

# Running from Liquidity Risk\*

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## **Abstract**

We analyse bank runs under uncertainty over asset liquidity, and show how the risk of losses upon premature loan termination produces an unique run equilibrium, where inefficient runs occur even under minimal fundamental risk. The model refines the bank run framework by introducing actual bankruptcy rules on mandatory stay, such that default occurs before all assets are paid out. Thus less liquid assets are not available for running depositors. As a result, asset liquidity risk has a concave effect on run incentives, unlike fundamental risk. Runs are rare when asset liquidity is abundant, become more frequent as it falls but decrease again under very low asset liquidity. The optimal social choice limits inessential runs by offering a higher rollover yield. However, the private choice minimizes funding costs, tolerating more frequent runs.

Key words: liquidity risk, bank runs, global games, demandable debt, mandatory stay.

JEL classification: .

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# 1 Introduction

The 2002-2007 credit boom was largely driven by real estate lending funded by very short term debt. The growing maturity mismatch was supported by the belief that loan securitization via asset backed securities (ABS) had made bank assets more liquid. Yet once some credit risk became apparent, ABS assets rapidly became illiquid, creating solvency concerns and ultimately propagating runs across intermediaries (Brunnermeier, 2009). With hindsight, ABS prices at the peak of the crisis fell way too low relative to their ultimate performance. Only a decisive intervention by central banks avoided a large scale of fire sales that would have devastated bank balance sheets.

This experience has led to sharper scrutiny of the degree of bank liquidity mismatch. While maturity transformation is at heart of bank intermediation, they create the possibility of self fulfilling runs (Diamond and Dybvig 1983, Goldstein and Pauzner 2005) under fundamental asset risk. In this paper we study whether there can also be a distinct effect of asset liquidity risk, defined as nonfundamental price risk associated with temporary market conditions. We study a simple context where all agents are risk neutral, and demandable debt serves for contingent transaction needs (as in Stein, 2012) rather than for an extreme liquidity demand by early-dying consumers (Diamond and Dybvig 1983). We establish an unique equilibrium where runs are driven by uncertainty over early liquidation value of bank assets, even if fundamental risk is arbitrarily small. This complements the result on inessential runs by Goldstein and Pauzner (2005), and relates it to the emerging literature on how market conditions create liquidity risk. This may occur when there is too little cash in the market to arbitrage mispricing, due to leverage and maturity mismatch choices by financial intermediaries (Brunnermeier and Petersen, 2009; Gromb and Vayanos, 2002).<sup>1</sup> Market participants may also suddenly have limited resources due to increased adverse selection or counterparty risk (Krishnamurthy 2010, Gorton and Ordoñez 2014). In general, liquidity risk may be infrequent but cause sharp losses,

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<sup>1</sup>Duffie and Strulovici (2009) study how a gradual flow of arbitrage capital in a search context causes temporary trading opportunities.

even when fundamental value risk is minimal.

To characterize precisely how asset liquidity affects run incentives, we introduce an accurate process of bank default. Traditional bank run models assume that in a run all assets are sold immediately to satisfy withdrawals, so at default all those who did not run receive nothing. In reality, a bank is declared insolvent as soon as its liquid reserves are depleted and no longer can immediately meet on-demand withdrawals. At that point a mandatory stay is triggered, interrupting asset sales and initiating an orderly liquidation process.<sup>2</sup> As a result of this efficient legal provision, illiquid assets are shared also with those who did not run.

This feature contribute to a surprising concave effect of liquidity risk on run frequency, quite unlike fundamental risk. Abundant liquid assets encourages rollover, as it supports confidence that the bank will be able to repay withdrawals. As asset liquidity falls, the appeal of running increases. However, because of mandatory stay the relative payoff of rollover also rises, since illiquid assets are not paid out ahead of default. Once liquid assets are very scarce, this effect dominates and the frequency of runs starts to decrease. Thus in equilibrium there is a concave, inverted U-shaped relation between asset liquidity and run frequency.<sup>3</sup>

Banks traditionally have held liquid buffers to back demandable debt, and most of their assets were short term. However, in recent decades the evolution of bank balance sheets has lead to increased liquidity risk, through the expansion of long term mortgage lending and the rise of secured funding.

Our approach considers the case of a single intermediary. An analysis extended to more banks could better describe the overall source of liquidity risk. It is already recognized that fire sales by some intermediary may cause financial constraints to become binding for others, and lead to further runs.

In the model there is a role for regulation, as the bank maximizes profits rather than consumer surplus. In a context of risk neutrality there is no need for liquidity insurance, so

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<sup>2</sup>Bankruptcy law was introduced precisely to solve the externality created when creditors grab or liquidate assets in an uncoordinated fashion, destroying value.

<sup>3</sup>Note that in equilibrium intermediaries with no liquid assets could never be funded with demandable debt, as they could not provide transaction services.

social welfare is here served by minimizing the chance of inessential runs. It is easy to show that as a higher rollover premium reduces the frequency of runs, a social planner would choose to offer large rents to those who do not withdraw. However, in general the private choice will not offer large rollover rents in order to contain average funding costs, thus inducing more frequent runs than socially optimal.<sup>4</sup> In our simple set up demandable debt and minimum cash reserves are justified by contingent transaction needs. Introducing a strong need for liquidity insurance under the extreme time preferences of Diamond and Dybvig (1983) would considerably complicate the analysis while not significantly altering the logic of our analysis.

The model adopts the framing for analysing unique run equilibria based on Goldstein and Pauzner (2005), and relies on their solution concept. Intuitively, adding interim asset liquidity risk increases the chance that depositors coordinate on a self protective but inefficient run. While we focus on liquidity risk, some (arbitrarily small) amount of fundamental risk is essential to obtain our result, as it ensures existence of a lower dominance region.

In a related paper (Matta and Perotti 2016) we study how banks may choose to ex ante allocate asset liquidity across lenders by their choice of secured (repo) funding. Repo debt may be designed to be absolutely safe and thus enables to reduce funding costs. However, its use shifts some nonfundamental risk to unsecured lenders. In the unique run equilibrium, the private choice of repo funding tends to increase the chance of unsecured debt runs.

## 2 The Basic Model

The economy lasts for three periods  $t = 0, 1, 2$ . It is populated by a bank and a continuum of risk neutral lenders indexed by  $i$ . The intermediary has access to a project that needs one unit of funding at  $t = 0$ . It raises funds from risk neutral lenders, each endowed with one unit. Lenders demand an expected return of  $\gamma > 1$ , reflecting their alternative storage option between  $t = 0$  and  $t = 2$ . As the mass of lenders is large, perfect competition prevails. A fraction  $\alpha$

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<sup>4</sup>This is equivalent to the result that a higher rate paid to early withdrawals, justified by classic liquidity insurance, results in more frequent runs (Goldstein and Pauzner, 2005)

of lenders face a contingent need of one unit at  $t = 1$  for the purpose of transacting. There is no aggregate uncertainty as transaction needs are identically and independently distributed. Their occurrence at  $t = 1$  is private information. If lenders are unable to obtain cash to satisfy their transaction needs, they shift to an alternative settlement technology and incur a transaction cost  $\tau$ .

