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Abstract

This article studies the impact of education and fertility in structural transformation and growth. In the model there are three sectors, agriculture, which uses only low-skill labor, manufacturing, that uses high-skill labor only and services, that uses both. Parents choose optimally the number of children and their skill. Educational policy has two dimensions, it may or may not allow child labor and it subsidizes education expenditures. The model is calibrated to South Korea and Brazil, and is able to reproduce some key stylized facts observed between 1960 and 2005 in these economies, such as the low (high) productivity of services in Brazil (South Korea) which is shown to be a function of human capital and very important in explaining its stagnation (growth) after 1980. We also analyze how different government policies towards education and child labor implemented in these countries affected individuals' decisions toward education and the growth trajectory of each economy.

JEL classification codes: J13, J24, O40, O41, O47, O57

Keywords: economic growth, structural transformation, education, fertility

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1 Introduction

In the last fifty years, some economies experienced episodes of rapid and sustained economic growth, while others had episodes of high growth followed by stagnation and even recession. South Korea and Brazil are examples of these two distinct paths.

The Brazilian economy experienced high productivity and output growth until the early 80's, in a clear catch-up process relative to the leading economies. The country presented one of the highest growth rates in the world and ceased to be predominantly rural to become urban, with production concentrated in manufacturing and service sector. However, in the early 80's, Brazil began to reverse this catch-up process and productivity began to fall[1].

South Korea, despite having levels of per capita output and relative productivity below Brazil until the mid-80's, experienced an uninterrupted growth process throughout the entire period (1960 to 2005). This was due to the high growth of the Korean average productivity, which can be summarized with the following fact: until the mid-60's, the productivity of a Korean worker was only 60% of the Brazilian, however in 2005 a worker in South Korea was on average more than twice as productive as the Brazilian. The exemplary Korean growth was also possible due to government policies which promoted education that enabled the formation of skilled labor for manufacturing and service sector. In contrast, a large part of Brazilian stagnation in the 80's and 90's can be explained by economic, educational and social policies.

The differences between the Brazilian and Korean productivities since mid-80's may be consequence of an accelerated or a meager growth of some productive sector. An hallmark of the economic development of countries, and documented in the literature, is the process of structural transformation. In this process, there is a displacement of resources between different productive sectors (agriculture, manufacturing and services) over time. In general, the economies initially undergo a reduction in the share of agricultural sector and an increase in the importance of manufacturing and service sector in the workforce and in the production. This process usually causes an increase in aggregate productivity of economies, since labor productivity in agriculture is usually smaller than in the other two sectors. In a second stage of structural transformation, the importance of manufacturing falls and the share of services sector (in terms of labor and value added) dominates the economy.

The literature[2] indicates the low growth in the service sector as the main cause for the stagnation of the Brazilian economy and the decline in relative productivity after the 80's. Therefore, the

[1] See Figure 1.

[2] See [Duarte and Restuccia, 2010] and [Silva and Ferreira, 2011].

question of what would be the cause of the poor performance of this sector naturally arises. The data indicate a massive reallocation of labor from agriculture toward the service sector, while the percentage of workers in manufacturing remained stable during the period 1960 to 2005 in Brazil. As in general, the labor employed in agriculture is of low qualification, an argument sustained in this paper is that the flow of unskilled labor to services is a possible cause for the poor performance of the sector. Regarding South Korea, we can say that almost all workers who left agriculture went toward the service sector, as pointed out by [Kim and Topel, 1995]. Thus, both economies went through a quite similar structural transformation process. However, sector growth was quite different: South Korea had in the period faster growth in manufacturing - at an annualized rate of 5.5% - as opposed to less than 3% in Brazil, and the service sector grew by 2% a year in Korea but virtually zero in Brazil[3].

To understand the dissimilar development experience of these two countries, we propose a structural transformation model that incorporates human capital decision, in which the trade-off between the quantity of children and the quality of education faced by parents is affected by government policies. In the model, there will be two types of workers, skilled and unskilled (or high- and low-skill), and three sectors, agriculture, manufacturing and services. Agriculture employs only unskilled labor; manufacturing, only skilled labor and the service sector employs both types. Therefore, the structural transformation is stylized, in the sense that there will be in a first moment reallocation of unskilled labor from agriculture to the services, and then reallocation of skilled labor from manufacturing to the service sector.

The type of qualification of each adult will be determined by the investment of his parents during his childhood. Parents chooses optimally the number of offsprings and their skill. Educational policy has two dimensions, it may or may not allow child labor and it subsidizes education expenditures.

The model is calibrated to the Brazilian and South Korean economies and is able to reproduce the main stylized facts observed between 1960 and 2005. In addition, we performed some counterfactual exercises, which measures the importance of educational policies to the accumulation of human capital and how it affects the process of structural transformation.

This article relates to several literatures. Regarding the structural transformation literature, this process was first documented as a stylized fact of economic growth by [Kuznets, 1973]. Our paper is closer to the recent literature that aims to understand episodes of accelerated growth, stag-

[3]It is worth noting that while Brazil remained closed to trade and adopted a policy of import substitution industrialization during almost the entire period, Korea opened the economy to international trade and implemented policies to stimulate exports and industrialization. However, this point is not emphasized in this study.

nation and decline stressing productivity differences across sectors and the reallocations of labor between them[4]. The article of [Timmer and de Vries, 2009] argues that the episodes of growth are explained by increases in productivity across sectors and concludes that improvements in productivity in the service sector are more important than the growth of productivity in manufacturing[5]. In [Duarte and Restuccia, 2010], the authors investigate the role of sectoral labor productivity in structural transformation and in the trajectory of aggregate productivity of 29 countries. They are able to find large differences in agriculture and service productivity but a minor difference in manufacturing sector. They also noted that the catch-up process in the productivity of manufacturing is responsible for half of the productivity gains, while the low productivity of the service sector explains all the experiences of stagnation and economic decline. [Silva and Ferreira, 2011] investigates the same issue of [Duarte and Restuccia, 2010] in six Latin American countries in the period of 1950-2003. They came to the conclusion that the service sector was generally responsible for the reversal of the catch-up process, which occurred since the 80's.

[Betts et al., 2013] and [Kim and Topel, 1995] study structural transformation in South Korea. [Betts et al., 2013] examines, quantitatively and qualitatively, the role of international trade and (distorting) trade policies in the reallocation of labor and production, using a model of three sectors and two countries (Korea and OCDE), with labor as the only factor of production. [Kim and Topel, 1995] analyzes the evolution of the labor market in South Korea during an episode of extraordinary economic growth (1970-1990), with industrialization and structural transformation process as their background. They found that the fraction of workers employed in agriculture fell by 30 percentage points in less than 20 years and there is no evidence that agricultural workers migrated to manufacturing. Indeed, employment growth in manufacturing occurred only by recruiting new entrants (with higher human capital) in the labor market, which tend to stay in this sector throughout their career.

This article is also related to the fertility and education literature, which began with the work of [Becker, 1960] and nowadays is quite extensive[6]. [Becker et al., 1990] studies the interaction of fertility and education decisions with economic development. It assumes endogenous fertility and a rising rate of return on human capital. There are two steady states: a Malthusian regime, where wages are constant and fertility is high, and a growth regime with rising wages and low fertility. The model, however, is not able to generate the transition between stagnation and

[4] [Herrendorf et al., 2013] presents an extensive literature review and recent advances.

[5] [Herrendorf and Valentinyi, 2012] also aims to answer which sectors are responsible for low productivity in developing countries.

[6] A major review of literature with relevant criticisms is presented in [Jones et al., 2010].

growth. [Doepke, 2004] is able to fill this gap. Furthermore, [Doepke, 2004] was probably the first to consider the role of fertility decisions in the discussion of growth and inequality[7]. In the paper, the transition from Malthusian stagnation to the modern growth regime can be generated from a relatively simple model, based on the trade-off between quantity and quality of children faced by individuals. This article also introduces particular government policies - which we follow closely in our paper - in order to produce different transition paths for each country. It also introduces to the model child labor, taken as an opportunity cost of the infant. The model is calibrated to Brazil and South Korea

[Mbiekop, 2013] develops a model of structural transformation that incorporates the choice of human capital. The model has just two productive sectors (agriculture and manufacturing) and the supply of skilled labor increases the productivity of manufacturing, which is intensive in this type of labor, while agriculture uses only unskilled labor. Results depend mainly on the educational composition of the workforce and so the paper focuses on human capital accumulation as an engine of economic development of African countries[8].

In certain aspects, this paper presents several contributions to these literatures. It incorporates fertility and education decisions, and consequently the formation of human capital to the structural transformation models. Thus, labor ceases to be purely homogeneous. Therefore, the characterization of the structural transformation process becomes richer. We use this framework to propose an explanation for the decline in service productivity pointed by [Duarte and Restuccia, 2010] and [Silva and Ferreira, 2011], since 1980 in Brazil (also in other Latin American economies). Furthermore, it is the first paper in the fertility and education literature to consider three consumer goods. Moreover, the analysis of the model takes place via a general equilibrium model, while [Erosa et al., 2010] and [Mbiekop, 2013] are based in partial equilibrium analysis. Similar to [Badel et al., 2013] we study human capital formation in a three sectors structural transformation model. However, we incorporate fertility decisions and abstract from migration and urbanization decisions, which is key to their analysis

This paper proceed as follows. In the next section, the data is described in details. In section 3, stylized facts of Brazilian and Korean economies and some motivations are presented. In section 4, the model and the equilibrium are described, taking into account the inclusion of public policies. In section 5, we present the analytical results of the model. In section 6, the model is calibrated

[7] See also [Moav, 2005] and [de la Croix and Doepke, 2003].

