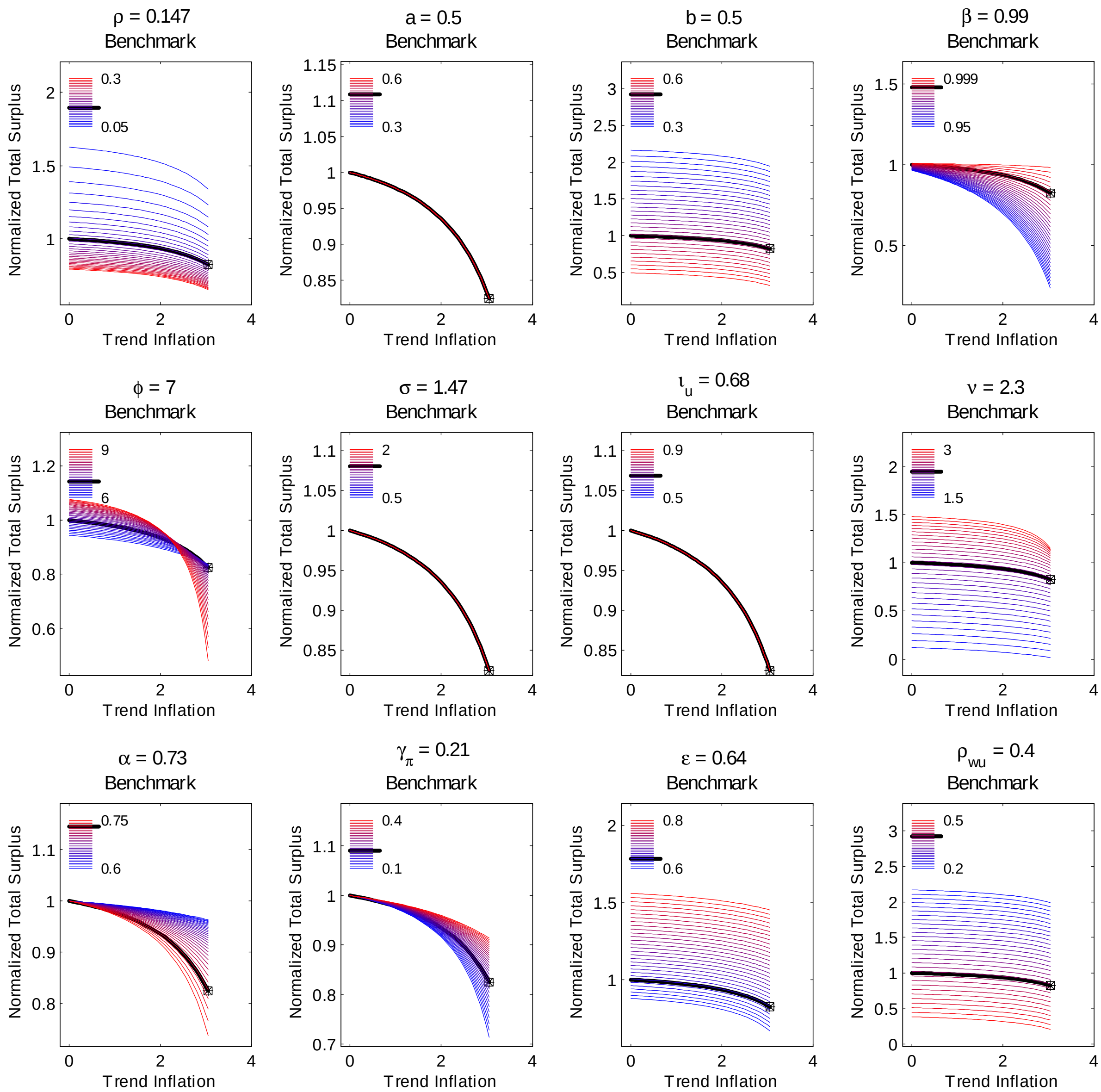

Figure 11: Comparative Statics - Total Surplus

Note: (Bold Line) theoretical total surplus $\bar{\mathfrak{s}}$ at benchmark calibration; (Color Gradient) Blue is total surplus at smallest parameter in interval, Red is total surplus at largest parameter in interval; (Square) theoretical total surplus at sample average $\bar{\pi}$=3.05.

With pure preference shocks (Figure 6), the equilibrium with flexible prices still generates small theoretical relative volatilities when compared to empirical assessments. Theoretical relative volatilities are about 1.3 times as large as in the case of pure technology shocks. The puzzle still exists for most labor market quantities, but is somehow mitigated as the equilibrium with flexible prices delivers relative volatilities about 2 times as large as in the case of pure technology shocks. And again, if $\bar{\pi} = 0$, theoretical relative volatilities are slightly larger than those obtained under flexible prices. As trend inflation rises, relative volatilities increase and the amplification lines seem to move in parallel to those under pure technology shocks.

With pure monetary policy shocks (Figure 7), the equilibrium with flexible prices does not respond, i.e. theoretical standard deviations are all zero under absence of nominal rigidities. Since the standard deviation of labor productivity is also zero under flexible prices, theoretical relative volatilities are not defined (i.e. 0/0). Yet, if nominal rigidities have a role, the equilibrium with $\bar{\pi} = 0$ already generates theoretical relative volatilities of the same order of magnitude as their empirical counterparts. Even though relative volatilities at $\bar{\pi} = 0$ are about 4 times as large as in the case of pure technology shocks, the amplification lines increase to about 10 times as large when inflation trend is $\bar{\pi} = 3.05$. This effect was predicted in the analysis on responses to monetary policy shocks, in Section 8.1.

In comparative statics analyses, relative volatilities change in the same direction as each parameter increase when the economy is hit either by technology or preference shocks. When hit by monetary policy shocks, the effect is switched for the labor elasticity ε in the production function and separation rate ρ. Moreover, changes in amplification lines might even not rise in parallel with those obtained with technology and preference shocks.

Increases in the separation rate ρ lead to very modest reductions in relative volatilities under technology and preference shocks. It leads, however, to reasonable increases under monetary policy shocks. Increases in the unemployment elasticity a in the matching function lead to reductions in relative volatilities under technology and preference shocks. It leads, however, to almost negligible reductions under monetary policy shocks. Increases in the workers bargaining power b lead to increases in relative volatilities under all types of shocks. Increases in the subjective discounting parameter β lead to reductions in relative volatilities under all types of shocks. Increases in the elasticity of substitution ϕ lead to increases in relative volatilities under all types of shocks. Increases in the reciprocal σ of the intertemporal elasticity of substitution lead to very modest reductions in relative volatilities under all types of shocks. Increases in the degree of habit persistence ι_u on consumption utility lead to increases in relative volatilities under all types of shocks. This effect is exacerbated under preference shocks, due to the direct effect on consumption.