- *Project*

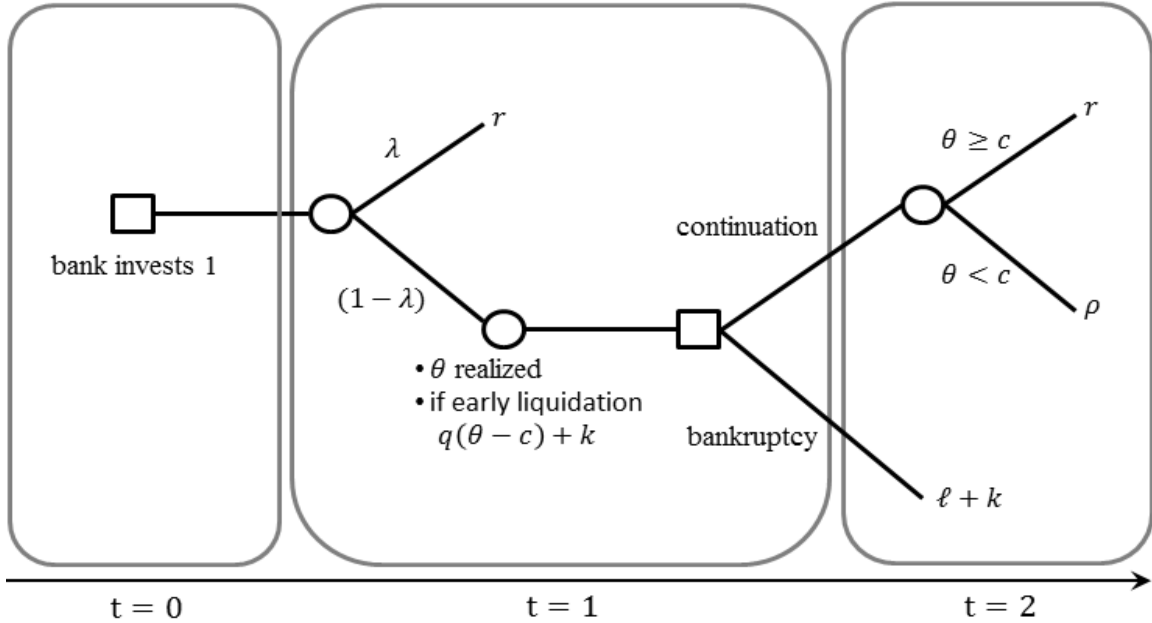
For each unit invested, the project generates a return of  $y_t(\omega)$  at  $t = 1, 2$ , where  $\omega \in \{H, L\}$  is the aggregate state. With probability  $\lambda$  the state is revealed at  $t = 1$  to be high ( $\omega = H$ ), and the project matures at  $t = 1$ :  $y_1(H) = r > \gamma$ . With probability  $1 - \lambda$ , the state is revealed to be low ( $\omega = L$ ), and the project matures only at  $t = 2$ . In this case, early liquidation at  $t = 1$  could be costly as the project has not fully developed its potential. The early liquidation value has a safe component  $k > \alpha$  plus an uncertain value  $\theta$ , drawn from a uniform distribution on  $[0, \bar{\theta}]$ , and realized at  $t = 1$ .  $\theta$  is not verifiable. Initiating the process of liquidating risky assets at  $t = 1$  involves a fixed cost  $c > 0$ , such that their net liquidation value is effectively  $q(\theta - c)$ , where  $q = 1$  for  $\theta \geq c$  and  $q = 0$  for  $\theta < c$ .

In addition, in the  $L$  state there is a chance of loss at time 2 when asset liquidity is very low, namely  $\theta < c$ . In this case the final return is only  $y_2(L) = \rho < 1$ . Note that as long as  $c$  is small, the project is almost always riskless in the  $L$  state provided it is allowed to mature. As  $c$  goes to zero, fundamental risk (and thus bank solvency risk) vanishes.

- *Bank Default and Orderly Liquidation*

Since all claims are safe in the high state, we focus on the low state  $\omega = L$ . The bank uses its reserves to meet repayments demanded at  $t = 1$ . Once liquid reserves are exhausted, the bank is forced to fire sales of illiquid assets. If remaining repayments are larger than the liquidation proceeds from an immediate sale, the bank is declared in default. At that point bankruptcy law forces a stay for all creditors, avoiding the cost  $c$  as well as fire sales, and

Figure 1: Project Timeline



enabling orderly resolution at  $t = 2$ .<sup>5</sup> Under orderly liquidation, asset liquidity risk is resolved and illiquid assets are worth  $\ell \geq 0$ . This is clearly an efficient legal provision as long as  $\ell \geq \theta$ . Unpaid creditors are then treated equally.

We now introduce some assumptions. We assume that

$$\frac{k(1-\alpha)}{k-\alpha}(1-\ell-k) \geq r-1 > \bar{\theta}-c+k-1 > 0,$$

such that: (1) the bank is capable of repaying lenders' investment outlays at  $t = 1$  for a sufficiently high liquidation value; (2) the continuation value is higher than the liquidation value for  $\theta \geq c$ ; (3) the value produced under orderly liquidation is not enough to fully repay all lenders ( $\ell + k < 1$ ), and is sufficiently low relative to the asset return  $r$ .

We further impose

$$\frac{\lambda r + (1-\lambda)(k+\ell) - \gamma}{\alpha(1-\lambda)} > \tau > \frac{r-1}{1-\alpha}.$$
<sup>6</sup>

<sup>5</sup>This differs from the standard assumption that repayments are met sequentially by selling all assets immediately.

<sup>6</sup>Note that this can be satisfied for  $\lambda$  sufficiently large.

The lower bound implies that lenders with transaction needs prefer to receive one unit at  $t = 1$  in the high state, rather than a pro rata share of the surplus  $\frac{r-\alpha}{1-\alpha}$  at  $t = 2$ . The upper bound implies that the project has positive NPV even if always liquidated under orderly resolution in the low state, provided lenders with transaction needs receive one unit at  $t = 1$  in the high state.

Finally, we assume  $k \leq \rho < \ell + k$  such that, for  $\theta < c$ , the value of the assets at  $t = 2$  is at least as great as its liquidation value at  $t = 1$ , but smaller than its value under orderly resolution at  $t = 2$ .

### 3 Runs under Liquidity Risk

In this section we first show that, as in Diamond and Dybvig (1983), demandable debt is optimal in our setting with common knowledge about asset liquidity. We then follow Goldstein and Pauzner (2005) and examine the efficiency properties of demandable debt in a setting of incomplete information.

#### 3.1 Benchmark: Common Knowledge

In our model, efficiency is achieved when lenders with liquidity needs are paid at least one unit at  $t = 1$  (so as to avoid the transaction cost  $\tau$ ) and when bankruptcy occurs if and only if  $\theta < c$  (since  $r > k + \ell > \rho$ ). As we will see, if lenders' transaction needs were observable as of  $t = 1$ , it would be possible to write a contract that achieves the first best. However, when transaction needs are privately observed, no such a contract exists. We consider contracts that maximize the monopolist bank's payoff subject to lenders' participation constraints.

- *First Best*

Although contracts can be contingent on transaction needs, they cannot be a function of  $\theta$ , as it is not verifiable. Thus, a contract that achieves the first best must indirectly induce bankruptcy

when  $\theta < c$ . One such contract is as follows: (1) lenders with transaction needs are paid one unit at  $t = 1$ ; (2) lenders without transaction needs are entitled to demand a payment of one unit at  $t = 1$ , with withdrawals being served sequentially until the bank runs out of reserves; (3) in the absence of bankruptcy, the fraction  $\phi$  of lenders without transaction needs not paid at  $t = 1$  are paid the minimum between  $d \in (1, \frac{r-\alpha}{1-\alpha}]$  and the pro rata share of the surplus at  $t = 2$ ,  $\min \left\{ \frac{qr+(1-q)\rho-\alpha-(1-\alpha)(1-\phi)}{(1-\alpha)\phi}, d \right\}$ , where  $d$  holds lenders to their participation constraint.