[8] [Erosa et al., 2010] presents a model of heterogeneous agents in order to quantify the importance of differences in human capital versus TFP to explain variations in per capita income across countries. The model has two sectors - manufacturing and services -, and fertility and education decisions, but does not study structural transformation.

with the intention to reproduce the allocations of different types of labor between productive sectors and the distribution of skills in population of the two countries. In section 7, the numerical results are presented, i.e., the results generated by the model are compared with the data and we perform some counterfactuals. Finally, section 8 concludes.

2 Data

The sample covers Brazil, South Korea and United States[9] and the years between 1955 and 2005. However, the period chosen for study is between 1960 and 2005. This occurs because the five early years of the sample period were chosen as the basis period for productivity in order to generate variability in productivity across the sectors at the next periods[10]. The series of value added and employment by sectors were taken from [McMillan and Rodrik, 2011]. The [McMillan and Rodrik, 2011] data is composed by the Groningen Growth and Development Centre (GGDC) database, which provides statistics of employment and real value added for 27 countries (among them, Brazil and South Korea) disaggregated into 10 sectors ([Timmer and de Vries, 2007] and [Timmer and de Vries, 2009]) with the inclusion of data from African countries, China and Turkey. The reason to use [McMillan and Rodrik, 2011] database is that the authors converted the value added to international dollars of 2000 and took into account the purchasing power parity (PPP).

The value added and employment data cover ten productive sectors, but in this paper they were grouped into three major sectors (agriculture, manufacturing and services), following the structural transformation literature. The ten production sectors are defined by the ISIC Rev.3[11] definitions and were grouped as follows: agriculture includes agriculture, forestry and fishing (01-05), manufacturing is composed by mining and extraction (10-14), manufacturing (15-37), utilities (40-41) and construction (45), and the service sector consists of wholesale and retail trade, hotels and restaurants (50-55), transport, storage and communication (60-64), finance, insurance and real estate (65-74), and community, social, personal and government services (75-99).

The hours worked data in the ten productive sectors was obtained from different sources for each country. For South Korea, this series was obtained easily from the EU KLEMS database[12]. However, this data is only available for the years 1970-2005. No data are available on hours

[9] This only for comparison of the productivity trajectory of Brazil and South Korea.
[10] It will be be explained with more details in the section Calibration.
[11] International Standard Industrial Classification of All Economic Activities, Rev.3.
[12] See [O'Mahony and Timmer, 2009].

worked in Brazil[13]. Thus we had to generate it from PNAD (Pesquisa Nacional por Amostra de Domicílio[14]), a household survey that covers all regions of Brazil and interviews more than 300.000 people every year since 1976[15].

Following the structural transformation literature, the productivity series was constructed as the ratio between the value added and the total number of hours worked by sector for each economy and for the period 1955-2005.

From the [Barro and Lee, 2010] database we obtained the share of the population percentage that has primary, secondary or tertiary education. These data are used as variables of human capital accumulation for each country and compared with the model results.

3 Stylized Facts

After World War II, Brazil was still a poor country and predominantly agricultural. However, between 1960 and 1980, the country underwent a profound transformation. During this period, the Brazilian economy grew very fast. Until the late 70s, there was a catch-up process relative to the United States (U.S.). As can be seen in Figure 1, the Brazilian income per capita was only 20% of U.S. income in 1960, but after 20 years it was already 35% of the U.S.. Note that the relative output per worker followed the same trend of income per capita, with only a small differential. However, since 1980 we can observe a drastic growth slowdown. The relative income per capita and output per worker began to fall and the convergence process was reversed. Only after the 2000s these variables showed signs of recovery, remaining practically constant until 2005[16].

South Korea is a successful example of economic growth. In 1960, the Korean economy was approximately half of Brazil (in terms of income per capita and output per worker), and in 2005, it was already half of the U.S. economy and twice as big as Brazil. From Figure 1, we can see that South Korea experienced a continuous process of catch-up[17]. During the 1960-2005 years, South Korea had one of the highest growth rates in the world. In a short period, the country became a developed and industrialized economy.

[13] There is a time series that covers only the metropolitan regions for the period 1992-2005 provided by LABORSTA.

[14] National Household Survey conducted every year in Brazil (see Appendix A and *www.ibge.gov.br/home/estatistica/populacao/trabalhoerendimento/pnad2005/* for more details).

[15] Since the period of interest is 1955-2005, we had to repeat for the previous years the data of 1970 and of 1976 for South Korea and Brazil, respectively.

[16] Note that relative productivity in Brazil by the end of the period was at the same level that in 1960, about 20%.

[17] Due to the Asian crisis of 1997, there was a decline in economic growth, which was reversed rapidly and in 1999, the country has returned to grow.

These two growth trajectories that initially were very similar but from 1980 became quite different pose the following question: what went wrong with Brazil? When the attention falls on government policies, a very important difference is education. While Brazil barely invested in human capital formation, South Korea adopted policies such as education subsidy and child labor restriction. When focusing in sectorial contribution to growth, there are indications that the service sector was they main culprit for the low growth of Brazil.

3.1 Structural Transformation

From Figure 2, we observe the evolution of the relative share of workers employed in agriculture, manufacturing and services in Brazil between 1960 and 2005[18]. In 1960, about 58% of Brazilian workers were still in agriculture, 17% in manufacturing and 25% in services. Over time, manufacturing participation in the labor force has had little change, growing only 2 percentage points between 1960 and 2005. However, there was an important reallocation of workers from agriculture toward services. In 2005, about 62% of the workforce was in the service sector and only 19% was in agriculture. Thus, we can say that the structural transformation occurred significantly between agricultural and service sectors.

Still in Figure 2, we observe the evolution of labor allocation across the productive sectors in South Korea. Similar to Brazil, there was a massive transfer of workers from agriculture to services: the share of labor of the agricultural sector fell from 69% in 1960 to 8% in 2005, while the service share rose from 21% to 63%. Regarding the share of workers in manufacturing, it grew until 1991, felt in the next seven years and then has stabilized since 1998. Although it appears that there was also a migration of workers from agriculture to industry, [Kim and Topel, 1995] shows, from a cohort analysis, that this kind of reallocation actually did not happen. In fact there was a change in the workforce composition in the sense that greater participation of workers in manufacturing was due to the entry of new individuals (with a higher education level) in the labor market who have decided to go to this sector.

Figure 3 shows the evolution of productivity[19] in the three sectors between 1960 and 2005 in both countries. In Brazil, there is a clear upward trend in the productivity of the agricultural and manufacturing sectors, and a rise followed by a fall in service productivity. Furthermore, throughout the period agricultural productivity is well below those of the other two sectors. Between 1960 and 1980, the three productive sectors showed significant growth, with agricultural and service

[18] In this initial analysis, we assumed homogeneous labor.

[19] Here defined as output per worker.

productivity growing at similar rates (2.6% per year) while manufacturing grew at a higher rate (4% per year). In this period there is a significant reallocation of workers from agriculture to services. Given that the service was more productive than agriculture, this reallocation led to an acceleration of the Brazilian aggregate productivity, which resulted in convergence. However, since 1980, service productivity fell significantly (a negative growth rate of 2.5% per year between 1980 and 2005) and manufacturing stagnated. Considering the increasing share of workers employed in the service sector (over 60% of Brazilian labor force in 2005) and the continuous fall of productivity in this sector, it is easy to understand the reversal of the catch-up process in the Brazilian economy. Thus, it seems that the inefficiency of the service sector is behind the fall in productivity growth and in per capita income.

In South Korea, one can observe a continuous productivity growth in all the three sectors during the entire period (1960-2005). Manufacturing experienced very high growth, 5.5% annualized. Agriculture also had a considerable growth in productivity (5.4% per year). The productivity growth of service sector was much lower, with a rate of about 2% per year. Although the performance of services has been less than the other two sectors, the growth of productivity contributed positively to reducing its economic gap relative to United States. In summary, while the service sector can be seen as the main responsible for Brazilian stagnation, Korean economic success can be explained by the good performance of all three productive sectors, in particular services, which employs the highest percentage of workers.

3.2 Education

In the post World War II, Brazil adopted an economic development project of import substitution, characterized by strong stimulus to investment in physical capital in manufacturing, nationalization of public utility services and steep barriers to international trade[20]. Education was not a priority so that the country experienced a low level of investment in the sector, particularly in public primary education. The low investment in education along with a very high rate of population growth (and high fertility rate, although declining, as can be seen in Figure 5) led to the relatively unskilled workforce observed in later years.

Although free and compulsory basic education is provided by law since 1930 in Brazil, in practice public schools are of poor quality and primary education does not reach many rural areas until now. The neglect of the government with the universalization of education over time can be

[20] For more details, see [Veloso et al., 2013].

perceived from the educational expenditures. Table 2 (taken from [Maduro Junior, 2007]) shows the low investment in education (as a proportion of GDP) by the government until the 1980 and the disproportion between investment per student in the primary and tertiary levels (in 1960, a student in tertiary education costs 117 times more than one in primary education). Moreover, for many years the Brazilian government was very tolerant with child labor: despite having established since its 1934 Constitution the minimum age of 14 for employment, this was never enforced in the country. In 1985, 18.7% of the children between ages 10 and 14 were in the labor market.[21]

South Korea, after the Korean War (1950-1953), instituted a plan of compulsory and free basic education, which led to a high enrollment rate already in 1960. Furthermore, the education control was gradually withdrawn from local administrations (provinces) and concentrated in the Ministry of Education, which became responsible for the administration of schools, the allocation of resources, the development of school curriculum, among other tasks. Restrictions on child labor were taken very seriously since the Korean independence. Although the country has signed only in 1991 the International Labor Organization (ILO) convention that rules out child labor under the age of 14, since 1960 the participation rates of Korean children in workforce were very low: 1.1% of children between 0 and 15 years in 1960 and only 0.3% in 1985, according to the ILO.