Increases in the reciprocal ν of the Frisch elasticity of labor supply lead to reductions in relative volatilities under all types of shocks. Increases in the degree α of price rigidity lead to increases in relative volatilities under all types of shocks. Increases in the degree γ_π of price indexation lead to reductions in relative volatilities under all types of shocks. Increases in the labor elasticity ε in the production function lead to very modest reductions in relative volatilities under preference shocks, and very modest increases under monetary policy shocks. Under technology shocks, due to the direct effect on the production function, relative volatilities experience reasonable reductions. Finally, increases in the steady state replacement ratio ϱ_{wu}, i.e. unemployment compensation over aggregate salary, lead to increases in relative volatilities.

8.3 Correlations

Tables 2, 3 and 4 show empirical and theoretical correlation matrices obtained under pure technology, preference and monetary policy shocks. For comparison, I show theoretical values obtained under $\bar{\pi} = 0$ and $\bar{\pi} = 3.05$. As expected, there is no pure shock able to totally explain the correlations. Theoretical correlations under those levels of trend inflation preserve the same sign, even though those for $\bar{\pi} = 3.05$ tend to approach better empirical ones. Finally, most theoretical correlations with hours per worker have the wrong sign when compared with empirical counterparts. This suggests that a adjusting cost must be in place when real-world firms adjust hours.

Table 2: Empirical and Theoretical Correlation Matrices (Pure Technology Shocks)

	$\widehat{\mathcal{Y}}_t$	$\hat{h}_t$	$\hat{H}_t$	$\hat{\mathsf{u}}_t^e$	$\hat{\mathsf{v}}_t$	$\hat{\pi}_t$
$\widehat{\mathcal{Y}}_t$	**1.00**					
$\hat{h}_t$	0.47 **-0.82** −0.61	**1.00**				
$\hat{H}_t$	0.82 **-0.66** −0.51	0.73 **0.92** 0.98	**1.00**			
$\hat{\mathsf{u}}_t^e$	−0.80 **-0.85** −0.71	−0.65 **0.99** 0.95	−0.95 **0.86** 0.87	**1.00**		
$\hat{\mathsf{v}}_t$	0.71 **0.71** 0.51	0.77 **-0.93** −0.88	0.90 **-0.98** −0.95	−0.90 **-0.87** −0.72	**1.00**	
$\hat{\pi}_t$	0.10 **-0.32** −0.39	−0.05 **0.73** 0.96	0.15 **0.92** 0.99	−0.19 **0.62** 0.84	0.05 **-0.87** −0.92	**1.00**

Note: In each cell $\overset{Emp}{\underset{Ze}{\mathbf{Tr}}}$, (Emp) is the empirical correlation, obtained with original data detrended as in Shimer (2005), i.e. Hodrick Prescott with smoothing parameter 10^5; (Tr) is the theoretical correlation, obtained under $\bar{\pi}$=3.05, when the model is hit only by technology shocks. (Ze) is the theoretical correlation, obtained under $\bar{\pi}$=0, when the model is hit only by technology shocks.

Table 3: Empirical and Theoretical Correlation Matrices (Pure Preference Shocks)

	$\widehat{\mathcal{Y}}_t$	$\hat{h}_t$	$\hat{H}_t$	$\hat{\mathsf{u}}_t^e$	$\hat{\mathsf{v}}_t$	$\hat{\pi}_t$
$\widehat{\mathcal{Y}}_t$	**1.00**					
$\hat{h}_t$	0.47 **0.89** 0.99	**1.00**				
$\hat{H}_t$	0.82 **1.00** 1.00	0.73 **0.89** 0.99	**1.00**			
$\hat{\mathsf{u}}_t^e$	−0.80 **0.47** 0.63	−0.65 **0.83** 0.74	−0.95 **0.48** 0.64	**1.00**		
$\hat{\mathsf{v}}_t$	0.71 **-0.88** −0.94	0.77 **-0.96** −0.88	0.90 **-0.88** −0.94	−0.90 **-0.75** −0.34	**1.00**	
$\hat{\pi}_t$	0.10 **0.94** 0.98	−0.05 **0.98** 1.00	0.15 **0.95** 0.98	−0.19 **0.72** 0.78	0.05 **-0.92** −0.85	**1.00**

Note: In each cell $\overset{Emp}{\underset{Ze}{\mathbf{Tr}}}$, (Emp) is the empirical correlation, obtained with original data detrended as in Shimer (2005), i.e. Hodrick Prescott with smoothing parameter 10^5; (Tr) is the theoretical correlation, obtained under $\bar{\pi}$=3.05, when the model is hit only by preference shocks. (Ze) is the theoretical correlation, obtained under $\bar{\pi}$=0, when the model is hit only by preference shocks.

Table 4: Empirical and Theoretical Correlation Matrices (Pure Monetary Policy Shocks)

	$\widehat{\mathcal{Y}}_t$	$\hat{h}_t$	$\hat{H}_t$	$\hat{\mathsf{u}}^e_t$	$\hat{\mathsf{v}}_t$	$\hat{\pi}_t$
$\widehat{\mathcal{Y}}_t$	**1.00**					
$\hat{h}_t$	0.47 **0.89** 0.99	**1.00**				
$\hat{H}_t$	0.82 **0.97** 1.00	0.73 **0.849** 1.00	**1.00**			
$\hat{\mathsf{u}}^e_t$	−0.80 **0.85** 0.97	−0.65 **1.00** 0.99	−0.95 **0.79** 0.98	**1.00**		
$\hat{\mathsf{v}}_t$	0.71 **-0.95** −0.87	0.77 **-0.93** −0.81	0.90 **-0.85** −0.85	−0.90 **-0.92** −0.75	**1.00**	
$\hat{\pi}_t$	0.10 **0.94** 0.98	−0.05 **0.70** 0.95	0.15 **0.88** 0.97	−0.19 **0.65** 0.92	0.05 **-0.87** −0.95	**1.00**

Note: In each cell $\overset{Emp}{\underset{Ze}{\mathbf{Tr}}}$, (Emp) is the empirical correlation, obtained with original data detrended as in Shimer (2005), i.e. Hodrick Prescott with smoothing parameter 10^5; (Tr) is the theoretical correlation, obtained under $\bar{\pi}$=3.05, when the model is hit only by monetary policy shocks. (Ze) is the theoretical correlation, obtained under $\bar{\pi}$=0, when the model is hit only by monetary policy shocks.

9 Conclusions

The literature has long agreed that the canonical DMP model with search and matching frictions in the labor market can deliver large volatility in unemployment, consistent with US data during the *Great Moderation* period (1985-2005), only if there is at least some wage stickiness. In this paper, I show that the canonical model can deliver nontrivial volatility in unemployment without wage stickiness.

By keeping average US inflation at a small but positive rate, monetary policy may be accountable for the standard deviations of labor market quantities to have achieved those large empirical levels. Solving the Shimer (2005) puzzle, the role of long-run inflation holds even for an economy with flexible wages, as long as it has staggered price setting and search and matching frictions in the labor market. I also find that monetary policy plays an important role in channeling how much of fluctuations are passed into variables in the labor and goods markets.

The major part of the large fluctuations in the labor market are generated by pricing multipliers, which magnify the fluctuations coming from the goods market. Positive trend inflation generates first order effects on the dispersion of relative prices, which quickly increase as trend inflation rises, reducing aggregate output and increasing aggregate disutility to work. This in turn, leads to reduced total surpluses in job matches. Therefore, fluctuations gets larger relative roles in affecting total surpluses, which amplifies volatilities in the labor market.