When  $\theta < c$ , the bank is known by all to be insolvent at  $t = 2$ . In this case, lenders without transaction needs have a strictly dominant strategy to withdraw at  $t = 1$ , since otherwise they can be paid at most  $\frac{k-\alpha+\ell}{1-\alpha} < 1$ . This results in bankruptcy, as cash reserves are not enough to repay all of them ( $k - \alpha < 1 - \alpha$ ).

For  $\theta \geq c + 1 - k$ , no bankruptcy can occur since the reserves and the interim value of illiquid assets are enough to pay all lenders. Thus it is a strictly dominant strategy for lender without transaction needs to wait and get a sure payment of  $d > 1$  at  $t = 2$  instead of getting one unit at  $t = 1$ .

For intermediate values of asset liquidity  $\theta \in [c, c + 1 - k)$ , there exists an efficient equilibrium. If no lender without transaction needs demands payment at  $t = 1$ , then bankruptcy does not occur, in which case they indeed prefer to wait for a payment of  $d > 1$  at  $t = 2$  to demanding a payment of one unit at  $t = 1$ . This establishes existence. However, there is another equilibrium that results in inefficient bankruptcy. If all lenders without transaction needs demand payment at  $t = 1$ , then bankruptcy results. In this case, they prefer their expected payoff of  $\frac{k-\alpha}{1-\alpha} + (1 - \frac{k-\alpha}{1-\alpha}) \frac{\ell}{1-k}$  than to not demand payment at  $t = 1$  and receive  $\frac{\ell}{1-k}$  with certainty at  $t = 2$ .

- *Second Best*

When transaction needs are not observable, the first best is not attainable. A simple application of the revelation principle shows that, if a contract achieves the first best, lenders with transaction needs are paid at least one unit at  $t = 1$ , while payments to other lenders



must ensure they are not better off pretending to have transaction needs. However, this is not feasible when  $\theta < c$ , since in this case the maximum payoff left after transaction needs are satisfied is  $\frac{k-\alpha+\ell}{1-\alpha} < 1$ . As a result, all lenders will claim to have transaction needs.

Thus, the second best maximizes the probability that lenders with transaction needs are paid at least one unit at  $t = 1$  when  $\theta < c$ , subject to the incentive compatibility and feasibility constraints. Since unpaid lenders are treated equally in bankruptcy, it is easy to show that in the second best both types of lenders receive one unit with probability  $k$  outside bankruptcy, and  $\frac{\ell}{1-k}$  under orderly liquidation with probability  $1 - k$ .<sup>7</sup>

We are thus led the following proposition:

**Proposition 1 (Optimality of Demandable Debt)** *The second best is implemented by the following demandable debt contract: (1) lenders are entitled to demand a payment of one unit at  $t = 1$ , with withdrawals being served sequentially until the bank runs out of reserves; (2) in the absence of bankruptcy, the fraction  $p$  of lenders not paid at  $t = 1$  are paid the minimum between  $d \in (1, \frac{r-\alpha}{1-\alpha}]$  and the pro rata share of the surplus at  $t = 2$ ,  $\min \left\{ \frac{qr+(1-q)\rho-(1-p)}{p}, d \right\}$ , where  $d$  holds lenders to their participation constraints.*

The contract described in Proposition 1 implements the following equilibria. For  $\theta < c$ , it is strictly dominant for all lenders to demand payments at  $t = 1$ , in which case bankruptcy results. For  $\theta \geq c+1-k$ , no bankruptcy occurs since the reserves and illiquid assets are enough to pay all lenders. In this case, it is strictly dominant for lenders with transaction needs to withdraw at  $t = 1$ , and for lender without transaction needs to wait and be paid at  $t = 2$ . For  $\theta \in [c, c+1-k)$ , it is strictly dominant for lenders with transaction needs to withdraw at  $t = 1$ . There exists an equilibrium in which only lenders with transaction needs withdraw, such that bankruptcy is avoided, but there is another equilibrium in which all lenders withdraw, resulting in inefficient bankruptcy.

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<sup>7</sup>This follows because equal treatment in bankruptcy also implies equal treatment outside bankruptcy, as lenders without transaction needs must not envy the expected payment received out of the reserves  $k$  by the other lenders. Thus, the probability that lenders with transaction needs receive at least one unit out of  $k$  is maximal when all lenders are paid exactly one unit with the same probability, which can be at most  $k$ .

Table 1: Payoffs of Lenders without Transaction Needs Conditional on  $\omega = L$

	$\alpha + (1 - \phi)(1 - \alpha) \leq q(\theta - c) + k$	$\alpha + (1 - \phi)(1 - \alpha) > q(\theta - c) + k$
roll over	$qd + (1 - q) \frac{\rho - \alpha - (1 - \alpha)(1 - \phi)}{(1 - \alpha)\phi}$	$\frac{\ell}{1 - k}$
withdraw	1	$\frac{k}{\alpha + (1 - \phi)(1 - \alpha)} + \left(1 - \frac{k}{\alpha + (1 - \phi)(1 - \alpha)}\right) \frac{\ell}{1 - k}$

From the analysis it is clear that while lenders with transaction needs always withdraw at  $t = 1$ , lenders without transaction needs face a coordination problem when  $\theta \in [c, c + 1 - k)$ . Their preferred action depends on the chance of bankruptcy, which in turn depends on the action of their fellow lenders. Table 1 summarizes their payoffs. As a consequence of the coordination problem, there are equilibria in which the second best is not achieved.

We now adopt the global game approach, which eliminates common knowledge to obtain a unique equilibrium. As in Goldstein and Pauzner (2005), we assume the bank finances the project with demandable debt — optimal under common knowledge — and show how it determines the probability of bankruptcy.

### 3.2 The Unique Equilibrium under Incomplete Information

Adopting the global game approach enables to solve for an unique equilibrium. This requires removing the assumption of common knowledge among lenders about asset liquidity risk. Under this approach it becomes possible to compute the probability of bankruptcy as incomplete information goes to zero, thus endogenizing the pricing of the demandable debt contract.