Different public policies of education and child labor implemented in Brazil and South Korea have produced, as expected, different results. Figure 6 shows the evolution of average years of schooling of the Brazilian and Korean population. In 1960, the difference between the two countries was only 1.4 years, but in 2005 this differential rose to 4.5 years. Tables 4 and 5 show the evolution of the percentage of Brazilian and Korean population with certain educational levels. In 1960 the percentage with no schooling was almost the same in the two economies, but over time the situation improved in Korea way more than in Brazil, so that in 2005, this figure was three times higher in Brazil than in South Korea. When we look to the proportion of people who have the primary, secondary and tertiary education, South Korea always had higher percentages in high educational levels (secondary and tertiary) and population growth in these two levels was also higher than in Brazil. Therefore, analyzing only the years of study of each economy, the Korean evolution was more favorable. We can also analyze enrollment ratios. Table 3 shows the evolution of enrollment rates in primary and secondary education levels. Although Brazil and Korea present primary rate very similar over the years, Korea has a large and growing advantage in the rates of secondary education, which further illustrates the better educational level of the

[21] And child labor is even more persistent among male children, with 25.3% of them in the Brazilian workforce in 1987 and 24.3% in 1990, according to [Doepke, 2004].

Korean population. Although the two countries have very similar primary enrollment rates, this measure overestimates educational level in Brazil, because it was not taken into consideration the quality of education[22].

4 Model

We consider a standard dynastic OLG economy extended with endogenous fertility and multiple production sectors. Indviduals live for two periods. The investments of young individuals (children) in human capital determine their skill levels when old. Old individuals (adults) decide how to allocate their time between child rearing and working. They must also choose in which of the three sectors of the economy to work: agriculture, manufactures and services. Adults can also choose how many children to have and how much to invest in their education. In our analysis we introduce education policies in a model that combines the main elements of Barro-Becker [XXX] and subsequent models in the quantity/quality trade-off literature and the non-homotheticity in consumption used in recent structural transformation models.[23]

In any given period, the utility V of an adult depends on a vector $c = (c_A,\ c_M, c_S)$ of consumption levels of agricutural goods, manufactures and services. It is also a function of the number of children and their utility when old. In our baseline model, we restrict our analysis to two types of children: low-skill[24] (denoted by the subindex L) and high-skill[25] (denoted by the subindex H).

Consider an adult with consumption c, n_L low-skill children and n_H high-skill children who are, respectively, expected to attain utilities $V_H^{'}$, $V_L^{'}$ when old. We assume that such an adult attains an utility level

$$V(c, n_H, n_L) = U\left(c\right) + \beta(n_H + n_L)^{-\varepsilon}[n_H V_H^{'} + n_L V_L^{'}],$$

where

$$U\left(c\right) = \frac{\left[v(c_A)(c_M)^b(c_S + \bar{c}_S)^{(1-b)}\right]^{1-\sigma}}{1-\sigma}.$$

Here $v(c_A) = 1$ if $c_A \geq \bar{c}_A$ and $v(c_A) = 0$ otherwise. For tractability, we assumed that food is not valued beyond a subsistance threshold $\bar{c}_A$. As for the other parameters: b governs the share of manufacturing vis-à-vis services; $\bar{c}_S \geq 0$ is a positive parameter that indicates that services are a

[22] For comparisons between the quality of education in the countries, see test results from PISA (Programme for International Student Assessment), conducted every three years since 2000.

[23] See [Doepke, 2004] and [Duarte and Restuccia, 2007] for leading examples of papers in these two lines of work.

[24] In this paper, unskilled and low-skill have the same meaning.

[25] In this paper, skilled and high-skill have the same meaning.

superior good; $0 < \sigma < 1$ is the inverse of the intertemporal elasticity of substitution; we assume $\sigma < 1$ to keep the values the utility being positive. Finally, ε governs the curvature of the parental altruism as a function of the quantity of children.[26] These parameters will play an important role for the transition dynamics of the economy.

Adults are endowed with one unit of time, which can be allocated between working in the marketplace or raising children (at home). Raising each child requires a fraction $0 < \phi < 1$ of time, regardless of the skills level of the parent. But parents not only decide on the number of children (quantity) but also on their education (quality). On one hand, parents can opt to keep (some of) the children unschooled, and they will be low-skill adults. On the other hand, parents can opt to invest in school for their children. Schooling a child requires teachers and only high-skill adults can supply teaching services. For a child to become a high-skill worker when old, he must also receive $0 < \phi_H < 1$ units of time from a teacher. Thus, the (opportunity) cost of just having a child varies proportionally with the skills of the the parent, but the absolute cost of providing him with schooling is the same for all parents regardless of their skills.

We consider two simple government policies. One is the extend to which child labor is allowed. In our stylized environment, children that are not attending school might remain idle (e.g. watching TV) or parents can put them to work. A working child can provide $0 < \phi_L < \phi$ units of low skill labor. Also, we assume that children who work do not attend school and become low-skill adult. As in Doepke (2004), we assume that restrictions imposed by governments imply that parents of unschooled children can extract at most $0 \leq \phi_L^g \leq \phi_L$ units of low-skill labor. Those restrictions are relevant since they can reduce the cost of having unschooled children.[27]

The other government policy we consider is a subsidy on education. Specifically, we assume that the government subsidizes a fraction $0 < \delta < 1$ of the schooling costs. These subsidies are financed with a proportional tax on earnings τ. We assume that the rate δ is exogenously given and that the tax rate evolves over time, according to the state of the economy, so that the government budget is balanced each period, as we explain further below. Both policies $\{\phi_L^g, \delta\}$ shape up the fertility and human capital investment decisions of parents, and the production of the three sectors in the economy.

There are three production sectors in the economy: agriculture (A), manufacturing (M), and services (S). We use agriculture goods as the numeraire, and p_M and p_S are the prices of manufacturing and services relative to that numeraire. Aside from the non-homotheticities in preferences,

[26] See [Becker et al., 1990].

[27] However, since $\phi_U < \phi$, the overall net cost of having a child is positive and fertility is strictly below the biological upper bound $1/\phi$.

the sectors have differences in terms of the skills required. First, the agricultural technology is produced with a constant returns to scale production function that only uses low-skill labor:

$$Y_A = Z_A L_{A,L}. \tag{1}$$

Here, Y_A is the total output of agricultural goods, Z_A is the (exogenously evolving) TFP in agriculture and $L_{A,L}$ the aggregate labor units supplied to agriculture. Second, for simplicity we assume that manufacturing goods are produced using only high-skill labor as input:

$$Y_M = Z_M L_{M,H}. \tag{2}$$

Again, $L_{M,H}$ denotes the number of hours of high-skill labor, and Z_M represents the manufacturing sector TFP of the period. These stark assumptions about factor intensities have been made for the sake of tractability, and very similar results would be obtained in more general settings as long as agriculture is highly intensive in low-skill labor and manufacture is highly intensive in high-skill labor.

Finally, we assume that services are produced using both, high-skill and low-skill labor. The service sector is comprise of very heterogenous industries, ranging from the services provided by low-skill workers such as maids, drivers, cleaning, security personnel and entry level clerical personnel, to health, finance, design and enginnering specialists. Likewise, whether high-skill and low-skill labor are complements or substitutes can vary widely across the different service sectors. At the aggregate level, perhaps, the safest assumption is that the elasticity of substitution is one. Therefore, we assume that the aggregate production for services is

$$Y_S = Z_S (L_{S,L})^{1-\alpha} (L_{S,H})^{\alpha}, \tag{3}$$

where Y_S is the aggregate output of services, $L_{S,L}$ and $L_{S,H}$ denotes the number of hours of low- and high-skill labor and Z_S denotes the TFP in that sector. Notice that the Cobb-Douglas assumption implies that Z_S is Hicks-neutral, so its growth over time by itself won't have an impact on the skill premium. Lastly, the parameter $0 < \alpha < 1$ is the high-skill labor output share in services.

Finally, we assume that all productivity terms grow exogenously, possibly at different rates:

$$Z_A^{'} = (1 + \gamma_A) Z_A, \tag{4}$$

$$Z_M^{'} = (1 + \gamma_M) Z_M, \tag{5}$$

$$Z_S^{'} = (1 + \gamma_S) Z_S. \tag{6}$$

where γ_A, γ_M, γ_S are positive and constant overtime.

4.1 Equilibrium

We take the government policies $\{\phi_L^g, \delta\}$ as constant over time and assume competitive labor, goods and education markets. The state of the economy is entirely given by the three productivity levels and the mass of low and high skill adults entering each period. As a shorthand, define $X \equiv (Z_A, Z_M, Z_S, N_L, N_H)$. Equilibrium prices and allocations, which are about to be explained, are functions of X. The recursive competitive equilibrium determines the allocation of labor across sectors and consumption across households, and the law of motion of X by determining the overall fertility and education decisions of households.

Production in each sector $j \in \{A, M, S\}$ is carried out by competitive firms that take goods prices $p_j(X)$, and the wages for both types of labor $w_L(X)$, $w_H(X)$ as given. Each firm maximize profits by choosing low- and high-skill labor units. Free entry and constant returns to scale imply that the size of firms is indetermined, so that aggregate demand for low- and high-skill labor $L_{j,L}$ and $L_{j,H}$ can be solved by

$$\max_{L_{j,L},\, L_{j,H}} p_j(X) Y_j - w_S(X) L_{j,S} - w_L(X) L_{j,L},$$

subject, respectively, to the production functions (1), (2) or (3).

How much high- and low-skill labor is allocated across sectors, is determined by the population of high- and low-skill workers, their demands decisions, and how much time they devote to having children and providing for their education.