Another important finding is that the Shimer (2005) puzzle is somehow mitigated when the economy is hit only by preference shocks. When the economy is hit by pure monetary shocks, amplifications are so large that the puzzle is no more.

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A Steady State and Composite Parameters

Let $\bar{\Pi} = 1 + \bar{\pi}$ denote the gross trend inflation, measured at the appropriate frequency. The composite parameters are

Table 5: Composite parameters

$\mu \equiv \frac{\theta}{\theta-1}$	; $\varrho \equiv \frac{1-\beta}{\beta} + \rho$	; $\omega \equiv \frac{1+\nu}{\varepsilon} - 1$	; $\tilde{\omega} \equiv \frac{1}{\varepsilon} - 1$	; $b^u \equiv 1 - (1-b)\,\varrho_{wu}$
$\bar{\alpha} \equiv \alpha \left(\bar{\Pi}\right)^{(\phi-1)(1-\gamma_\pi)}$	; $\bar{\vartheta} \equiv \left(\bar{\Pi}\right)^{(1+\phi\omega)(1-\gamma_\pi)}$	; $\tilde{\vartheta} \equiv \left(\bar{\Pi}\right)^{(1+\phi\tilde{\omega})(1-\gamma_\pi)}$	; $\mathfrak{C}_0 \equiv \frac{1-\bar{\alpha}}{1-\bar{\alpha}\bar{\tilde{\vartheta}}}$	
$\mathfrak{C}_1 \equiv \frac{1-\bar{\alpha}}{1-\bar{\alpha}\bar{\vartheta}}$	; $\mathfrak{C}_2 \equiv \frac{1-\bar{\alpha}\beta}{1-\bar{\alpha}\bar{\vartheta}\beta}$	; $\mathfrak{C}_3 \equiv \frac{1-\alpha}{1-\bar{\alpha}}$	; $\mathfrak{C}_4 \equiv \frac{\mathfrak{C}_1}{\mathfrak{C}_2}$	
$\bar{\kappa} \equiv \frac{\left(1-\bar{\alpha}\bar{\vartheta}\beta\right)(1-\bar{\alpha})}{\bar{\alpha}(1+\phi\omega)}$	; $\bar{\kappa}_{\varpi} \equiv \frac{(1-\bar{\alpha})}{(1+\phi\omega)}$	; $\mathfrak{P}_1 \equiv \frac{(\phi-1)(1-\gamma_\pi)}{(1-\phi_{\mathfrak{i}})(\phi_\pi-1)}$		
$\mathfrak{S}_1 \equiv \frac{\varepsilon}{\chi\mu(1-\iota_{\mathfrak{u}})^\sigma}$	; $\mathfrak{M}_1 \equiv \frac{(1+\nu)(1-b)}{\left(1-\frac{b\varepsilon}{\mu}\right)}$	; $\mathfrak{M}_2 \equiv \frac{\frac{(1-b)}{b^u}\frac{\varepsilon}{\mu}\left((1-\varrho_{wu})-\frac{\mathfrak{C}_4}{\mathfrak{M}_1}\right)}{\left(\frac{\varrho}{\rho}+(1-\rho)\frac{b}{b^u}\frac{\eta}{\rho}\left(\frac{\beta(1-b)\eta}{k}\right)^{\frac{1-a}{a}}(\bar{\mathfrak{s}})^{\frac{1-a}{a}}\right)}$		

Let $\bar{\epsilon}$ and $\overline{\mathcal{A}}$ denote the (arbitrary) positive steady state levels of the preference and technology shocks. The steady state levels $\bar{Y}$, $\bar{C}$, $\bar{\theta}$, $\bar{\mathsf{p}}$, $\bar{\mathsf{q}}$, $\bar{\mathsf{v}}$ and $\bar{\mathsf{n}}$ are then found by solving the non-linear system:

$$\varrho\frac{k}{\bar{\mathsf{q}}} = \frac{(1-b)\varepsilon}{\mu}\left(1-\frac{\mathfrak{C}_4}{\mathfrak{M}_1}\right)\frac{\bar{Y}}{\bar{\mathsf{n}}} - b\,(1-\rho)\,k\bar{\theta} - (1-b)\,w^u \quad ; \ \bar{Y} = \bar{C} + k\bar{\mathsf{v}} \quad ; \ \bar{\mathsf{v}} = \frac{\rho\bar{\mathsf{n}}\bar{\theta}}{\bar{\mathsf{p}}}$$

$$\bar{Y}^\omega\bar{C}^\sigma = \frac{\mathfrak{S}_1(\mathfrak{C}_3)^{\frac{(1+\phi\omega)}{(\phi-1)}}}{\mathfrak{M}_1(\mathfrak{C}_2)}\bar{\epsilon}_{\mathfrak{u}}\overline{\mathcal{A}}^{(1+\omega)} \quad ; \ \bar{\mathsf{n}} = \frac{\bar{\mathsf{p}}}{\rho+(1-\rho)\bar{\mathsf{p}}} \quad ; \ \bar{\mathsf{p}} = \eta\bar{\theta}^{(1-a)} \quad ; \ \bar{\mathsf{q}} = \eta\bar{\theta}^{(-a)}$$

The remaining levels are

$$\bar{\mathcal{P}}^{-\phi(1+\omega)} = \mathfrak{C}_1\left(\mathfrak{C}_3\right)^{\frac{-(1+\phi\omega)}{(\phi-1)}} \quad ; \ \bar{h} = \frac{\bar{H}}{\bar{\mathsf{n}}} \quad ; \ \bar{H} = \left(\frac{\bar{Y}}{\overline{\mathcal{A}}}\right)^{(1+\tilde{\omega})}\bar{\mathcal{P}}_H^{-\phi(1+\tilde{\omega})} \quad ; \ \bar{\mathsf{u}} = \frac{\rho\bar{\mathsf{n}}}{\bar{\mathsf{p}}} \quad ; \ \bar{I} = \frac{\bar{\Pi}}{\beta}$$

$$\bar{\mathcal{P}}_H^{-\phi(1+\tilde{\omega})} = \mathfrak{C}_0\left(\mathfrak{C}_3\right)^{\frac{-(1+\phi\tilde{\omega})}{(\phi-1)}} \quad ; \ \bar{w} = \frac{\varepsilon}{\mu}\frac{\bar{Y}}{\bar{\mathsf{n}}} - \varrho\frac{k}{\bar{\mathsf{q}}} \quad ; \ \bar{\mathfrak{s}} = \frac{\bar{\mathfrak{u}}}{b} = \frac{\bar{\mathfrak{j}}}{(1-b)} = \frac{k}{\beta(1-b)\bar{\mathsf{q}}} \quad ; \ \bar{Q} = \frac{\beta}{\bar{\Pi}}$$

$$\bar{\upsilon} = \frac{\chi}{1+\nu}\left(\frac{\bar{Y}}{\overline{\mathcal{A}}}\right)^{(1+\omega)}\bar{\mathcal{P}}^{-\phi(1+\omega)} \quad ; \ \left(\bar{Y}^n\right)^\omega\left(\bar{C}^n\right)^\sigma = \frac{\mathfrak{S}_1}{\mathfrak{M}_1}\bar{\epsilon}_{\mathfrak{u}}\overline{\mathcal{A}}^{(1+\omega)} \quad ; \ \bar{\mathsf{u}}^e = 1 - \bar{\mathsf{n}}$$