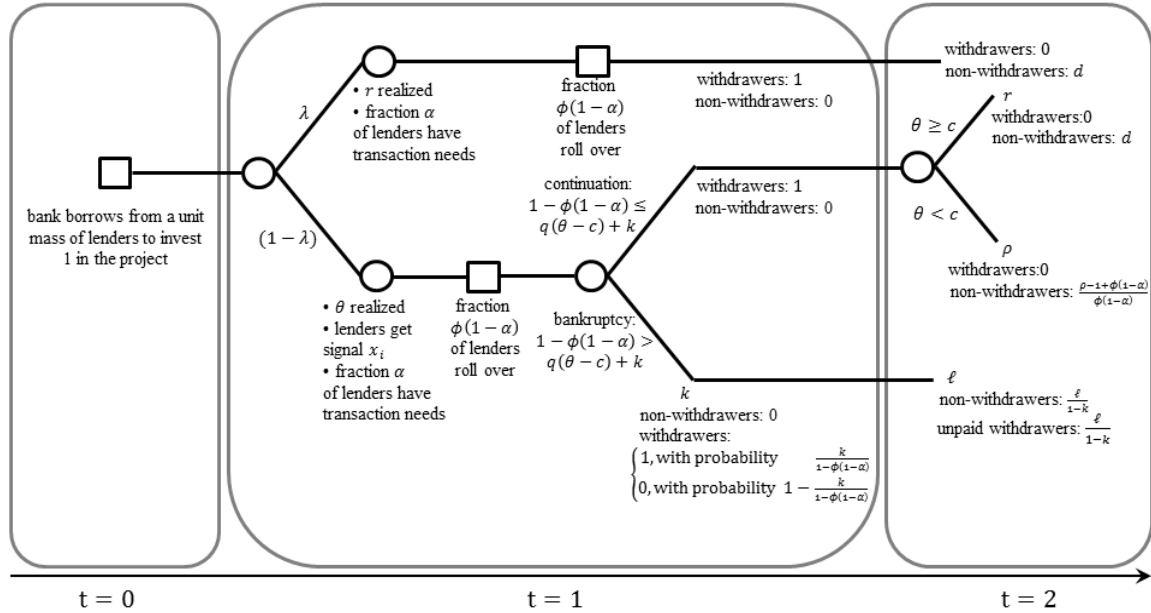
Accordingly, we now assume that in the low state  $\omega = L$  the bank observes  $\theta$ , while lenders receive individual noisy signals on the value of  $\theta$ , the early liquidation value of assets above the safe component  $k$ .

Let this signal be given by

$$x_i = \theta + \sigma \eta_i, \tag{1}$$

where  $\sigma > 0$  is an arbitrarily small scale parameter and  $\eta_i$  are i.i.d. across players and uniformly distributed over  $[-\frac{1}{2}, \frac{1}{2}]$ .

Figure 2: Game Timeline



Once lenders receive their signal, they face a complex coordination problem. Their decision to roll over depends on their beliefs about both liquidity risk  $\theta$  and on the fraction  $\phi$  of lenders who roll over (strategic uncertainty). The unique equilibrium is in switching strategies around a common cutoff  $\theta^*$ : all lenders run for signals below the threshold and roll over otherwise. Uniqueness of equilibrium can be established along the lines of the solution offered by Goldstein and Pauzner (2005) for global games that violate global strategic complementarity,<sup>8</sup> but satisfy a single crossing property. Specifically, the lenders' net rollover payoff is positive if the fraction of lenders who roll over is above a certain threshold, and negative otherwise.<sup>9</sup> The next two subsections sketch the derivation of the equilibrium and compare it to the results in the literature.

<sup>8</sup>This arises because lenders' incentive to roll over is not monotonically increasing in the fraction of lenders who roll over.

<sup>9</sup>Their results rely on the assumption that the noise terms are uniformly distributed. If we relax this assumption by allowing any noise distribution satisfying the monotone likelihood ratio property, our equilibrium is still unique among monotonic strategies (Morris and Shin, 2003).

### 3.2.1 Equilibrium Runs

Let  $\Pi_L^R(\phi, \theta)$  be the net rollover payoff relative to running. We have

$$\Pi_L^R(\phi, \theta) = \begin{cases} qd + (1 - q) \frac{\rho - \alpha - (1 - \alpha)(1 - \phi)}{(1 - \alpha)\phi} - 1, & \text{if } \alpha + (1 - \phi)(1 - \alpha) \leq q(\theta - c) + k \\ -\frac{k}{\alpha + (1 - \phi)(1 - \alpha)} \left(1 - \frac{\ell}{1 - k}\right), & \text{if } \alpha + (1 - \phi)(1 - \alpha) > q(\theta - c) + k \end{cases}. \quad (2)$$

Suppose lenders follow a monotone strategy with a cutoff  $\kappa$ , rolling over if their signal is above  $\kappa$  and withdraw otherwise. Lender  $i$ 's expectation about the fraction of rollover lenders, conditional on  $\theta$  is simply the probability that any lender observes a signal above  $\kappa$ , that is,  $1 - \frac{\kappa - \theta}{\sigma}$ . This proportion is less than  $z$  if  $\theta \leq \kappa - \sigma(1 - z)$ , assessed by each lender  $i$  under the conditional distribution of  $\theta$  given his signal  $x_i$ .

As in the literature of global games, when  $\sigma \rightarrow 0$  strategic uncertainty dominates over uncertainty about  $\theta$  this probability equals  $z$  for  $x_i = \kappa$ .<sup>10</sup> That is, the threshold type believes that the proportion of lenders that roll over follows the uniform distribution on the unit interval. The equilibrium cutoff can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about  $\phi$ . Formally, it is the unique  $\theta^*$  such that  $\int_0^1 \Pi_L^R(\phi, \theta^*) d\phi = 0$ .

This leads us to our first main result.

**Proposition 2 (Run Cutoff)** *In the limit  $\sigma \rightarrow 0$ , the unique equilibrium at  $t = 1$  has lenders following monotone strategies with threshold  $\theta^*$  given by*

$$\theta^* = e^{\alpha \frac{d-1}{k(1-\frac{\ell}{1-k})} - W\left(\frac{d-1}{k(1-\frac{\ell}{1-k})} e^{\alpha \frac{d-1}{k(1-\frac{\ell}{1-k})}}\right)} + c - k, \quad (3)$$

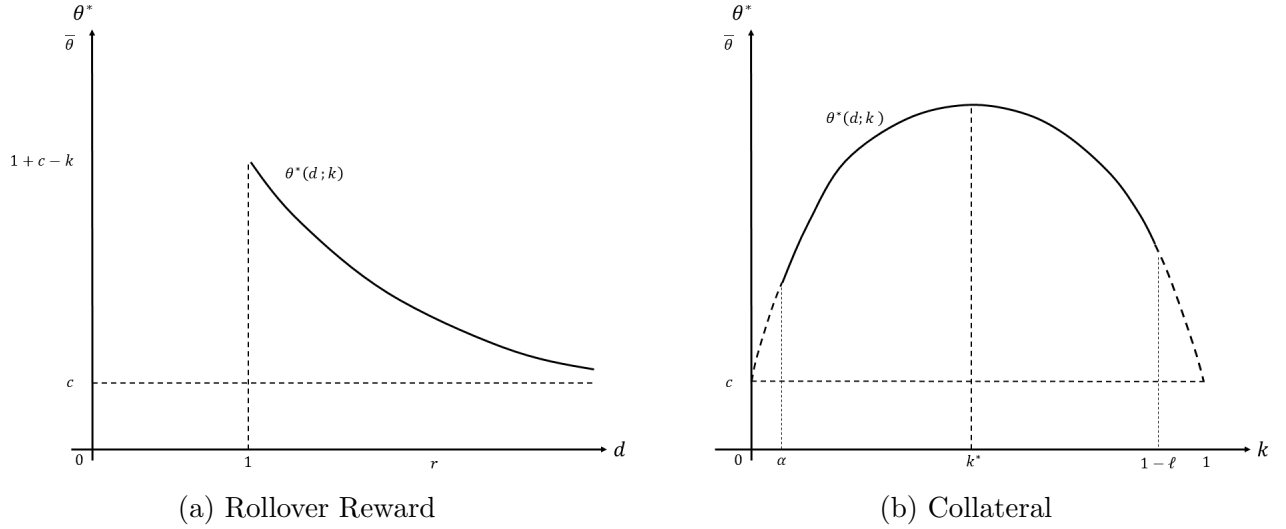
where all lenders roll over if  $\theta > \theta^*$  and do not roll over if  $\theta < \theta^*$ .<sup>11</sup>

Proposition 1 allows us to derive how the probability of bankruptcy relates to the bank's

<sup>10</sup>See Morris and Shin (2003) for a comprehensive discussion of the global games literature.