Consider now the problem of a household with skills $i \in \{L, H\}$. The economy's state X, and its law of motion $X' = \Lambda(X)$, and the government policies are fixed by $\{\phi_L^g, \delta\}$. Each household takes as given wage rates $\{w_L(X), w_H(X)\}$, the price of goods, $\{p_M(X), p_S(X)\}$, and the tax rate $\tau(X)$ that each period balances the budget of the government (as we explain shortly). Given those prices and policies, the household attains an overall utility level V_i defined by the Bellman Equation

$$V_i(X) = \max_{\{c_A, c_M, c_S, n_L, n_H\}} \left\{ U[c_A, c_M, c_S] + \beta(n_H + n_L)^{-\varepsilon} [n_H V_H(X') + n_L V_L(X')] \right\},$$

subject to the budget constraint

$$c_A + p_M(X) c_M + p_S(X) c_S + \phi_H (1-\delta) w_H(X) n_H \leq [1 - \tau(X)] \{ w_i(X) [1 - \phi(n_L + n_H)] + \phi_L^g w_L(X) n_L \}. \tag{7}$$

The left-hand side of the constraint is the value of all the goods acquired in the market, including, if any, the education services for their children. Notice that the cost of education services is net of government subsidies. The right hand side includes all the earnings of the household, including, if any, the earnings of working unschooled children.[28]

Finally, for the government to balance its budget every period, the proportional tax on all labor earnings must be equal to

$$\tau(X) = \frac{\delta\phi_H N_H'(X)}{L_H(X) + \phi_H N_H' + L_L(X)\frac{w_L(X)}{w_H(X)}}, \tag{8}$$

This tax rate results from equating the total outlay of education subsidies $\delta\phi_H w_H(X) N_H'(X)$ with the taxes collected by imposing a rate $\tau(X)$ on the aggregate labor earnings $L_H(X)w_H(X) + \phi_H N_H'(X)\, w_H(X) + L_L(X)w_L(X)$. Here, we have defined L_H and L_L as the total high- and low-skill units of labor supplied to produced goods, which is detailed below, and includes, if any, unschooled child labor. Hence, $\phi_H N_H'(X)\, w_H(X)$ represents the costs labor earnings of the high-skill workers providing education services.

We now have all the elements needed to define and then characterize an equilibrium in this economy.

Definition 1 *Let $j \in \{A, M, S\}$ index the sectors and $i, k \in \{L, H\}$ index the skill/school levels of the population. Given an initial state $X_0 \in \mathbb{R}^5_+$, exogenous growth rates $\{\gamma_A, \gamma_M, \gamma_S\}$ and government policies $\{\delta, \phi^g_L\}$, an* ***equilibrium*** *in this economy is:* (a) *a law of motion $\Lambda : \mathbb{R}^5_+ \to \mathbb{R}^5_+$, for the state X;* (b) *price and wage functions $p_j : \mathbb{R}^5_+ \to \mathbb{R}_+$, and $w_i : \mathbb{R}^5_+ \to \mathbb{R}_+$,* (c) *labor allocations $L_{j,i} : \mathbb{R}^5_+ \to \mathbb{R}_+$, and consumption and fertility decisions, $c_{i,j} : \mathbb{R}^5_+ \to \mathbb{R}_+$ and $n_{i,k} : \mathbb{R}^5_+ \to \mathbb{R}_+$; and* (d) *a tax function $\tau : \mathbb{R}^5_+ \to [0, 1]$, such that:* ***(i)*** *for any X in the current period, the state X' in the next period is given by $X' = \Lambda(X)$;* ***(ii)*** *given prices $\{w_i(\cdot), p_j(\cdot)\}$, (a) the allocations $\{L_{j,i}(\cdot)\}$ solve the firms problem; (b) the allocations $\{c_{i,j}(\cdot), n_{i,k}(\cdot)\}$ solve the household problem; (c) the goods and labor markets clear; (d) the budget constraint of the government balances (i.e. the tax rate is given by (8)); and (iii) the transition $\Lambda(\cdot)$ is given by the growth rates by $\{\gamma_A, \gamma_M, \gamma_S\}$ and the fertility and education decisions $\{n_{i,k}(\cdot)\}$.*

We now proceed to characterize the equilibrium. It turns out that, except for the fertility decisions, we can solved for all equilibrium objects in closed form. Moreover, a straightforward property of our assumed preferences imply that fertility decisions and the transition function $\Lambda(\cdot)$ have a very simple property. Such property is very useful to determine the restributional impact of policies and how they shape the demographic and productivity dynamics of countries.

[28] For concreteness, notice that we are assuming that child labor is subject to the same tax rate as adult labor. Such assumption is as unappealing as the alternative of no taxes on child labor, but the consequences of choosing one over another fall greatly into the calibration of ϕ^g_L.

Take as given any $X \in \mathbb{R}^5_{++}$, and start with the demand for labor by firms in all three sectors. From our simplifying assumptions, $L_{A,H} = L_{M,L} = 0$. First, given that agriculture is our numeraire ($p_A = 1$), the low-skill wage is given by the productivity in that sector,

$$w_L(X) = Z_A. \tag{9}$$

Second, the first order condition of producers of manufacturing sector provide a direct link between the wages of high-skill workers and the price of manufactures

$$w_H(X) = p_M(X) Z_M. \tag{10}$$

Finally, the service sector hires high- and low-skill labor in the following amounts

$$w_H(X) = p_S(X) Z_S \alpha \left[\frac{L_{S,L}(X)}{L_{S,H}(X)}\right]^{1-\alpha}, \tag{11}$$

and

$$w_L(X) = p_S(X) Z_S (1-\alpha) \left[\frac{L_{S,H}(X)}{L_{S,L}(X)}\right]^{\alpha}. \tag{12}$$

In the equilibrium of our simple setting, low-skill workers must be indifferent between working for services or agriculture. Likewise, high-skill workers must be indifferent between working in services or manufacturing. Then, in an interior equilibrium with positive production of services[29], expressions (9) and (12) imply that the price of services in terms of agricultural goods must be

$$p_S(X) = (1-\alpha)^{-1} \frac{Z_A}{Z_S} \left[\frac{L_{S,L}(X)}{L_{S,H}(X)}\right]^{\alpha}, \tag{13}$$

while expressions (10) and (11) imply that the price of services in terms of manufactures must be

$$\frac{p_S(X)}{p_M(X)} = \alpha^{-1} \frac{Z_M}{Z_S} \left[\frac{L_{S,H}(X)}{L_{S,L}(X)}\right]^{1-\alpha}. \tag{14}$$

Now consider the demand side. We first fully characterize the intra-period equilibrium conditions on prices, wages and labor allocation across the three sectors, take as given arbitrary fertility and education decisions $n_{i,k}$. We then partially characterize the behavior of households in terms of fertility and education decisions $n_{i,k}$ and the implied transition $X' = \Lambda(X)$.

First, define the fraction of adults of type i who have children of type k, as a function of state X: $\lambda_{i,k}(X)$. Note that the sum of these fractions must be equal to one for each type of parent (adult)

[29] Our utility function U implies that agriculture and manufactures are always produced, but the production of services might be zero. If so $L_{S,L}(X) = L_{S,H}(X) = 0$, and the equilibrium $p_S(X)$ can be anywhere within an interval.

$$\lambda_{H,H}(X) + \lambda_{H,L}(X) = \lambda_{L,H}(X) + \lambda_{L,L}(X) = 1.$$

Now, consider the fertility and education decisions $n_{i,k}$ as given. This means, that each low -skill household supplies $[1 - \phi(n_{L,L} + n_{L,H})] + \phi_L^g n_{L,L}$ units of low-skill labor. In turn, each high-skill household provides $[1 - \phi(n_{H,L} + n_{H,H})]$ units of high-skill labor and $\phi_L^g n_{H,L}$ units of low-skill labor. Therefore, the aggregate supply of high- and low-skill labor, L_H and L_L, available for the production of consumption goods are, respectively, given by

$$\begin{aligned} L_H &= N_H [1 - \phi(\lambda_{H,L} n_{H,L} + \lambda_{H,H} n_{H,H})] - \phi_H (N_H \lambda_{H,H} n_{H,H} + N_L \lambda_{L,H} n_{L,H}), \\ L_L &= N_L [1 - \phi(\lambda_{L,L} n_{L,L} + \lambda_{L,H} n_{L,H})] + \phi_L^g (N_H \lambda_{H,L} n_{H,L} + N_L \lambda_{L,L} n_{L,L}), \end{aligned} \tag{15}$$

where the high-skill labor required to educate the next crop of high-skill workers is subtracted in the first expression, while the low-skill labor supplied by children is added in the second expression. In terms of L_H and L_L, the first order conditions of producers from the three sectors lead to simple equilibrium of prices and quantities. First, since $p_A = 1$, the wage of low-skill labor is entirely determined by the productivity of the agricultural sector,

$$w_L(X) = Z_A.$$

Second, from the assumed form for the demand for agricultural goods, the low-skill labor used in that sector is

$$L_{A,L} = \frac{\bar{c}_A}{Z_A}(N_L + N_H).$$

The remaining low-skill workers are employed in the service sector

$$L_{S,L} = L_L - \frac{\bar{c}_A}{Z_A}(N_L + N_H).$$

In a similar vein, if $L_{M,H} \leq L_H$ units of high-skill labor are allocated to manufactures, then the remaining $L_H - L_{M,H}$ workers are in services. Then, from equation (14), the indifference between high-skill workers about working for the manufactures or services sectors imply

$$\frac{p_S(X)}{p_M(X)} = \alpha^{-1} \frac{Z_M}{Z_S} \left[\frac{L_H - L_{M,H}}{L_{S,L}} \right]^{1-\alpha}. \tag{16}$$

The other determinant of relative prices and of how labor is allocated across sectors is the demand. Given our assumption of Stone-Geary preferences it is easy to show that each household of skill level i will equate the marginal rate of substitution between manufactures $c_{M,i}$ and services $c_{S,i}$, i.e.

$$\frac{(1-b)\, c_{M,i}}{b\,[c_{S,i} + \bar{c}_S]} = \frac{p_S(X)}{p_M(X)}.$$

Since Stone-Geary preferences are Gorman aggregable, the equilibrium relative prices $p_S(X)/p_M(X)$ must satisfy

$$\frac{p_S(X)}{p_M(X)} = \frac{(1-b)\,Z_M L_{M,H}}{b\left[Z_S(L_{L,S})^{1-\alpha}(L_H - L_{M,H})^{\alpha} + \bar{c}_S(N_L+N_H)\right]}. \tag{17}$$

Then, equating (16) and (17) and simplifying, leads to the condition

$$\left[1+\frac{\alpha(1-b)}{b}\right]L_{H,M} = L_H + \frac{\bar{c}_S(N_L+N_H)}{Z_S\left[L_L - \frac{\bar{c}_A}{Z_A}(N_L+N_H)\right]^{1-\alpha}}\left[L_H - L_{H,M}\right]^{1-\alpha}.$$

Clearly, given any level L_H, as long as $L_L > \frac{\bar{c}_A}{Z_A}(N_L+N_H)$, there is a uniquely defined allocation $0 < L_{M,H} < L_H$ of high-skill labor between manufactures and services. As a function of $L_{M,H}$, the LHS is strictly increasing and is zero when $L_{M,H}=0$; the RHS is strictly decreasing in $L_{H,M}$, and when $L_{H,M}=0$ its value is positive. Moreover, when all labor is allocated to manufacturing, $L_{M,H}=L_H$, then the LHS is strictly greater than the RHS.