B Calibrating Nuisance Parameters $[\eta, w^u, k, \chi]$

From steady state values $\left[\bar{h}, \varrho_{wu}, \bar{\mathsf{u}}^e, \bar{\theta}^e\right]$, compute:

$$(1)\ \bar{\mathsf{n}} = 1 - \bar{\mathsf{u}}^e \quad (2)\ \bar{\mathsf{u}} = 1 - (1-\rho)\,\bar{\mathsf{n}} \quad (3)\ \bar{\mathsf{p}} = \tfrac{\rho\bar{\mathsf{n}}}{\bar{\mathsf{u}}} \quad (4)\ \bar{\mathsf{p}}^e = \tfrac{\rho\bar{\mathsf{n}}}{\bar{\mathsf{u}}^e}$$

$$(5)\ \bar{\theta} = \tfrac{\bar{\theta}^e}{(1+\bar{\mathsf{p}}^e)} \quad (6)\ \bar{\mathsf{q}} = \tfrac{\bar{\mathsf{p}}}{\bar{\theta}} \quad (7)\ \eta = \bar{\mathsf{q}}\bar{\theta}^a \quad (8)\ \bar{Y} = \overline{\mathcal{A}}\left(\bar{h}\bar{\mathsf{n}}\right)^{1/(1+\tilde{\omega})}\bar{\mathcal{P}}_H^{\phi}$$

Solve the linear system for $[w^u, \bar{w}, k]$:

$$w^u - \varrho_{wu}\bar{w} = 0 \quad ; \quad (1-b)\,w^u - \bar{w} + b\,(1-\rho)\,\bar{\theta}k = -\tfrac{\varepsilon}{\mu}\left(b + \tfrac{(1-b)\mathfrak{C}_4}{\mathfrak{M}_1}\right)\tfrac{\bar{Y}}{\bar{\mathsf{n}}} \quad ; \quad \bar{w} + \tfrac{\varrho}{\bar{\mathsf{q}}}k = \tfrac{\varepsilon}{\mu}\tfrac{\bar{Y}}{\bar{\mathsf{n}}}$$

Compute $\bar{\mathcal{Y}} = \bar{C} = \bar{Y} - k\bar{\mathsf{v}}$ and $\chi = \frac{\varepsilon}{\mu(1-\iota_{\mathfrak{u}})^{\sigma}}\frac{(\mathfrak{C}_{3,t})^{\frac{(1+\phi\omega)}{(\phi-1)}}}{\mathfrak{M}_1(\mathfrak{C}_{2,t})}\left(\bar{\epsilon}_{\mathfrak{u}}\right)\left(\overline{\mathcal{A}}\right)^{(1+\omega)}\left(\bar{Y}\right)^{-\omega}\left(\bar{C}\right)^{-\sigma}$.

C Calibration Set

Following, I summarize the calibration set. See Section 7 for more details.

Table 6: Calibration Set

Parameter		Value	Source
Subjective discount factor	β	0.99	Cooley and Prescott (1995)
Technology shock inertia	$\phi_{\mathfrak{a}}$	0.95	Cooley and Prescott (1995)
Elasticity to hours (production function)	ε	0.64	Cooley and Prescott (1995)
Matching elasticity to unemployment	a	0.50	Shimer (2005)
Workers' bargaining power	b	a	Hosios (1990)
Elasticity of substitution leading to price markup μ=1.17	ϕ	7	Ravenna and Walsh (2011)
Reciprocal of intertemp elastic substitut	σ	1.47	Smets and Wouters (2007)
Habit persistence	$\iota_{\mathfrak{u}}$	0.68	Smets and Wouters (2007)
Reciprocal of the Frisch elasticity	ν	2.30	Smets and Wouters (2007)
Price stickiness	α	0.73	Smets and Wouters (2007)
Price indexation	γ_{π}	0.21	Smets and Wouters (2007)
Taylor rule inertia (t-1)	ϕ_{i1}	1.05	Coibion and Gorodnichenko (2011)
Taylor rule inertia (t-2)	ϕ_{i2}	-0.13	Coibion and Gorodnichenko (2011)
Taylor rule response to expected inflation	ϕ_{π}	2.20	Coibion and Gorodnichenko (2011)
Taylor rule response to GDP growth	ϕ_{gy}	1.56	Coibion and Gorodnichenko (2011)
Taylor rule response to GDP gap[42]	ϕ_y	0.02	Coibion and Gorodnichenko (2011)
Separation rate	ρ	0.147	Observed Average (Section 3)
Trend inflation level	$\bar{\pi}$	3.05	Observed Average (Section 3)

Setting Nuisance Parameters $[\eta, w^u, k, \chi]$

Variable at Steady State		Value	Source
Aggregate hours per worker	$\bar{h}$	1	Author's Discretion
Replacement ratio	ϱ_{wu}	0.40	Ravenna and Walsh (2008)
End-of-period unemployment rate	$\bar{u}^e$	0.057	Observed Average (Section 3)
End-of-period tightness rate	$\bar{\theta}^e$	1.85	Observed Average (Section 3)

D The log-linearized model

For any variable $\mathcal{X}_t$, the hatted representation $\widehat{\varkappa}_t \equiv \log\left(\mathcal{X}_t/\bar{\mathcal{X}}\right)$ is its log-deviation from its steady state level $\bar{\mathcal{X}}$.

1. Beveridge equations:

$$\begin{array}{lll}
\widehat{\mathsf{n}}_t = (1-\rho)\,\widehat{\mathsf{n}}_{t-1} + \rho\widehat{\mathsf{m}}_t & ;\ \widehat{\mathsf{u}}_t = -\frac{(1-\rho)\bar{\mathsf{p}}}{\rho}\widehat{\mathsf{n}}_{t-1} & ;\ \widehat{\mathsf{p}}_t = (1-a)\,\widehat{\theta}_t \\
\widehat{\mathsf{m}}_t = (1-a)\,\widehat{\theta}_t + \widehat{\mathsf{u}}_t & ;\ \widehat{\mathsf{v}}_t = \widehat{\theta}_t + \widehat{\mathsf{u}}_t & ;\ \widehat{\mathsf{q}}_t = -a\widehat{\theta}_t \\
\widehat{\mathsf{v}}_t^e = (1-\rho)\,\widehat{\mathsf{u}}_t^e - \frac{1}{a}\widehat{\mathsf{q}}_t^f & ;\ \widehat{\mathsf{p}}_t^e = \widehat{\mathsf{m}}_t - \widehat{\mathsf{u}}_t^e & ;\ \widehat{\theta}_t^e = \widehat{\mathsf{v}}_t^e - \widehat{\mathsf{u}}_t^e \\
\widehat{\mathsf{u}}_t^e = -\frac{\bar{\mathsf{p}}}{\rho}\widehat{\mathsf{n}}_t & ;\ \widehat{\mathsf{q}}_t^e = \widehat{\mathsf{m}}_t - \widehat{\mathsf{v}}_t^e & ;\ \widehat{\mathsf{q}}_t = \widehat{\mathsf{q}}_{t-1}^f
\end{array}$$