<sup>11</sup> $W(\cdot)$  is known as the Lambert W function and is the inverse function of  $y = xe^x$  for  $x \geq -1$ .

Figure 3: Run Cutoff



funding policy. Recall that a lower  $\theta^*$  is desirable, as it implies less frequent runs.

**Corollary 1 (Yield and Collateral Effects on Stability)** *The run threshold  $\theta^*$  has the following properties:*

- (i) *It is strictly decreasing and strictly convex in the roll over premium  $d$ , and strictly concave in collateral value  $k$ .*
- (ii) *There exists a cutoff  $k^* \in (0, 1 - \sqrt{\ell})$  such that it is strictly decreasing in  $k$  for  $k \geq k^*$  and strictly increasing in  $k$  for  $k < k^*$ .*

From (3), the signal  $\theta^*$  makes the threshold lender indifferent, as the recovery ratio in a run balances the rollover premium  $d - 1$ . More asset liquidity  $k$  has a stabilizing “probability” effect, the likelihood that the bank has enough liquid assets to repay withdrawals. The second term reflects a “relative payoff” effect, the net rollover gain when there is no bankruptcy relative to expected losses in a default. This second term is directly affected under our bankruptcy assumption.

The comparative statics offer some intuitive insight (see Figure 2) . A higher rollover premium  $d$  improves the payoff of rolling over for a given chance of default, and unambiguously

reduces the probability of runs.<sup>12</sup> However, a large rollover reward reduces the return to the bank in all solvent states. This observation is essential to understand a bank’s pricing incentives.

An increase in asset liquidity  $k$  has a more complex effect. It has an unambiguous linear “probability” effect. Higher asset liquidity reduces the chance that the bank runs out of reserves in a run, which leads to a lower  $\theta^*$ . This effect is equivalent to having better fundamentals. But there is also a “relative payoff” effect, as less liquidity here decreases the expected payoff of both rollover and run strategies. The probability effect is dominant as assets become more very liquid at  $t = 1$ , so runs are less frequent. As asset liquidity declines, runs are increasingly frequent. However, as runners can only be paid out of liquid assets, as these decline their relative payoff drops, producing a hump-shaped relationship.<sup>13</sup> Thus asset liquidity risk has a distinct effect than fundamental value, which has a monotonic effect on run frequency.

In our setup, runs are inefficient if and only if  $\theta \geq c$ . Therefore, a direct implication of Proposition 1 is:

**Corollary 2** *Almost all runs are inefficient as  $c \rightarrow 0$ , in which case the probability of runs is bounded away from zero:  $\theta^* \geq e^{\alpha \frac{d-1}{k(1-\frac{\ell}{1-k})} - W\left(\frac{d-1}{k(1-\frac{\ell}{1-k})} e^{\alpha \frac{d-1}{k(1-\frac{\ell}{1-k})}\right)} - k > 0$ .*

Corollary 2 says that runs are frequent and almost always inefficient when uncertainty is essentially driven by asset liquidity risk (i.e., for vanishing fundamental risk). To focus on runs caused by liquidity risk, henceforth we take  $c$  to be arbitrarily small.

### 3.2.2 Comparison to Pure Fundamental Risk

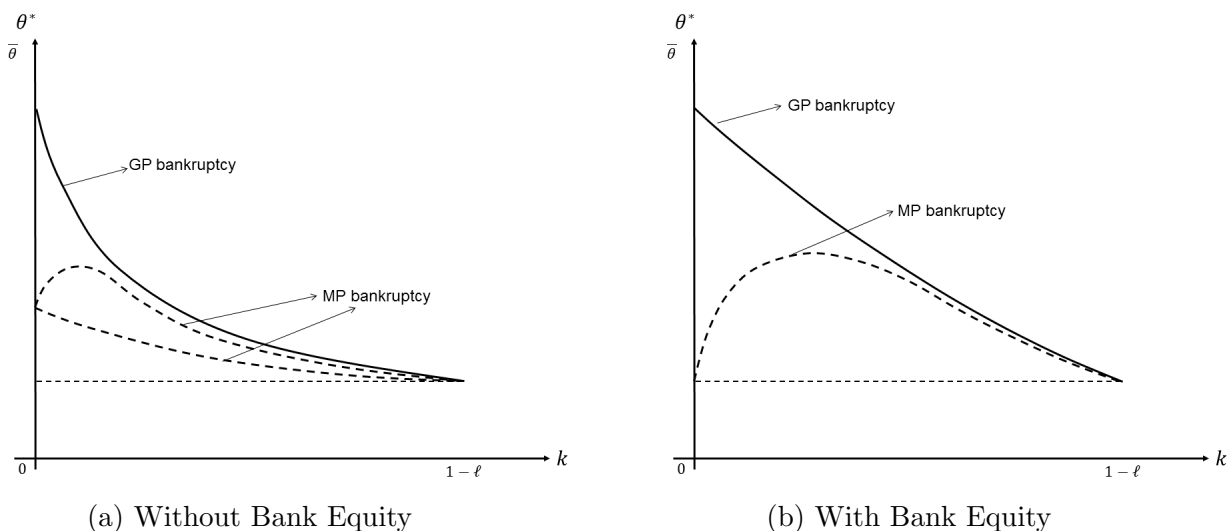
As our setup represents a departure from the standard bank-run model, it is important that we understand the differential effects of our key novel elements. Goldstein and Pauzner (2005) adds fundamental risk to the seminal work of Diamond and Dybvig (1983) and obtain an unique

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<sup>12</sup>This is the specular effect of a higher short term rate in Diamond and Dybvig 1983, which increases runs while providing more liquidity insurance.

<sup>13</sup>This effect would be stronger if more  $k$  implied lower proceeds  $\ell$  in the orderly liquidation process

Figure 4: Run Cutoff



run equilibrium. They provide conditions under which demand-deposit contracts subject to a strict sequential service — agents that withdraw are paid until the bank runs out of assets — provide better liquidity risk sharing. In our set up all agents are risk neutral, although demand deposits are needed to avoid some illiquidity cost. Our setup expands on two fundamental aspects. First, we examine the effect of asset liquidity risk partly unrelated to fundamental risk. Second, our realistic description of the bankruptcy process introduces a mandatory stay, so that in default illiquid assets are also available by those that did not run. We also assume the bank maximizes profit rather than welfare, trading off funding costs and financial stability. In the social optimum in Goldstein and Pauzner (2015), rollover lenders receive all residual value. In our setup the social optimum may be obtained as the choice of a regulator, seeking to correct any inefficiency in the private bank funding choice.

Figure 4 shows the run cutoff as a function of asset liquidity under pure fundamental risk.<sup>14</sup> Figure 4(a) maps the run threshold under zero bank equity as in Goldstein and Pauzner (2015), while Figure 4(b) depicts the run threshold when banks maximize equity value. Each figure illustrates the run cutoffs under both bankruptcy procedures: the solid lines assume strict sequential service (GP), whereas the dashed lines assume mandatory stay (MP).

<sup>14</sup>Figure 4 illustrates results formally derived in Matta and Perotti, 2015b, available upon request.

Several conclusions can be drawn from Figure 4. First, under the sequential service constraint the incidence of runs is always decreasing in asset liquidity  $k$ . Second, this result is not qualitatively affected by the assumption on bank equity. Finally, under profit maximizing banks, mandatory stay produces a first increasing, then decreasing quasiconcave function of asset liquidity  $k$ .<sup>15</sup>

In summary, introducing asset liquidity risk next to the new bankruptcy procedure results in the concavity (which implies, but is not implied by quasiconcavity) of the run threshold. We consider next its implications for the private and social planner choice of funding.