Here: discussion of comparative statics. ERE.

We now examine the determination of $n_{i,k}$, the fertility and human capital decisions of children. Let $e_i \equiv \sum_j p_j c_{j,i}$ denote the expenditure in goods of a household with skills i. The budget constraint can be re-written as

$$e_i = \bar{w}_i(X) - E_i,$$

where $\bar{w}_i(X) \equiv [1-\tau(X)]\,w_i(X)$ are the net-of taxes potential (or full) labor market earnings of the household and $E_i \equiv [1-\tau(X)]\{[w_i(X)\phi(n_L+n_H)] - \phi_L^g w_L(X)n_L\} + \phi_H(1-\delta)\,w_H(X)n_H$, is the amount of resources spent in children.

Following Doepke (2004), the fertility/schooling decisions can be solved as a problem of choosing the total amount spend in children and the compostion of the family between high- and low-skill. To this end, first notice that the period, indirect utility function can be written as

$$\bar{U} = U(\bar{w}_i(X) - E_i; X) = \frac{1}{1-\sigma}\left\{\left[\frac{b(\bar{w}_i(X) - E - \bar{c}_A + p_S(X)\bar{c}_S)}{p_M(X)}\right]^b\left[\frac{(1-b)(\bar{w}_i(X) - E - \bar{c}_A + p_S(X)\bar{c}_S}{p_S(X)}\right.\right.$$

Second, we write what are the full prices of high- and low-skill children. The former cost the parent

$$p_{i,H}(X) = \phi\bar{w}_i(X) + (1-\delta)\,\phi_H w_H(X),$$

i.e. the time cost plus the cost of education, both net of taxes and subsidies. Similarly, each low-skill child costs the parent

$$p_{i,L}(X) = \phi\bar{w}_i(X) - \phi_L^g \bar{w}_L(X),$$

i.e. the time cost plus minus the earnings from child labor, both net of taxes. Obviously, $E = n_L p_{i,L}(X) + n_H p_{i,H}(X)$.

Adults take the alternative future utilities $V_L(X')$ and $V_H(X')$ of their children as given. Since we are assuming that $\sigma < 1$, we can restrict our analysis to the cases in which $0 < V_L(X') \leq V_H(X')$, where the second inequality holds because high-skill individuals can always opt for the occupations and fertility choices of the low-skilled. We can rewrite the problem as one choosing total expenditure E and the fraction f of E that is spent on raising high-skill children. Thus, the number of high-skill children is $n_H = f \times E/p_{i,H}$, and the number of low-skill children is $n_L = (1-f) \times E/p_{i,L}$. Plugging those expressions, the fertility/human capital problem of an adult can be written as:

$$\max_{\substack{0\leq E\leq \bar{w}_i,\\ 0\leq f\leq 1}} \left\{ U(\bar{w}_i(X) - E;\, X) + \beta E^{1-\varepsilon} \left(\frac{f}{p_{i,H}} + \frac{(1-f)}{p_{i,L}} \right) \left[\frac{f V_H(X')}{p_{i,H}} + \frac{(1-f) V_L(X')}{p_{i,L}} \right] \right\}. \quad (18)$$

Under these preferences, low skill and high skill children are perfect substitutes, i.e. the indifference curves are straight lines for the parents. The relative prices $p_{i,H}$, $p_{i,L}$ and the utilities $V_L(X')$, $V_H(X')$ determine the family composition in a stark form.

Proposition 2 *For all $\{E, f\}$, that attains the maximum in (18), the solution is $f = 0$ or $f = 1$, i.e., the problem of adult has only corner solutions.*

Proof. *See Appendix B.* ■

Adults choose to have only one type of child so that there is never both types of children in the same family. Given this result, it is possible to determine the optimal number of children by assuming that parents have a single type k, where $k \in \{H, L\}$. While it is not possible to explicitly solve the equation above for n_k[30], the separability of preferences imply that children are superior goods:

Proposition 3 *Conditional on the type $k \in \{H, L\}$ of children, the optimal number of children n_k is increasing in the futute utility $V_k(X')$ and the net of taxes full income $\bar{w}_i(X)$.*

Proof. *See Appendix B* ■

Therefore, children are a normal good. Another important property of the adult's problem is described in the following proposition:

[30] These problems are straightforward to computed, as described in detail in Appendix D.

Proposition 4 *An adult is indifferent between both types of children if, and only if, the costs and utilities of children satisfy the following condition:*

$$\frac{V_H(X')}{(p_{i,H})^{1-\varepsilon}} = \frac{V_L(X')}{(p_{i,L})^{1-\varepsilon}} \tag{19}$$

If an adult is indifferent, the total expenditure on children does not depend on the type of child chosen.

Proof. *See Appendix B.* ■

The above propositions generate important implications for the mobility between generations. Since in equilibrium $w_H > w_L$, nurturing high-skill children is relatively cheaper for high-skill parents than for low-skill parents

$$\frac{p_{H,H}}{p_{H,L}} < \frac{p_{L,H}}{p_{L,L}}. \tag{20}$$

High-skill parents have a "comparative advantage" in the "production" of high-skill children, while the low-skill parents have a "comparative advantage" in the "production" of low-skill children. Moreover, as the relative price of children differs between the types of parents, it is not possible that both types of adults remain indifferent between the two types of kids at the same time. And given the comparative advantage, in equilibrium there will be always high-skill parents who have high-skill children, as well as there will be always low-skill parents who have low-skill children.

We have to define what kind of parent will be indifferent between the two types of kids. Since the interest of this model is to generate upward intergenerational mobility, this equilibrium path occurs only when low-skill adults are indifferent between the two types of children. In this case, all high-skill parents have high-skill children, while the low-skill parents have both types of children. We have then the following corollary.

Corollary 5 *For any X such that $w_H(X) > w_L(x)$, it must be true that:*

- *A positive fraction of high-skill adults have high-skill children and a positive fraction of low-skill adults have low-skill children:*

$$\lambda_{H,H}(X), \lambda_{L,L}(X) > 0.$$

- *Only one type of adult can be indifferent between the two types of children:*

$$\lambda_{H,L}(X) > 0 \quad \Longrightarrow \quad \lambda_{L,H}(X) = 0,$$

$$\lambda_{L,H}(X) > 0 \quad \Longrightarrow \quad \lambda_{H,L}(X) = 0.$$

Add: *Proposition. Impact on policy. Given* $\{p_k^t, w_i^t, \tau^t\}$ *the effect of changes in policies, equilibrium with upward mobility: (a) child labor, increase welfare of low skill, increase fertility; (b) education subsidies, increasse welfare and fertility of high skill.*

Discussion of taxes and GE.

5 Calibration

We need to determine the value of the preference parameters σ, β, ε, b, $\bar{c}_A$, and $\bar{c}_S$; the parameters representing the cost of raising a child (ϕ), educational cost (ϕ_H), child labor productivity (ϕ_L), government subsidy to education (δ); and coefficient of the production function of services (α). Moreover, we have to find the time series of productivity of each sector A_j, where $j \in \{A, M, S\}$.

To find the productivity series, first we have to define the amounts of high- and low-skill labor employed in the service sector. Given our very stylized production functions, we have to use a not so rigorous definition of high-skill labor: individuals with the secondary level education (incomplete) are considered highly skilled. Had we opted, for instance, to associate highly skilled with college education the number of high-skill labor in 1960 would be very small and way smaller than the observed labor in manufacturing in both countries. The evolution of the percentage of high-skill individuals in the two countries is given by Table 6, that uses data from [Barro and Lee, 2010][31].

Returning to the issue of the amount of low and high-skill labor employed in the service sector, first we need to make some points clear. Although the definition described below seems somewhat arbitrary, it was the only one found so that comparable data between Brazil and Korea (from [McMillan and Rodrik, 2011]) could be used. We classify the subsectors which comprise the service sector as skilled or unskilled. When we analyze the wage differential of the service subsectors in relation to manufacturing (assumed to employ only high-skill labor), we can see from [Medeiros, 2012] that the workers employed in transport, storage and communication

[31] Note that in the model a period comprises 25 years. However, there are only data available, for both countries, from 1960 to 2005.