2. Aggregate Surplus and Aggregate wage and job creation curves:

$$\begin{array}{rl}
(1-\iota_{\mathrm{u}})\,\hat{C}_t^{ad} = & \hat{C}_t - \iota_{\mathrm{u}}\hat{C}_{t-1} \qquad ; \qquad \hat{u}_t' = \hat{\epsilon}_{\mathrm{u},t} - \sigma\hat{C}_t^{ad} \\
\bar{w}\hat{w}_t = & b\left[\frac{\varepsilon\bar{Y}}{\mu\bar{\mathsf{n}}}\left(\hat{Y}_t - \hat{\mathsf{n}}_t\right) - \frac{(1-\rho)\bar{\mathsf{p}}}{a}\frac{k}{\bar{\mathsf{q}}}\widehat{\mathsf{q}}_t^f\right] + (1-b)\,\frac{(1+\nu)\bar{\upsilon}}{\bar{u}'\bar{\mathsf{n}}}\left(\hat{\upsilon}_t - \hat{u}_t' - \hat{\mathsf{n}}_t\right) \\
\frac{k}{\bar{\mathsf{q}}}\hat{\mathsf{q}}_t^f = & \beta E_t\left[(1-\rho)\,\frac{k}{\bar{\mathsf{q}}}\hat{\mathsf{q}}_{t+1}^f - \frac{\varepsilon\bar{Y}}{\mu\bar{\mathsf{n}}}\left(\hat{Y}_{t+1} - \hat{\mathsf{n}}_{t+1}\right) + \bar{w}\hat{w}_{t+1} + \frac{k}{\beta\bar{\mathsf{q}}}\left(\hat{\imath}_t - \hat{\pi}_{t+1}\right)\right] \\
(1-b)\,\bar{\mathfrak{s}}\hat{\mathfrak{s}}_t = & -(1-\rho)\,\frac{k}{\bar{\mathsf{q}}}\hat{\mathsf{q}}_t^f + \frac{\varepsilon\bar{Y}}{\mu\bar{\mathsf{n}}}\left(\hat{Y}_t - \hat{\mathsf{n}}_t\right) - \bar{w}\hat{w}_t
\end{array}$$

3. Aggregates and productivity:

$$\begin{array}{ll}
\hat{\upsilon}_t = (1+\omega)\left(\hat{Y}_t - \widehat{\mathcal{A}}_t - \phi\hat{\mathcal{P}}_t\right) & ;\ \hat{\mathcal{P}}_t = \bar{\alpha}\bar{\vartheta}\hat{\mathcal{P}}_{t-1} - \frac{\left(\bar{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) \\
\hat{H}_t = (1+\tilde{\omega})\left(\hat{Y}_t - \widehat{\mathcal{A}}_t - \phi\hat{\mathcal{P}}_{Ht}\right) & ;\ \hat{\mathcal{P}}_{Ht} = \bar{\alpha}\tilde{\vartheta}\hat{\mathcal{P}}_{Ht-1} - \frac{\left(\tilde{\vartheta}-1\right)\bar{\alpha}}{(1-\bar{\alpha})}\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) \\
\hat{h}_t = \hat{H}_t - \widehat{\mathsf{n}}_t & ;\ \hat{\wp}_t = \hat{Y}_t - \hat{H}_t
\end{array}$$

4. IS curve and market clearing identity:

$$\hat{x}_t^c = E_t\hat{x}_{t+1}^c - \frac{1}{\sigma}\left(\hat{\imath}_t - E_t\hat{\pi}_{t+1} - \hat{r}_t^n\right) \quad ;\ \hat{Y}_t = \mathfrak{s}_c\hat{C}_t + (1-\mathfrak{s}_c)\,\hat{\mathsf{v}}_t^e$$

where $\hat{x}_t^c \equiv \frac{\hat{C}_t^{ad}}{\hat{C}_t^{ad^n}}$, $\mathfrak{s}_c \equiv \frac{\bar{C}}{\bar{Y}}$ and $\hat{r}_t^n \equiv \left(\hat{\epsilon}_{\mathrm{u},t} - E_t\hat{\epsilon}_{\mathrm{u},t+1}\right) - \sigma\left(\hat{C}_t^n - E_t\hat{C}_{t+1}^n\right)$ is the real interest rate under flexible prices.

5. Labor-augmented Generalized New Keynesian Phillips Curve (LaGNKPC):

$$\begin{array}{rl}
\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) = & \beta E_t\left(\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{ind}\right) + \bar{\kappa}\left(\omega\hat{x}_t + \sigma\hat{x}_t^c\right) + \left(\bar{\vartheta}-1\right)\bar{\kappa}_{\varpi}\beta E_t\hat{\varpi}_{t+1} + \hat{\varsigma}_t \\
\hat{\varpi}_t = & \bar{\alpha}\bar{\vartheta}\beta E_t\hat{\varpi}_{t+1} + \phi\,(1+\omega)\left(\hat{\pi}_t - \hat{\pi}_t^{ind}\right) + \left(1-\bar{\alpha}\bar{\vartheta}\beta\right)\left(\omega\hat{x}_t + \sigma\hat{x}_t^c\right) \\
+ & \left(\hat{x}_t - \hat{x}_{t-1}\right) - \sigma\left(\hat{x}_t^c - \hat{x}_{t-1}^c\right) \\
\hat{\varsigma}_t = & \bar{\alpha}\bar{\vartheta}\beta E_t\hat{\varsigma}_{t+1} + \left(\bar{\vartheta}-1\right)\beta E_t\hat{\xi}_{t+1} \\
\hat{\xi}_t = & \bar{\kappa}_{\varpi}\,(1+\omega)\left[\left(\hat{Y}_t^n - \hat{Y}_{t-1}^n\right) - \left(\widehat{\mathcal{A}}_t - \widehat{\mathcal{A}}_{t-1}\right)\right]
\end{array}$$

Under flexible prices, use: $\omega\hat{Y}_t^n + \sigma\hat{C}_t^{ad^n} = \hat{\epsilon}_{\mathrm{u},t} + (1+\omega)\,\widehat{\mathcal{A}}_t$

6. Monetary policy: $\hat{\imath}_t = \phi_i\hat{\imath}_{t-1} + (1-\phi_i)\left[\phi_\pi E_t\hat{\pi}_{t+1} + \phi_{gy}\left(\hat{y}_t - \hat{y}_{t-1}\right)\right] + \hat{\epsilon}_{\mathrm{i},t}$

7. Technology Shock: $\widehat{\mathcal{A}}_t = \phi_{\mathfrak{a}}\widehat{\mathcal{A}}_{t-1} + \hat{\epsilon}_{\mathfrak{a},t}$

E Proof of Proposition 2

I start the proof by showing that, in this model economy, the dynamics of all labor market variables are completely determined when the dynamics of job openings v_t^e is set. This result is important for showing a strong result on monetary policy trade-off by the end of this section.