### 3.3 The Pricing of Unsecured Debt

This section examines the bank's initial funding choice  $d$ . Because the project has positive NPV for any funding choice, we can focus on the stability tradeoff, excluding other effects of its financing structure.

The ex ante expected payoff of lenders as a function of its face value  $d$  is

$$V_L(d) = \lambda[(1 - \alpha)d + \alpha] + (1 - \lambda) \left\{ \frac{\bar{\theta} - \theta^*(d)}{\bar{\theta}} [(1 - \alpha)d + \alpha] + \frac{\theta^*(d)}{\bar{\theta}} (k + \ell - \alpha\tau) \right\}$$

The bank's expected payoff can be written as the return of the project of a solvent bank  $r$  net of financing costs and the expected deadweight loss  $DW(d)$ :

$$\begin{aligned} V_B(d) &= \lambda[r - (1 - \alpha)d - \alpha] + (1 - \lambda) \left( \frac{\bar{\theta} - \theta^*(d)}{\bar{\theta}} \right) [r - (1 - \alpha)d - \alpha] \\ &= r - V_L(d) - DW(d), \end{aligned} \tag{4}$$

where  $DW(d)$  is the total payoff lost in the event of bankruptcy, that is

$$DW(d) = (1 - \lambda) \frac{\theta^*(d)}{\bar{\theta}} [r - (k + \ell - \alpha\tau)]. \tag{5}$$

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<sup>15</sup>In this case we can establish that the threshold is neither concave nor convex.



### 3.3.1 Socially Optimal Pricing

As a benchmark, we characterize the optimal financing contract chosen by a social planner. The social planner chooses the face value  $d$  that maximizes the aggregate payoff subject to the participation constraint of the bank and its lenders:

$$\max_d r - DW(d) \tag{6}$$

subject to

$$V_B(d) \geq 0, V_L(d) \geq \gamma.$$

In other words, the optimal financing policy minimizes the chance of runs (a deadweight loss) subject to agents' participation constraints. Since  $-DW(d)$  is increasing in  $d$ , the social planner would increase  $d$  as much as possible.

Increasing  $d$  above lenders' breakeven level offers lenders some rent to encourage rollover, henceforth defined as "rollover rent". The maximum rollover rent is reached when the bank's participation constraint is binding at  $d = \frac{r-\alpha}{1-\alpha}$ , as all asset value is promised to depositors rolling over. Since the bank's participation constraint binds, it follows that the lenders' participation constraint does not bind. Proposition 2 characterizes the socially optimal financing policy.

**Proposition 3 (Optimal Funding)** *The socially optimal financing contract requires the bank to offer the maximum possible rollover rent ( $d^o = \frac{r-\alpha}{1-\alpha}$ ).*

Intuitively, the social planner care about minimizing losses due to early withdrawals, and thus boosts the incentive to roll over by offering the maximum possible roll over yield.

### 3.3.2 Private Pricing

The bank's problem is to choose the rollover reward  $d$  that maximize its payoff subject to the participation constraint:

$$\begin{aligned} \max_d V_B(d) & \tag{7} \\ \text{subject to} & \\ V_L(d) \geq \gamma. & \end{aligned}$$

In making this choice, the bank trades off the cost of financing  $d$  against the expected deadweight loss from runs.

**Proposition 4 (Private Inefficiency)** *The probability of bankruptcy under the socially optimal funding structure is always lower than under the private funding choice:  $\theta^*(d^o) < \theta^*(d^*)$ .*

While the social planner minimizes the probability of runs by choosing the maximum feasible rollover value  $d^o = \frac{r-\alpha}{1-\alpha}$ , the private choice of  $d^*$  is lower than the social optimum value, leading to a higher threshold  $\theta^*(d^*) > \theta^*(d^o)$  and thus more frequent runs.

Proposition 4 characterizes the optimal private funding choice.

**Proposition 5 (Private Pricing)** *The bank's financing policy is characterized as follows:*

- (i) *The privately optimal choice of  $d^*$  either holds lenders to their participation constraint, or leads to a positive rollover rent characterized by  $-\frac{\partial DW(d^*)}{\partial d} = \frac{\partial V_L(d^*)}{\partial d}$ .*
- (ii) *There exists a cutoff  $\lambda_1 \in [0, 1)$  such that, if  $\lambda > \lambda_1$ , the bank offers no rollover rents to its lenders.*

The face value  $d$  balances lower funding costs against a higher deadweight loss. When  $\lambda$  is sufficiently high, runs are rare so the private choice of funding is a corner solution.

## 4 Conclusion

The recent crisis highlighted how asset illiquidity plays a critical role in propagating distress. This paper examines run incentives under asset liquidity risk. We obtain a unique run threshold equilibrium, where runs may occur even when the fundamental value of bank assets are almost certainly safe. In order to describe precisely the allocation of liquidity risk inherent in run incentives, we improve the stylized bank run framework used in the literature with a correct characterization of the bank default process. Existing run models assume that withdrawals are met by asset sales until none are left. In reality, less liquid assets cannot be sold immediately without huge losses. To avoid a hasty termination of real projects, bankruptcy law forces an automatic stay on all lenders once the borrower runs out of liquid assets. Remaining assets are then sold under orderly resolution, limiting fire sales of very illiquid assets. This precise allocation of liquidity risk produces a surprising result. Unlike the case of pure fundamental risk, the chance of a bank run is not monotonic in asset liquidity risk.

# Appendix

**Proof of Proposition 1.** For  $\theta < c$ , it is strictly dominant for all lenders to demand payments at  $t = 1$ , in which case bankruptcy results. In the event of bankruptcy, lenders without transaction needs that withdraw receive an expected payment of at least  $k + (1 - k) \frac{\ell}{1-k}$ , while those that do not receive  $\frac{\ell}{1-k}$  with certainty. In the absence of bankruptcy, those that demand payments at  $t = 1$  are paid one unit, while the others will receive a pro rata share of the surplus at  $t = 2$ , which is at most  $\rho < 1$ . Demanding payments at  $t = 1$  is even more attractive for lenders with transaction needs as it allows the avoidance of the transaction cost  $\tau$  with probability  $k$ .

For  $\theta \geq c + 1 - k$ , no bankruptcy occurs since the reserves and illiquid assets are enough to pay all lenders. In this case, it is strictly dominant for lenders without transaction needs to wait and receive at least  $\min\{r, d\} > 1$  at  $t = 2$ . It is also strictly dominant for lenders with transaction needs to withdraw at  $t = 1$ , since a payment of one unit at  $t = 1$  is greater than  $\frac{r-\alpha}{1-\alpha} - \tau$  — this follows from  $\tau > \frac{r-1}{1-\alpha}$  — their maximum payoff if they wait until  $t = 2$ .