(60-64) or in financial activities, insurance, real estate and administrative services (65-74) receive a higher wage than those in manufacturing. And since wage reflects the labor productivity, we argue that the workers in these two sectors are all highly skilled. But when we analyze the wholesale and retail trade, hotels and restaurants (50-55) subsector, we realize a negative (and statistically significant) differential. So we conclude that the low-skill labor is predominant in this subsector. It remains to consider personal services (social community, personal and government services (75-99)) subsector. Note that it is quite heterogeneous, but we only have data available for the countries in this level of aggregation (from GGDC database). Although education and health are activities that pay a higher wage (relative to manufacturing), they are relatively smaller (in terms of labor) than the other two (other community, social and personal services, and private households with employed persons). And since these two sectors have a negative wage differential, we considered the whole personal services as a sector that employs only low-skill labor. Although all the above analysis has been done with Brazilian data (PNAD), we assumed earlier that both countries have the same production functions, so we argue that these labor definitions are also valid for Korea.

From the definition above of high- and low-skill labor, service productivity is obtained according to (3). Before this, however, we must first determine the value of α. This parameter represents the share of high-skill workers in the total income of the service sector. We used data from PNAD, the Brazilian household survey, and two different methodologies to calculate α. In the first methodology, we considered workers with less than nine years of education as lowly skilled. We then multiply the number of individuals in this group by their average income[32] to obtain total income of low-skill workers, and divided it by the income of the entire services sector.

In the second methodology, we classified sectors as low-skill or high-skill using a definition close to that discussed in the previous paragraph. For each low-skill sector we multiply the number of workers (that were considered to be all lowly skilled) by the average labor income of the sector. We then added the income of all low-skill sectors and divided by the total income of the service sector. We found that the participation of low-skill labor in the total income of services to vary between 0.35 to 0.41, depending on the methodology or whether we included public sector or not. We set $\alpha = 0.40$. We found similar values when using surveys form different years.

In the literature, productivity dispersion across sectors is crucial for the process of structural transformation to contribute to productivity growth. Thus, for each country and each period the

[32] Actually the PNAD does not have information on the exact years of schooling of an individual, but only on intervals of schooling - e.g., "one or less year of education" or "between six and nine years of education". We multiply the total number of individuals by the average income of each interval.

average productivity for the period 1955-1959 is normalized to one, i.e., $A_{A,55-59} = A_{M,55-59} = A_{S,55-59} = 1$. Denoting γ_j as the productivity growth rate of the sector j for in a single period, the productivity of the first (1960-1982) and second (1983-2005) periods are respectively $A_{j,60-82} = (1 + \gamma_j)A_{j,55-59}$ and $A_{j,83-05} = (1 + \gamma_j)^2 A_{j,55-59}$, where $A_{j,60-82}$ is the average productivity of period 1960-1982 and $A_{j,83-05}$ is the average productivity of period 1983-2005.

Most of the parameters are the same for both countries. The intertemporal preference parameters (σ, β, and ε) were calibrated following [Doepke, 2004], while the intratemporal preference parameters followed [Herrendorf et al., 2011] and [Duarte and Restuccia, 2007]. The parameters of manufacturing (b) and services ($1 - b$) weights followed the calibration of [Herrendorf et al., 2011].

Since a developed economy generally spends a smaller fraction of their income in the agricultural goods, the parameters $\bar{c}_A$ and $\bar{c}_S$ depend on the level of development of each country. Therefore, these two parameters were calibrated for each country (Brazil and Korea) and each period (first and second). The parameter $\bar{c}_A$ was computed so that the model reproduced the share of low-skill labor employed in agriculture. And $\bar{c}_S$ was chosen to minimize the distance between the product and the consumption of service.

The calibration of education costs parameters follows [Doepke, 2004]. Furthermore, the fraction of the education cost paid by the government was chosen to be 0.5, as in [Doepke, 2004]. And we assumed that in Korea there was a partially restriction on child labor initially, but in the second period this type of labor was completely abolished. For Brazil, it was assumed no education subsidy by the government and no restrictions on child labor over the years.

6 Numerical Results

6.1 Benchmark Economies

The model reproduces closely the allocations of the different types of labor across the three productive sectors, at the initial (1960-1982) and final (1983-2005) periods. Table 7 presents the results for the initial period. The proportion of low-skill labor employed in agriculture generated by the model is slightly below, but very close to the data in the two countries. The proportion of high-skill labor in manufacturing is slightly above but close to the actual values. Table 8 presents the results for the final period. The allocation of low-skill labor in agriculture produced by the model are only 1 p.p. higher than the actual value in the two economies. With respect to the allocation of high-skill labor in manufacturing, the model is able to reproduce the data in Brazil and misses by only 5 p.p. in Korea. Therefore, the process of structural transformation of the two economies is nicely explained by the model.

Results regarding the proportion of high-skill individuals in the economies are not so good as the labor allocations. This problem occurs for both countries. From Tables 7 and 8, we can observe for Brazil that there is an overestimation of this variable by 21 p.p. initially, but it drops to just 9 p.p. at the final period. For South Korea, there is also a considerable overestimation (difference of 20 p.p.), but it falls to only 4 p.p. at the final period and the model reproduces very well the data. This high deviation, especially at the initial period, may be due to the fact that all workers employed in manufacturing were considered highly skilled. And since manufacturing was initially more developed in the two countries and employed proportionally more workers, this could have led tho the overestimation of high-skill individuals by the model. But the model seems to reproduce more closely the final period of the two economies, especially the Korean.

Given the distribution of high-skill individuals and the allocations of the different types of labor in productive sectors, we can also obtain with the benchmark model the growth rate of the per capita output. The Brazilian benchmark economy produces a growth rate of 36% between the two periods and the Korean per capita output grows by 232%. According to the Penn-World Table Brazil basically stagnate in the period (a negative growth of 8%) and Korean grew by 207%. Hence, although overestimating slightly growth in both cases, the model reproduces the order of magnitude and the relative figures very well. We will use these figures to compare the effect of different types of counterfactuals exercises in the growth of the economies at the next subsection.

6.2 Counterfactuals

The first class of these counterfactuals is to check what would happen to the economy of each country if other public policies had been adopted. In Table 9 we analyze the case in which Brazil subsidizes education and restricts child labor, following Korean second period policies. Initially this policy change does not affect the economy, since the values of the variables of interest are very similar to those in Tables 7 and 8. However at the second period, the proportion of high-skill individuals increases substantially, going from 36% in benchmark model to 60%. This result indicates that the policies implemented by Korea would produce a considerable accumulation of human capital if they had been implemented in Brazil.

This policy also does not produce major changes at the first period in the allocation of labor between sectors. However at the next period it is able to prevent a massive reallocation of low-skill workers from agriculture to the service sector. Therefore, the low-skill labor now represents less than half of all labor employed in the service sector, i.e., high-skill labor becomes the majority in the sector. Furthermore, if these public policies were implemented in Brazil, the growth rate of per capita output would be 57% between the two periods, i.e., 21 p.p. above the growth of the Brazilian benchmark economy (36%). Hence, in an annual base Brazil would grow almost 1% faster, without changing the path of sectorial productivities, had he adopted better educational policies.

In the case of Korea (Table 10), we run an experiment in which policies are such that there is no education subsidy and no child labor restriction. Initially, there is just a small decrease in the proportion of high-skill individuals compared to the benchmark model, and the sector allocations remains virtually unchanged. However, in the second period there is a considerable decrease in the proportion of high-skill individuals in the economy, dropping from 74% in the benchmark model to 42%. In addition, between the first and second period the number of skilled workers grows very little.

Due to this low human capital accumulation, Korea would end up following a trajectory very similar to that observed in Brazil, with a massive migration of low-skill labor from the agricultural sector to services. In this case the period growth rate of the Korean economy falls to only 112%, about half of the growth in the benchmark economy (232%). The smaller supply of high-skill labor negatively affects the production of manufactured goods and services, that are growing fast in the period and becoming dominant.

In a second group of counterfactual exercises countries switch sectorial productivities with each other. When Brazil is given South Korean productivities, from Table 11 we see that the distribution of high-skill individuals in the economy increases in both periods when compared with the benchmark results shown in Tables 7 and 8 (increase of 11 p.p. and 8 p.p. respectively). The distribution of skilled and unskilled labor (in percentage terms) is quite similar to those of the Korean benchmark model. Furthermore, the period growth rate of the Brazilian economy would be 125%, i.e., about three times larger than the growth rate of the benchmark economy.

With South Korea experiencing the Brazilian sectorial productivities, results in Table 12 shows that although the percentage of highly qualified individuals has fallen relative to the benchmark model (67% and 74%, respectively), this decrease is small, especially when compared with the results of the other counterfactual exercises for Korea. The reallocation of high- and low-skilled labor across sectors in the second period is such that two thirds of the low-skill labor is allocated in services, and 60% of high-skill labor goes to manufacturing, a result very similar to the benchmark simulation of Brazil. Moreover, with Brazilian productivities, South Korea output grows only 49% between the two periods, way less than in the benchmark economy (232%).

When comparing the growth rates of the two classes of counterfactuals with those of the benchmark economies, one can see that productivity in manufacturing explains a large portion of the output growth, while the low productivity levels in services (which in this case is the result of a massive allocation of unskilled labor to this sector) play a key role in explaining episodes of growth slowdown or stagnation. The different roles played by the secondary and tertiary sectors had already been discussed in the literature (e.g., [Duarte and Restuccia, 2010]), but not the role of human capital accumulation. In the counterfactual exercise in which Brazil implements South Korean educational policies, its period growth rate increases by 20 p.p. This is the result of a greater human capital accumulation, which reverted the poor performance of the service sector. Similarly when we assume that South Korea adopts Brazilian educational policies, the proportion of second period high-skill workers fall from 74% in the benchmark simulation to just 42%, leading to a fall in the growth rate of output of approximately 50% (from 232% to 112%).