Lemma 1 *Provided that the initial state* $(\mathsf{n}_0, \mathsf{v}_0^e)$ *at* $t = 0$ *is known, once the dynamics of end-of period job openings* v_t^e *is set, the path of all remaining labor market quantities are completely determined.*

Proof. Once v_{t-1}^e is set, total openings available at the beginning of period t is $\mathsf{v}_t = \mathsf{v}_{t-1}^e$. Since beginning-of-period unemployment rate is $\mathsf{u}_t = 1 - (1-\rho)\,\mathsf{n}_{t-1}$, the number of matches during this period is set $\mathsf{m}_t = \eta \mathsf{v}_t^{1-a}\mathsf{u}_t^a$. Therefore, the dynamics of aggregate measures of employed and unemployed workers are completely determined once the dynamics of v_t^e is set:

$$\mathsf{n}_t = (1-\rho)\,\mathsf{n}_{t-1} + \eta \mathsf{v}_t^{1-a}\left[1 - (1-\rho)\,\mathsf{n}_{t-1}\right]^a \quad ; \ \mathsf{u}_t = 1 - (1-\rho)\,\mathsf{n}_{t-1} \quad ; \ \mathsf{u}_t^e = 1 - \mathsf{n}_t$$

The remaining labor markets quantities are then directly determined as $\mathsf{m}_t = \eta \mathsf{v}_t^{1-a}\mathsf{u}_t^a$, $\theta_t \equiv \mathsf{v}_t/\mathsf{u}_t$, $\mathsf{p}_t \equiv \mathsf{m}_t/\mathsf{u}_t$ and $\mathsf{q}_t \equiv \mathsf{m}_t/\mathsf{v}_t$. ■

Lemma 1 implies that v_t^e has the flavor of a sufficient statistics to summarize the dynamics in the labor market. Therefore, in the following results, I only refer to v_t^e.

Corollary 3 *Provided that the initial state* $(\mathsf{n}_0, \mathsf{v}_0^e)$ *at* $t = 0$ *is known, once the dynamics of aggregate consumption* C_t *and output* Y_t *are set, the path of all labor market quantities are completely determined.*

Proof. Once C_t and Y_t are known, the aggregate market clearing condition determines $\mathsf{v}_t^e = \frac{1}{k}(Y_t - C_t)$. Therefore, using Proposition 1, all remaining labor market quantities are also completely determined. ■

Now, I generalize Alves (2014) main result on the lack of divine coincidence[43] in the standard New-Keynesian model. I show that under general circumstances monetary policy faces a trade-off in simultaneously stabilizing the (gross) inflation rate $\Pi_t \equiv 1 + \pi_t$, (gross) output gap $X_t \equiv 1 + x_t$, (gross) consumption gap $X_t^c \equiv 1 + x_t^c$, and all remaining labor market gaps, summarized by $X_t^v \equiv 1 + x_t^v$. Indeed, stabilizing all of those variables requires that monetary authority stabilizes the inflation rate at exactly zero percent or firms follow an exact full indexation mechanism $(\gamma_\pi = 1)$ when not optimally resetting their prices. Otherwise, monetary policy is able to stabilize at most only one of them.

The following proposition states the policy trade-off results for the nonlinear model. Therefore, they are robust to any log-linearized approximation and do not depend on assuming that distortions are sufficiently close to zero.

Proposition 2 *With staggered price setting* $(\alpha > 0)$ *and partial indexation* $(\gamma_\pi \neq 1)$, *there exists a trade-off in stabilizing the inflation rate* π_t, *output gap* x_t, *consumption gap* x_t^c, *and labor market gap* x_t^v *whenever the monetary policy chooses a non-zero rate for stabilizing* π_t: *it is impossible for* π_t *and anyone of the three gaps (i.e.* x_t, x_t^c *or* x_t^v*) to simultaneously have zero variance if* π_t *is stabilized at* $\bar{\pi} \neq 0$. *If* $\bar{\pi} = 0$ *or* $\gamma_\pi = 1$, *there is no stabilization trade-off and the divine coincidence holds.*

Proof. Assume, with no loss of generality, that the monetary authority stabilizes the (gross) inflation rate at a discretionary level $\Pi_t = \bar{\Pi}$, $\forall t$. It implies that $\Pi_t^{ind} = \bar{\Pi}^{\gamma_\pi}$, $\forall t$.

[43] See Blanchard and Gali (2007) for details on the divine coincidence property.

From (5) and (6), the optimal relative resetting price p_t^*/P_t and the ratio N_t/D_t remain at their steady state levels:

$$\frac{p_t^*}{P_t} = \left(\frac{1-\alpha}{1-\bar{\alpha}}\right)^{\frac{1}{(\theta-1)}} \quad ; \quad \frac{N_t}{D_t} = (\mathfrak{C}_3)^{\frac{(1+\phi\omega)}{(\phi-1)}} \tag{15}$$

where, again, $\mathfrak{C}_3 \equiv \frac{1-\alpha}{1-\bar{\alpha}}$.

In what follows, I show that there is a contradiction whenever anyone of the three gaps is also chosen to be stabilized after inflation is fixed at $\bar{\pi} \neq 0$ and $\gamma_\pi \neq 1$. As I show, the contradiction is only absent in the particular cases of $\bar{\pi} = 0$ or $\gamma_\pi = 1$.

(**A**) Suppose, by contradiction, that X_t^c is stabilized at level $\bar{X}^c$, $\forall t$. It implies that $(C_t - \iota_{\mathrm{u}} C_{t-1}) = \bar{X}^c \left(C_t^n - \iota_{\mathrm{u}} C_{t-1}^n\right)$, whose solution is $C_t = \bar{X}^c C_t^n$.[44] Therefore, the aggregate consumption and the stochastic discount factor evolve, in equilibrium, according to

$$C_t = \bar{X}^c C_t^n \quad ; \; Q_t = \frac{\beta}{\bar{\Pi}} \left(\frac{\mathrm{u}_{\mathrm{u},t}}{\mathrm{u}_{\mathrm{u},t-1}}\right) \left(\frac{C_t^{ad^n}}{C_{t-1}^{ad^n}}\right)^{-\sigma}$$

where $C_t^{ad^n} \equiv C_t^n - \iota_{\mathrm{u}} C_{t-1}^n$.