For  $\theta \in [c, c + 1 - k)$ , it is strictly dominant for lenders with transaction needs to withdraw at  $t = 1$ . In the event of bankruptcy, lenders with transaction needs that demand payments at  $t = 1$  receive an expected payment of at least  $k + (1 - k) (\frac{\ell}{1-k} - \tau)$ , while those that do not receive  $\frac{\ell}{1-k} - \tau$  with certainty. In the absence of bankruptcy, those that demand payments at  $t = 1$  are paid one unit, while those that do not receive at most  $\frac{r-\alpha}{1-\alpha} - \tau < 1$ . For lenders without transaction needs, however, their preference depends on the actions played by other lenders. There exists an equilibrium in which only lenders with transaction needs withdraw. Under these strategies, bankruptcy does not occur. Lenders without transaction needs prefer their payment of  $d > 1$  at  $t = 2$  to demanding a payment of one unit at  $t = 1$ . This establishes existence. However, there is another equilibrium that results in inefficient bankruptcy. If all lenders demand payment at  $t = 1$ , then bankruptcy results. In this case, lenders without transaction needs prefer their expected payoff of  $k + (1 - k) \frac{\ell}{1-k}$  than to not demand payment at  $t = 1$  and receive  $\frac{\ell}{1-k}$  with certainty at  $t = 2$ . ■

**Proof of Proposition 2.** Goldstein and Pauzner (2000) and Morris and Shin (2003) prove this result for a general class of global games, including those where  $\theta$  is drawn from a uniform distribution on  $[\underline{\theta}, \bar{\theta}]$ , the noise terms  $\eta_i$  are i.i.d. across players and drawn from a uniform distribution on  $[-\frac{1}{2}, \frac{1}{2}]$ , and that satisfy the following additional conditions: (i) for each  $\theta$ , there exists  $\phi^* \in \mathbb{R} \cup \{-\infty, \infty\}$  such that  $\Pi_U^R(\phi, \theta) > 0$  if  $\phi > \phi^*$  and  $\Pi_U^R(\phi, \theta) < 0$  if  $\phi < \phi^*$ ; (ii)  $\Pi_U^R(\phi, \theta)$  is nondecreasing in  $\theta$ ; (iii) there exists a unique  $\theta^*$  that satisfies  $\int_0^1 \Pi_U^R(\phi, \theta^*) d\phi = 0$ ; (iv) there exists  $\bar{D}$  and  $\underline{D}$  with  $\sigma < \min\{\bar{\theta} - \bar{D}, \underline{D} - \underline{\theta}\}$ , and  $\epsilon > 0$  such that  $\Pi_U^R(\phi, \theta) \leq -\epsilon$  for all  $\phi \in [0, 1]$  and  $\theta \leq \underline{D}$  and  $\Pi_U^R(\phi, \theta) > \epsilon$  for all  $\phi \in [0, 1]$  and  $\theta \geq \bar{D}$ ; and (v) continuity of  $\int_0^1 w(\phi) \Pi_U^R(\phi, x) d\phi$  with respect to signal  $x$  and density  $w$ . Except for (iii),  $\Pi_U^R(\phi, \theta)$  clearly satisfies (i), (ii), (iv) and (v).

We now show that (iii) is also satisfied. Let  $\Delta(\theta; d) \equiv \int_0^1 \Pi_U^R(\phi, \theta) d\phi$ . Since  $\Delta(\theta; d) < 0$

for all  $d$  and  $\theta < c$ , then if  $\theta^*$  exists it must be that  $\theta^* \geq c$ . Moreover, since  $\Delta(\theta; d)$  is strictly increasing in  $\theta$  for  $\theta \geq c$ , we must show that  $\Delta(c; d) \leq 0$  for all  $d$  (otherwise for some  $d$  we have  $\Delta(\theta; d) \geq \Delta(c; d) > 0$  for all  $\theta \geq c$  and no  $\theta^*$  would satisfy  $\Delta(\theta^*; d) = 0$ ). We also have that (a)  $\Delta(c; d)$  is strictly increasing in  $d$ , (b)  $d$  is bounded by  $\frac{r-\alpha}{1-\alpha}$  (in which case the bank's participation constraint binds), and (c)  $\Delta(c; \frac{r-\alpha}{1-\alpha}) = k \left(1 - \frac{\ell}{1-k}\right) \ln \frac{k \frac{k-\alpha}{1-\alpha} \frac{r-1}{1-\alpha}}{e^{-\frac{\ell}{1-k}}}$   $\leq (<) 0$  if  $e^{-\frac{\frac{k-\alpha}{k} \frac{r-1}{1-\alpha}}{1-\frac{\ell}{1-k}}} \geq (>) k$ . Therefore, for  $\frac{k-\alpha}{1-\ell-k} \frac{r-1}{1-\alpha} \leq 1$  we have

$$e^{-\frac{\frac{k-\alpha}{k} \frac{r-1}{1-\alpha}}{1-\frac{\ell}{1-k}}} > 1 - \frac{k-\alpha}{k} \frac{r-1}{1-\alpha} = (1-k) \left(1 - \frac{k-\alpha}{k} \frac{r-1}{1-\ell-k}\right) + k \geq k,$$

which implies that for all  $d$  we have  $\Delta(c; d) \leq \Delta(c; \frac{r-\alpha}{1-\alpha}) < 0$ . In addition, for all  $d$  we have  $\Delta(\theta; d) > 0$  for  $\theta$  sufficiently large such that there exists  $\theta^* > c$  that satisfies  $\Delta(\theta^*; d) = 0$ . Finally, there is a unique such  $\theta^*$  as  $\Delta(\theta; d)$  is strictly increasing in  $\theta$  for  $\theta \geq c$ .

For the derivation of the cutoff  $\theta^*$ , note that condition  $\int_0^1 \Pi_U^R(\phi, \theta^*) d\phi = 0$  is equivalent to

$$k \left(1 - \frac{\ell}{1-k}\right) \ln(\theta^* - c + k) + (\theta^* - c + k - \alpha)(d-1) = 0. \quad (\text{A.1})$$

After some algebra, (A.1) can be rewritten as

$$\frac{d-1}{k \left(1 - \frac{\ell}{1-k}\right)} e^{\alpha \frac{d-1}{k \left(1 - \frac{\ell}{1-k}\right)}} = \left[ \alpha \frac{d-1}{k \left(1 - \frac{\ell}{1-k}\right)} - \ln(\theta^* - c + k) \right] e^{\alpha \frac{d-1}{k \left(1 - \frac{\ell}{1-k}\right)} - \ln(\theta^* - c + k)}. \quad (\text{A.2})$$

Let  $W(\cdot)$  be the inverse function of  $y = xe^x$  for  $x \geq -1$  (known as the Lambert W function), that is,  $x = W(y)$ . Combined with (A.2) this implies

$$\theta^* = e^{\alpha \frac{d-1}{k \left(1 - \frac{\ell}{1-k}\right)} - W\left(\frac{d-1}{k \left(1 - \frac{\ell}{1-k}\right)} e^{\alpha \frac{d-1}{k \left(1 - \frac{\ell}{1-k}\right)}\right)} + c - k,$$

which establishes the result. ■

**Proof of Corollary 1.** First, let us establish a couple of results relative to the W function.

Implicitly differentiating  $y = W(y) e^{W(y)}$  results in

$$W' = \frac{W}{(W+1)y} = \frac{e^{-W}}{1+W} > 0,$$

$$W'' = W'^2 \left( \frac{-2-W}{1+W} \right) < 0.$$

These results, along with the definitions  $x(d, k) \equiv \alpha \frac{d-1}{k(1-\frac{\ell}{1-k})}$  and  $z(x(d, k)) \equiv x - W\left(\frac{x}{\alpha} e^x\right)$ , allow us to compute

$$\begin{aligned} \frac{\partial \theta^*}{\partial d} &= e^z \frac{\partial z}{\partial d} \\ &= e^z \frac{\alpha}{k(1-\frac{\ell}{1-k})} \left[ 1 - W' \left( \frac{x}{\alpha} e^x + \frac{e^x}{\alpha} \right) \right] \\ &= e^z \frac{\alpha}{k(1-\frac{\ell}{1-k})} \frac{x-W}{(W+1)x} < 0, \end{aligned}$$

where the inequality follows from the fact that  $x - W < 0$ , which is implied by (A.2). We further have that

$$\begin{aligned} \frac{\partial^2 \theta^*}{\partial d^2} &= e^z \left( \frac{\partial z}{\partial d} \right)^2 + e^z \frac{\partial^2 z}{\partial d^2} \\ &= e^z \left( \frac{\partial z}{\partial d} \right)^2 + e^z \left[ \frac{\alpha}{k(1-\frac{\ell}{1-k})} \right]^2 \left[ \frac{x-W}{(W+1)^2 x^2} - (x-W) \frac{W'x \left( \frac{x}{\alpha} e^x + \frac{e^x}{\alpha} \right) + W+1}{(W+1)^2 x^2} \right] \\ &> 0. \end{aligned}$$