Note however, that results in the second group of counterfactuals are such that the growth rate in Brazil, 125%, is much greater than in the first counterfactual. This so because South Korean manufacturing productivity is very high in the period (1983-2005) and leads to a large gain in the Brazilian overall productivity and growth. Note, however, that the impact in this case is large not only because of the direct effect of the very high TFPs of South Korea on Brazilian output but because of an indirect effect on human capital formation: the proportion of high-skill labor jumps from 31% in the first period to 42% and in the second period it goes from 35% to 44%. Human capital accumulation is still very important.

The third class of counterfactuals is to verify the impact of each public policy separately[33]. In the case of Brazil, from Table 13, it is possible to see that child labor restriction has a very significant impact on the economy. The percentage of high-skill individuals grows with this policy compared with the benchmark economy, from 31% to 48% initially, and in the second period, the growth is even higher, from 35% to 61%. There is also a greater concentration of low-skill labor in agriculture. Manufacturing maintains the same labor concentration than in the benchmark economy in both periods. The impact of education subsidy in the economy is smaller, the increase in the proportion of high-skill individuals is just 7 p.p. in the first period and 10 p.p. in the second. Moreover, in the two periods there is a higher concentration of low-skill labor in agriculture, and the percentage of high-skill labor remains stable and very close to that of the benchmark economy.

In the case of South Korea, the absence of child labor restrictions has an impact of only 1 p.p.

[33] Note, however, that it is difficult to consider the two policies separately because child labor restriction must be incentive compatible and there is a clear link between the cost of education and child labor. So the results presented below must be seen with a caveat in mind.

in the percentage of high-skill individuals in the economy (see Table 14). This fact results from the hypothesis adopted in the first period of the benchmark economy that child labor is partially banned in South Korea. In the second period, when this kind of work is totally prohibited, the impact of relaxing this restriction is significant, amounting to a drop of 20 percentage points. The allocations of the two types of labor in the first period remains similar to the benchmark economy, but in the second period, there is a larger concentration of low-skill individuals in the service sector. Similarly to Brazil, the impact of education subsidy is smaller when considering its effects on the formation of high-skill individuals. A no-subsidy policy produces a decrease in the formation of human capital of, respectively, 8 p.p. and 5 p.p. in the first and second periods.

7 Conclusion

In the second half of the last century, Brazil and South Korea had episodes of accelerated economic growth. However, since the 80's, the Brazilian economy has stagnated and the inefficient service sector is pointed by the structural transformation literature as the responsible. The hypothesis sustained is that the negative growth of labor productivity in the service sector is due to migration of low-skill workers from agriculture to service sector.

We have proposed a model that combines the elements of the structural transformation literature to the microeconomic approach of education and fertility choices. Furthermore, we considered in the model public policies of education subsidy and child labor restrictions. The model provided a good distribution of qualification levels in the population of the two countries, as well as the labor allocations between sectors. With counterfactual exercises, we concluded that policies of education subsidy and child labor restriction are essential for the accumulation of human capital in the economies, with the latter having a higher impact. The greater participation of high-skill individuals in the economy is able to avoid a greater allocation of low-skill workers in the service sector. Furthermore, comparing the growth paths (growth rates) produced by the counterfactual exercises with those produced by the benchmark economies we had the same conclusions pointed by [Duarte and Restuccia, 2010] and [Silva and Ferreira, 2011]. Therefore, using a micro-foundations approach, this article was able to compare and explain the processes of transformation structural, which were initially very similar between Brazil and South Korea, but from a certain point became different.

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A Appendix: Data

The average number of hours worked per year in each of the nine productive sectors (ISIC Rev. 3) over the period of interest (1955-2005) for Brazil was built from the PNAD database. The PNAD is a national household survey conducted every year in Brazil since 1976.

The geographical scope, the concept of work (occupied person) and the classification of economic activities have changed over the years. Regarding the geographical scope, since 1976 PNAD have already covered the Northeast, Southeast and South, the Federal District and metropolitan areas of the North and of others federation units of the Midwest Region. This coverage is reasonably maintained homogeneous until 2003. In 2004, the PNAD was implemented in rural areas of Rondônia, Acre, Amazonas, Roraima, Pará and Amapá and achieved full coverage of the Brazilian national territory[34]. To avoid discontinuities in the data and maintain homogeneous geographical coverage across the years, from 2003 onward we removed the rural areas of the states of the North from the sample.

Regarding the concept of work, there was a big change from 1992[35]. Until 1990, IBGE considered as occupied the person with 10 years or older who had some paid or unpaid work for at least 15 hours during the reference week. But from 1992, instead of the 15 hours required above, the required minimum became 1 hour per week. In order to maintain an harmony between the two concepts of occupied person, we used the definition of the years prior to 1992 for the entire period.

In addition to changes in the geographical scope and in the concept of work, there was also changes in classifications of activities over time. To maintain a homogeneous classification of sectors and avoid breaks in data series, we followed the definitions described in Table 17.

To have the average hours worked by sector data, first we obtained the total number of employed persons in each sector and the total hours worked. From these two series, the average hours became as the result of the ratio of the total hours by the number of persons employed in each production sector.

[34] For details, see *www.ibge.gov.br/home/estatistica/populacao/trabalhoerendimento/pnad2005/*.

[35] See [Silva and Grossi, 1997].

B Appendix: Proofs

Proof of Proposition 2. (Taken from [Doepke, 2004])

To show that there are no interior solutions, first assume that the optimal E_i is already determined.

Given this E and the fact that the function to be maximized is twice continuously differentiable in f, if there is an interior solution, the optimal f will have to satisfy first and second order conditions.

The strategy of proof will be find the unique f which solves the first order condition and then show that this f does not satisfy the second order condition.

We will name the function to be maximized $V_i(.)$, the first and second derivatives of V_i with respect to f are:

$$\frac{\partial V_i}{\partial f} = \beta E_i^{1-\varepsilon}\left[-\varepsilon\left(\frac{1}{p_{i,H}} - \frac{1}{p_{i,L}}\right)\left(\frac{f}{p_{i,H}} - \frac{1-f}{p_{i,L}}\right)^{-\varepsilon-1}\left(\frac{fV_H}{p_{i,H}} - \frac{(1-f)V_L}{p_{i,L}}\right) + \right.$$
$$\left. + \left(\frac{f}{p_{i,H}} - \frac{1-f}{p_{i,L}}\right)^{-\varepsilon}\left(\frac{V_H}{p_{i,H}} - \frac{V_L}{p_{i,L}}\right)\right],$$

$$\frac{\partial^2 V_i}{\partial^2 f} = \beta E_i^{1-\varepsilon}\left[\varepsilon(1+\varepsilon)\left(\frac{1}{p_{i,H}} - \frac{1}{p_{i,L}}\right)^2\left(\frac{f}{p_{i,H}} - \frac{1-f}{p_{i,L}}\right)^{-\varepsilon-2}\left(\frac{fV_H}{p_{i,H}} - \frac{(1-f)V_L}{p_{i,L}}\right) - \right.$$
$$\left. -2\varepsilon\left(\frac{1}{p_{i,H}} - \frac{1}{p_{i,L}}\right)\left(\frac{f}{p_{i,H}} - \frac{1-f}{p_{i,L}}\right)^{-\varepsilon-1}\left(\frac{V_H}{p_{i,H}} - \frac{V_L}{p_{i,L}}\right)\right].$$

Assuming that the first order condition is equal to zero, i.e., $\frac{\partial V_i}{\partial f} = 0$, the optimal f is:

$$f = \frac{\varepsilon\left(\frac{V_L}{p_{i,H}} - \frac{V_L}{p_{i,L}}\right) - \left(\frac{V_H}{p_{i,H}} - \frac{V_L}{p_{i,L}}\right)}{(1-\varepsilon)p_L\left(\frac{V_H}{p_{i,H}} - \frac{V_L}{p_{i,L}}\right)\left(\frac{1}{p_{i,H}} - \frac{1}{p_{i,L}}\right)}.$$

Now we will plug this value of f into the second derivative to verify that it is positive, and, consequently, the critical point is not a maximum.

To verify that $\frac{\partial^2 V_i}{\partial^2 f} > 0$, it suffices to show that:

$$(1+\varepsilon)\left(\frac{1}{p_{i,H}} - \frac{1}{p_{i,L}}\right)\left(\frac{f}{p_{i,H}} - \frac{1-f}{p_{i,L}}\right)^{-1}\left(\frac{fV_H}{p_{i,H}} - \frac{(1-f)V_L}{p_{i,L}}\right) - 2\left(\frac{V_H}{p_{i,H}} - \frac{V_L}{p_{i,L}}\right) < 0.$$

Plugging the optimal f in the above expression and performing some algebric operations, we can obtain:

$$(1+\varepsilon)\frac{1}{p_{i,L}}\left(\frac{V_L}{p_{i,H}}-\frac{V_L}{p_{i,L}}\right)<\frac{1}{p_{i,L}}\left[\varepsilon\left(\frac{V_L}{p_{i,H}}-\frac{V_L}{p_{i,L}}\right)-\left(\frac{V_H}{p_{i,H}}-\frac{V_L}{p_{i,L}}\right)\right]+2\frac{1}{p_{i,L}}\left(\frac{V_H}{p_{i,H}}-\frac{V_L}{p_{i,L}}\right)$$

$$\Rightarrow\left(\frac{V_L}{p_{i,H}}-\frac{V_L}{p_{i,L}}\right)<\left(\frac{V_H}{p_{i,H}}-\frac{V_L}{p_{i,L}}\right)$$

$$\Rightarrow V_L<V_H.$$

Note that this condition is always satisfied, because otherwise ($V_L > V_H$) there would be only unskilled children in population because they are already cheaper and would generate greater utility. Therefore, we have $\frac{\partial^2 V_i}{\partial^2 f} > 0$ and there is no interior solution.

□

Proof of Proposition 3.