Plugging the last result into (5), we conclude that N_t and D_t evolve according to

$$N_t = \left(1 - \frac{b\varepsilon}{\mu}\right) \left(\bar{X}^c\right)^\sigma (X_t)^\omega + \bar{\alpha}\vartheta\beta E_t \left[\left(\frac{\epsilon_{\mathrm{u},t+1}}{\epsilon_{\mathrm{u},t}}\right) \left(\frac{C_{t+1}^{ad^n}}{C_t^{ad^n}}\right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t}\right) N_{t+1}\right] \tag{16}$$

$$D_t = \left(1 - \frac{b\varepsilon}{\mu}\right) + \bar{\alpha}\beta E_t \left[\left(\frac{\epsilon_{\mathrm{u},t+1}}{\epsilon_{\mathrm{u},t}}\right) \left(\frac{C_{t+1}^{ad^n}}{C_t^{ad^n}}\right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t}\right) D_{t+1}\right] \tag{17}$$

Equations (15) and (16) imply that:

$$D_t = \left(1 - \frac{b\varepsilon}{\mu}\right) (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}} \left(\bar{X}^c\right)^\sigma (X_t)^\omega + \bar{\alpha}\vartheta\beta E_t \left[\left(\frac{\epsilon_{\mathrm{u},t+1}}{\epsilon_{\mathrm{u},t}}\right) \left(\frac{C_{t+1}^{ad^n}}{C_t^{ad^n}}\right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t}\right) D_{t+1}\right] \tag{18}$$

while (17) implies that:

$$\bar{\alpha}\beta E_t \left[\left(\frac{\epsilon_{\mathrm{u},t+1}}{\epsilon_{\mathrm{u},t}}\right) \left(\frac{C_{t+1}^{ad^n}}{C_t^{ad^n}}\right)^{-\sigma} \left(\frac{Y_{t+1}}{Y_t}\right) D_{t+1}\right] = D_t - \left(1 - \frac{b\varepsilon}{\mu}\right) \tag{19}$$

Using the last two results, I obtain a simpler relation between D_t and X_t:

$$(\vartheta - 1) D_t = \left(1 - \frac{b\varepsilon}{\mu}\right) \left(\vartheta - (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}} \left(\bar{X}^c\right)^\sigma (X_t)^\omega\right) \tag{20}$$

Note that, as the (gross) inflation rate remains fixed at $\Pi_t = \bar{\Pi}$, υ_t and H_t fast converge to their static equilibrium levels:

$$\upsilon_t = \frac{\chi}{1+\nu} (\mathfrak{C}_1) (\mathfrak{C}_3)^{\frac{-\phi(1+\omega)}{(\phi-1)}} \left(\frac{Y_t}{\mathcal{A}_t}\right)^{(1+\omega)} \quad ; \; H_t = (\mathfrak{C}_0) (\mathfrak{C}_3)^{\frac{-\phi(1+\tilde{\omega})}{(\phi-1)}} \left(\frac{Y_t}{\mathcal{A}_t}\right)^{(1+\tilde{\omega})}$$

where, again, $\mathfrak{C}_0 \equiv \frac{1-\bar{\alpha}}{1-\bar{\alpha}\bar{\vartheta}}$ and $\mathfrak{C}_1 \equiv \frac{1-\bar{\alpha}}{1-\bar{\alpha}\vartheta}$.

(**i**) Consider the case in which π_t is stabilized at a non-zero level $\bar{\pi} \neq 0$ and

[44] Note that the condition for habit-adjusted output gap stabilization implies $(1 - \iota_{\mathrm{u}} L) C_t = \bar{X}^c (1 - \iota_{\mathrm{u}} L) C_t^n$, where L is the lag operator. Therefore, the solution for $\iota_{\mathrm{u}} < 1$ is $C_t = \bar{X}^c C_t^n$.

indexation is partial ($\gamma_\pi \neq 1$). This implies that $\bar{\Pi} \neq 1$, $\vartheta \neq 1$ and $\bar{\alpha} \neq \alpha$. Plugging (20) into (17) and using $X_t \equiv \frac{Y_t}{Y_t^n}$, I obtain an equation that completely determines the dynamics of Y_t as a function of exogenous shocks:[45]

$$1 = (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}} \left(\bar{X}^c\right)^\sigma \left(\frac{Y_t}{Y_t^n}\right)^\omega + \bar{\alpha}\beta E_t\left[\left(\frac{\epsilon_{\mathrm{u},t+1}}{\epsilon_{\mathrm{u},t}}\right)\left(\frac{C_{t+1}^{ad^n}}{C_t^{ad^n}}\right)^{-\sigma}\left(\frac{Y_{t+1}}{Y_t}\right)\left(\vartheta - (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}}\left(\bar{X}^c\right)^\sigma\left(\frac{Y_{t+1}}{Y_{t+1}^n}\right)^\omega\right)\right]$$

Finally, the dynamics of vacancy opening is determined by the market clearing condition, i.e. $\mathsf{v}_t^e = \frac{1}{k}\left(Y_t - \bar{X}^c C_t^n\right)$. It is important to note that the dynamics of Y_t and v_t^e, as determined by the last results, are completely independent of labor market parameters.

Using Lemma 1, the dynamics of all remaining labor market quantities are then completely determined. Therefore, I compute the ratio $\frac{\upsilon_t/\mathsf{n}_t}{u_t'}$ as follows:

$$\frac{\upsilon_t/\mathsf{n}_t}{u_t'} = \frac{\chi}{(1+\nu)}(\mathfrak{C}_1)(\mathfrak{C}_3)^{\frac{-\phi(1+\omega)}{(\phi-1)}}\left(\frac{1}{\mathcal{A}_t}\right)^{(1+\omega)}\left(\frac{1}{\epsilon_{\mathrm{u},t}}\right)\left(\bar{X}^c C_t^{ad^n}\right)^\sigma (Y_t)^{(1+\omega)}\frac{1}{\mathsf{n}_t}$$

The aggregate salary and job creation curves, on the other hand, imply an independent and different relation between Y_t, labor market variables and exogenous shocks:

$$\frac{k}{\mathsf{q}_t^f} = E_t Q_{t+1}^\pi\left[(1-b)\left(\frac{\varepsilon}{\mu}\frac{Y_{t+1}}{\mathsf{n}_{t+1}} - (1+\nu)\frac{\upsilon_{t+1}/\mathsf{n}_{t+1}}{u_{t+1}'} - w^u\right) - b(1-\rho)k\theta_{t+1}^f + (1-\rho)\frac{k}{\mathsf{q}_{t+1}^f}\right]$$

Hence, the last result describes a contradiction, for it implies that Y_t and labor market variables should follow dynamics completely different from the ones previously determined. Therefore, it is impossible for x_t^c and π_t to simultaneously have zero variance if π_t is stabilized at $\bar{\pi} \neq 0$.

(ii) Consider the case in which π_t is stabilized at the zero level $\bar{\pi} = 0$, or firms follow an exact full indexation mechanism ($\gamma_\pi = 1$). This implies that $\bar{\Pi} = 1$, $\vartheta = 1$ and $\bar{\alpha} = \alpha$. Using (18) and (19), it is easy to verify that there is no contradiction in assuming that the consumption gap can be stabilized when the inflation rate is stabilized at $\bar{\pi} = 0$. In this case, there is no stabilization trade-off and the divine coincidence holds. Moreover, stabilizing the consumption gap automatically stabilizes the output gap at the zero level $\bar{\pi} = 0$, for (20) implies that X_t must remains constant at $X_t = \bar{X} = \left(\bar{X}^c\right)^{-\frac{\sigma}{\omega}}$.