Finally, we have

$$\begin{aligned} \frac{\partial \theta^*}{\partial k} &= e^z \frac{\partial z}{\partial k} - 1 \\ &= -e^z \alpha \frac{d-1}{k^2(1-\frac{\ell}{1-k})^2} \left[ 1 - \frac{\ell}{(1-k)^2} \right] \frac{x-W}{(W+1)x} - 1. \end{aligned}$$

If  $(1-k)^2 \leq \ell$ , then  $\frac{\partial \theta^*}{\partial k} < 0$  and  $\frac{x-W}{(W+1)x}$  is decreasing in  $k$ , which implies  $\frac{\partial^2 \theta^*}{\partial k^2} < 0$ . If  $(1-k)^2 > \ell$ , then  $\frac{\partial \theta^*}{\partial k}$  is positive for  $k$  close enough to 0 and  $\frac{x-W}{(W+1)x}$  is increasing in  $k$ , which implies that  $\frac{\partial^2 \theta^*}{\partial k^2} < 0$ . Therefore, there exists  $k^* \in (0, 1 - \sqrt{\ell})$  such that  $\frac{\partial \theta^*}{\partial k} = 0$ , with  $\frac{\partial \theta^*}{\partial k} > 0$

for  $k < k^*$  and  $\frac{\partial \theta^*}{\partial k} < 0$  for  $k > k^*$ . ■

**Proof of Corollary 2.** This result is shown in the proof of Proposition 1. ■

**Proof of Proposition 3.** The aggregate payoff  $r - DW(d)$  is clearly increasing in  $d$ . The bank's payoff is strictly concave in  $d$  as

$$\bar{\theta} \frac{\partial V_B^2(d)}{\partial d^2} = 2(1-\lambda)(1-\alpha) \frac{\partial \theta^*}{\partial d} - (1-\lambda)[r - (1-\alpha)d - \alpha] \frac{\partial^2 \theta^*}{\partial d^2} < 0,$$

which in turn implies  $V_B(d)$  is either (1) decreasing or (2) increasing and then decreasing since

$$\bar{\theta} \frac{\partial V_B(d)}{\partial d} = -(1-\alpha)[\bar{\theta} - (1-\lambda)\theta^*] - (1-\lambda)[r - (1-\alpha)d - \alpha] \frac{\partial \theta^*}{\partial d}$$

is negative for  $d = \frac{r-\alpha}{1-\alpha}$ . If  $\frac{\partial V_B(d)}{\partial d} \leq 0$  for all  $d$ , then  $V_B(d)$  is monotone decreasing. If for some  $d'$  we have  $\frac{\partial V_B(d')}{\partial d} > 0$ , then there exists  $d'' \in (d', \frac{r-\alpha}{1-\alpha})$  such that  $\frac{\partial V_B(d'')}{\partial d} = 0$ . Since  $V_B(d)$  is strictly concave in  $d$ ,  $\frac{\partial V_B(d)}{\partial d} > 0$  for  $d < d''$  and  $\frac{\partial V_B(d)}{\partial d} < 0$  for  $d > d''$ . Moreover, the bank's participation constraint binds when  $d = \frac{r-\alpha}{1-\alpha}$ , which implies  $V_B(d) < 0$  for all  $d > \frac{r-\alpha}{1-\alpha}$ . Therefore, the social planner chooses the maximum feasible rollover rent  $d^o = \frac{r-\alpha}{1-\alpha}$ . ■

**Proof of Proposition 4.** Suppose that  $\theta^o(d^o) \geq \theta^*(d^*)$ . Since we assume the project has positive NPV, the bank's payoff under (7) is greater than zero. But then a contract with  $d$  marginally greater than  $d^*$  satisfies both participation constraints in (6) and results in  $\theta^o(d^o) \geq \theta^*(d^*) > \theta^*(d)$ . But this contradicts  $d^o$  being a solution to (6). ■

**Proof of Proposition 5.** The first order necessary conditions (FOC) are

$$-\frac{\partial DW(d)}{\partial d} = \frac{\partial V_L(d)}{\partial d} (1-\mu), \quad (\text{A.3})$$

$$\mu [V_L(d) - \gamma] = 0, \quad (\text{A.4})$$

$$V_L(d) \geq \gamma, \quad (\text{A.5})$$

$$\mu \geq 0. \quad (\text{A.6})$$

Since  $V_B(d)$  is strictly concave (see Proof of Proposition 2), any  $d$  satisfying the FOC is a global maximizer, which shows (i).

For (ii), note that

$$\frac{\partial DW(d)}{\partial d} = \frac{(1-\lambda)}{\bar{\theta}} \frac{\partial \theta^*(d)}{\partial d} [r - (k + \ell - \alpha\tau)], \quad (\text{A.7})$$

$$\frac{\partial V_L(d)}{\partial d} = 1 - \alpha - \frac{(1-\lambda)}{\bar{\theta}} \left\{ (1-\alpha)\theta^*(d) + \frac{\partial \theta^*(d)}{\partial d} [(1-\alpha)d + \alpha - (k + \ell - \alpha\tau)] \right\}. \quad (\text{A.8})$$

Consider  $\mu = 0$ . As  $\lambda$  gets close to 1, the left- and right-hand sides of (A.3) approach 0 ((A.7) approximates 0) and  $1 - \alpha$  ((A.8) converges to  $1 - \alpha$ ), respectively. Since the derivative of the right-hand side of (A.3) with respect to  $\lambda$  is greater than that of the left-hand side, which is negative, there are only two possibilities: either the left-hand side of (A.3) is smaller than the right-hand side for all  $\lambda \geq \lambda_1 = 0$ , or there exists  $\lambda(d) \in (0, 1)$  such that the left-hand side of (A.3) is smaller than the right-hand side if  $\lambda > \lambda(d)$  and at least as great if otherwise. If the former is true for all  $d$ , then (A.3) can only be satisfied if  $\mu > 0$ . Suppose there exists  $d$  such that the latter is true and denote  $Y$  the set of all such  $d$ . If  $\lambda > \lambda_1 = \sup\{\lambda(d) : d \in Y\}$ , then (A.3) can only be satisfied if  $\mu > 0$ . Combining these two possibilities we deduct that there exists a cutoff  $\lambda_1 \in [0, 1)$  such  $\mu > 0$  if  $\lambda > \lambda_1$ , which in turn implies that  $V_L(d) - \gamma = 0$  (from (A.4)). ■



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