Note that the stabilization of the consumption and output gaps does not necessarily ensure stabilization of labor market gaps. The the market clearing condition implies that $\mathsf{v}_t^e = \frac{1}{k}\left(\bar{X}Y_t^n - \bar{X}^c C_t^n\right)$, which is not proportional to its level under flexible prices $\mathsf{v}_t^{en} = \frac{1}{k}\left(Y_t^n - C_t^n\right)$, i.e. $X_t^v = \bar{X}\varsigma_t^n - \bar{X}^c\left(\varsigma_t^n - 1\right)$, where $\varsigma_t^n \equiv \frac{Y_t^n}{Y_t^n - C_t^n}$. Stabilization of the labor market gap x_t^v requires that monetary policy additionally sets $\bar{X}^c$ to unity. In this case, we have automatic stabilization of the inflation rate and the three gaps, i.e. $\bar{x} = \bar{x}^c = \bar{x}^v = 0$, once inflation is stabilized at $\bar{\pi} = 0$.

(B) Suppose, by contradiction, that X_t is stabilized at level $\bar{X}$, $\forall t$. It implies that the aggregate output and the stochastic discount factor evolve, in equilibrium, according to

$$Y_t = \bar{X}Y_t^n \quad ; \; Q_t = \frac{\beta}{\bar{\Pi}}\left(\frac{\mathrm{u}_{\mathrm{u},t}}{\mathrm{u}_{\mathrm{u},t-1}}\right)\left(\frac{C_t^{ad}}{C_{t-1}^{ad}}\right)^{-\sigma}$$

where $C_t^{ad} \equiv C_t - \iota_{\mathrm{u}} C_{t-1}$.

[45]Recall that $\epsilon_{\mathrm{u},t}$ is the exogenous preference shock, while C_t^n and Y_t^n are functions of the exogenous preference $\epsilon_{\mathrm{u},t}$ and technology $\mathcal{A}_t$ shocks.

(i) Consider the case in which π_t is stabilized at a non-zero level $\bar{\pi} \neq 0$ and indexation is partial ($\gamma_\pi \neq 1$). Paralleling the steps done in item ($\mathbf{A} - \mathbf{i}$), it is easy to derive an equation that completely determines the dynamics of C_t as a function of exogenous shocks:

$$1 = \left(\bar{X}\right)^{\omega} (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}} \left(\frac{C_t^{ad}}{C_t^{ad^n}}\right)^{\sigma} + \bar{\alpha}\beta E_t\left[\left(\frac{\mathfrak{u}_{\mathfrak{u},t+1}}{\mathfrak{u}_{\mathfrak{u},t}}\right)\left(\frac{C_{t+1}^{ad}}{C_t^{ad}}\right)^{-\sigma}\left(\frac{Y_{t+1}^n}{Y_t^n}\right)\left(\vartheta - (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}}\left(\bar{X}\right)^{\omega}\left(\frac{C_{t+1}^{ad}}{C_{t+1}^{ad^n}}\right)^{\sigma}\right)\right]$$

Again, the dynamics of vacancy opening is determined by the market clearing condition, i.e. $\mathsf{v}_t^e = \frac{1}{k}\left(\bar{X}Y_t^n - C_t\right)$, which in turn implies that the dynamics of all remaining labor market quantities are then completely determined.

Similarly as before, the equation resulting from the aggregate salary and job creation curves describes a contradiction, for it now implies that C_t and labor market variables should follow dynamics completely different from the ones previously determined. Therefore, it is impossible for x_t and π_t to simultaneously have zero variance if π_t is stabilized at $\bar{\pi} \neq 0$.

(ii) Consider the case in which π_t is stabilized at the zero level $\bar{\pi} = 0$, or firms follow an exact full indexation mechanism ($\gamma_\pi = 1$). Following the same steps as those of item ($\mathbf{A} - \mathbf{ii}$), we can easily show that there is no stabilization trade-off and the divine coincidence holds in this case. Moreover, automatic stabilization of the inflation rate and the three gaps, i.e. $\bar{x} = \bar{x}^c = \bar{x}^v = 0$, requires that monetary policy additionally sets $\bar{X}$ to unity once inflation is stabilized at $\bar{\pi} = 0$.

(C) Suppose, by contradiction, that X_t^v is stabilized at level $\bar{X}^v$, $\forall t$. It implies that the aggregate vacancy openings and the stochastic discount factor evolve, in equilibrium, according to

$$\mathsf{v}_t^e = \bar{X}^v \mathsf{v}_t^{en} \quad ; \; Q_t = \frac{\beta}{\bar{\Pi}}\left(\frac{\mathfrak{u}_{\mathfrak{u},t}}{\mathfrak{u}_{\mathfrak{u},t-1}}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$$

(i) Consider the case in which π_t is stabilized at a non-zero level $\bar{\pi} \neq 0$ and indexation is partial ($\gamma_\pi \neq 1$). Paralleling the steps done in item ($\mathbf{A} - \mathbf{i}$), I find now an equation similar to the ones previously derived in items ($\mathbf{A} - \mathbf{i}$) and ($\mathbf{B} - \mathbf{i}$):

$$1 = (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}} \left(\frac{C_t^{ad}}{C_t^{ad^n}}\right)^{\sigma}\left(\frac{Y_t}{Y_t^n}\right)^{\omega} + \bar{\alpha}\beta E_t\left[\left(\frac{\mathfrak{u}_{\mathfrak{u},t+1}}{\mathfrak{u}_{\mathfrak{u},t}}\right)\left(\frac{C_{t+1}^{ad}}{C_t^{ad}}\right)^{-\sigma}\left(\frac{Y_{t+1}}{Y_t}\right)\left(\vartheta - (\mathfrak{C}_3)^{\frac{-(1+\phi\omega)}{(\phi-1)}}\left(\frac{C_{t+1}^{ad}}{C_{t+1}^{ad^n}}\right)^{\sigma}\left(\frac{Y_{t+1}}{Y_{t+1}^n}\right)^{\omega}\right)\right]$$

This equation must be solved with the market clearing condition, i.e. $Y_t = C_t + k\bar{X}^v \mathsf{v}_t^{en}$, to uniquely determine the dynamics of C_t and Y_t as a function of exogenous shocks. Again, Lemma 1 implies that the dynamics of all remaining labor market quantities are then completely determined.

Similarly as before, the equation resulting from the aggregate wage and job creation curves describes a contradiction, for it now implies that C_t, Y_t and labor market variables should follow dynamics completely different from the ones previously determined. Therefore, it is impossible for x_t^v and π_t to simultaneously have zero variance if π_t is stabilized at $\bar{\pi} \neq 0$.

(ii) Consider the case in which π_t is stabilized at the zero level $\bar{\pi} = 0$, or firms follow an exact full indexation mechanism ($\gamma_\pi = 1$). Following the same steps as those of item ($\mathbf{A} - \mathbf{ii}$), we can easily show that there is no stabilization trade-off and the divine coincidence holds in this case. Again, automatic stabilization of the inflation rate and the three gaps, i.e. $\bar{x} = \bar{x}^c = \bar{x}^v = 0$, requires that monetary policy additionally sets $\bar{X}^v$ to unity once inflation is stabilized at $\bar{\pi} = 0$. ■