Differentiating totally the first order condition, we obtain the following derivatives:

$$\frac{dn_k}{dV_k}=\frac{n_k}{\varepsilon V_k+(1-\sigma)p_{i,k}n_kV_k\left[\frac{b^2}{p_Mc_M}+\frac{(1-b)^2}{p_Sc_S}\right]}>0,$$

and:

$$\frac{dn_k}{dw_i}=\frac{(1-\tau)\,(1-\sigma)n_k}{\varepsilon\left[\frac{b^2}{p_Mc_M}+\frac{(1-b)^2}{p_Sc_S}\right]^{-1}+(1-\sigma)p_{i,k}n_k}>0.$$

□

Proof of Proposition 4.

Consider the problem in which an adult chooses the total education cost E_i, so that the number of children is given by $n_{i,k} = E_i/p_{i,k}$ (since each parent chooses only one type of child):

$$\max_{0\leq E_i\leq\bar{w}_i/p_{i,k}}\left\{\bar{U}+\beta(E_i/p_{i,k})^{1-\varepsilon}V_k\right\}.$$

This problem can be rewritten as:

$$\max_{0\leq E_i\leq\bar{w}_i/p_{i,k}}\left\{U+\beta(E)^{1-\varepsilon}\frac{V_k}{(p_{i,k})^{1-\varepsilon}}\right\}.$$

Since the costs and the utilities appear only in the last term of the above problem, an adult is indifferent between the two types of child if, and only if:

$$\frac{V_H}{p_{i,H}^{1-\varepsilon}} = \frac{V_L}{p_{i,L}^{1-\varepsilon}}.$$

Note that this condition does not depend on the wage and the adults face the same maximization problem regardless the type of child. Thus, the optimal total education cost does not depend on the type of the children and the higher cost of skilled children is equivalent to a fewer number of children.

□

C Appendix: Equilibrium

Another way of characterizing the equilibrium is described below.

First, we have to find the aggregate consumption of agricultural, manufacturing and service goods. The aggregate consumption of the good j, where $j \in \{A, M, S\}$, is given by

$$C_j \equiv \int c_j(i) di. \tag{21}$$

Substituting the optimal choices for each individual consumer in the above equation and assuming that there is a continuum of individuals in each period, we obtain the aggregate consumption of each good:

- Agricultural

$$C_A \equiv \int c_A(i) di = \int \bar{c}_A di \qquad \Rightarrow \qquad C_A = \bar{c}_A (N_H + N_L); \tag{22}$$

- Manufacturing

$$C_M \equiv \int c_M(i) di = \int \frac{b}{p_M}[e_i - p_A \bar{c}_A + p_S \bar{c}_S] di \qquad \Rightarrow$$

$$C_M = \frac{b}{p_M} \int e_i di + \frac{b}{p_M}[-p_A \bar{c}_A + p_{Se} \bar{c}_{Se}](N_H + N_L); \tag{23}$$

- Services

$$C_S \equiv \int c_S(i) di = \int \frac{(1-b)}{p_S}[e_i - p_A \bar{c}_A + p_S \bar{c}_S] - \bar{c}_S di \qquad \Rightarrow$$

$$C_S = \frac{(1-b)}{p_S} \int e_i di + \left[\frac{(1-b)}{p_S}[-p_A \bar{c}_A + p_S \bar{c}_S] - \bar{c}_S \right] (N_H + N_L). \tag{24}$$

Note that $\int e_i di$ is the total income in the economy after the total expenditure on raising children and represents the total disposable income to consumption of the skilled workers and the teachers $(L_H w_H + \phi_H N_H^{'} w_H)$, and the unskilled workers $(L_L w_L)$.

$$\int e_i di = (1-\tau) \left(L_H w_H + \phi_H N_H^{'} w_H + L_L w_L \right). \tag{25}$$

Therefore, the aggregate consumption of manufacturing and service goods is given by

$$C_M = \frac{b}{p_M} \left[(1-\tau) \left(L_H w_H + \phi_H N_H^{'} w_H + L_L w_L \right) + (-p_A \bar{c}_A + p_S \bar{c}_S)(N_H + N_L) \right], \tag{26}$$

$$C_S = \frac{(1-b)}{p_S} \left[(1-\tau) \left(L_H w_H + \phi_H N_H^{'} w_H + L_L w_L \right) + (-p_A \bar{c}_A + p_S \bar{c}_S)(N_H + N_L) \right] - \bar{c}_S (N_H + N_L). \tag{27}$$

The solution of the maximization problem of adults is obtained from the value function iteration. With this method, we can determine the number of children of each type that low-skill parents want to have, and the number of high-skill children that high-skill parents choose, as well as the fraction of low-skill parents who have high-skill children. With these informations, the measure of high-skill (N_H) and low-skill (N_L) people of the next period are easily calculated, and the labor supply of both types are obtained by (15).

We can now determine the equilibrium of the model. From the equilibrium in the agricultural market, we obtain the amount of low-skill labor employed in this sector from equations (1)and (22)

$$Y_A = C_A \qquad \Rightarrow \qquad Z_A L_{A,L} = \bar{c}_A(N_H + N_L) \qquad \Rightarrow$$
$$\Rightarrow \qquad L_{A,L} = \frac{\bar{c}_A}{Z_A}(N_H + N_L). \tag{28}$$

Initially we assume that $w_H(X) > w_L(X)$ and then we verify if it is satisfied. The remainining low-skill labor is employed in the service sector

$$L_{A,L} + L_{S,L} = L_L \qquad \Rightarrow \qquad L_{S,L} = L_L - \frac{\bar{c}_A}{Z_A}(N_H + N_L). \tag{29}$$

Substituting (29) into (11), we obtain the high-skill labor employed in the service sector as a function of w_H and p_S

$$L_{S,H} = \left[\frac{\alpha Z_S p_S}{w_H}\right]^{1/(1-\alpha)} \left[L_L - \frac{\bar{c}_A}{Z_A}(N_H + N_L)\right]. \tag{30}$$

And substituting (30) into $L_{M,H} + L_{S,H} = L_H$, we have the amount of skilled labor employed in manufacturing as a function of w_H and p_S

$$L_{M,H} = L_H - \left[\frac{\alpha A_S p_S}{w_H}\right]^{1/(1-\alpha)} \left[L_L - \frac{\bar{c}_A}{Z_A}(N_H + N_L)\right]. \tag{31}$$

Since we already have the allocation of labor between sectors as a function of w_H, now we have to determine this wage[36]. For this, we simply replace the labor allocations $L_{S,H}$ and $L_{S,L}$, and the wage $w_L = Z_A$ in (12)

$$w_H = \left[p_S \alpha^\alpha (1-\alpha)^{1-\alpha} Z_S Z_A^{-(1-\alpha)}\right]^{1/\alpha}. \tag{32}$$

With the wage depending only on the price p_S, we need to substitute it in the labor allocations $L_{S,H}$ and $L_{M,H}$ to obtain all the variables of interest as functions of only the price p_S

$$L_{S,H} = \left[\frac{Z_A}{p_S(1-\alpha)Z_S}\right]^{1/\alpha} \left[L_L - \frac{\bar{c}_A}{Z_A}(N_H + N_L)\right], \tag{33}$$

[36] Note that the wage of low-skill workers is already determined by (9): $w_L = Z_A$.

$$L_{M,H} = L_H - \left[\frac{Z_A}{p_S(1-\alpha)Z_S}\right]^{1/\alpha}\left[L_L - \frac{\bar{c}_A}{Z_A}(N_H + N_L)\right]. \tag{34}$$

Thus, we are able to obtain the production of service goods just by replacing $L_{S,H}$ and $L_{S,L}$ in (3)

$$Y_S = \frac{Z_A Z_S}{p_S(1-\alpha)}\left[L_L - \frac{\bar{c}_A}{Z_A}(N_H + N_L)\right]. \tag{35}$$

Now, simply imposing the market-clearing condition in the service market we find the equilibrium price p_S

$$Y_S = C_S \qquad \Rightarrow \qquad \frac{Z_A Z_S}{p_S(1-\alpha)}\left[L_L - \frac{\bar{c}_A}{Z_A}(N_H + N_L)\right] =$$

$$= \frac{(1-b)}{p_S}\left[(1-\tau)\left(L_H w_H + \phi_H N_H^{'} w_H + L_L w_L\right) + (-p_A\bar{c}_A + p_S\bar{c}_S)(N_H + N_L)\right] - \bar{c}_S(N_H + N_L).$$

Substituting w_S defined at (32) and $w_L = Z_A$ in the above equation, we obtain the expression which determines implicitly the equilibrium price p_S

$$\frac{A_A A_{Se}}{p_{Se}(1-\alpha)}\left[L_U - \frac{\bar{c}_A}{A_A}(N_S + N_U)\right] = \tag{36}$$

$$= \frac{(1-b)}{p_{Se}}\left\{(1-\tau)\left(\left[L_H + \phi_H N_H^{'}\right]\left[p_S\alpha^{\alpha}(1-\alpha)^{1-\alpha}Z_S Z_A^{-(1-\alpha)}\right]^{1/\alpha} + L_L Z_A\right) + \right.$$
$$\left. + \left(-p_A\bar{c}_A + p_S\bar{c}_S\right)(N_H + N_L)\right\} - \bar{c}_S\left(N_H + N_L\right)$$

With the equilibrium price p_S, we substitute it in (32) to find w_H. And with w_H, we obtain p_M from (10). Moreover, we just replace p_S in (33) and (34) to find the labor allocations $L_{S,H}$ and $L_{M,H}$. Thus, the equilibrium of the model is determined. Since the equilibrium was established in the labor, agricultural, and services market, the Walra's Law guarantees the equilibrium in manufacturing sector.

D Appendix: Algorithm

Computing the equilibrium of the model:

1. Form a vector of possible values for the equilibrium price p_S and for each of these values, perform the procedure below.

2. Guess the following initial values:

   (a) The measure of high- and low-skill individuals in the economy;

   (b) The number of high- and low-skill children that each type of parent decides to have;

   (c) The fraction of low-skill parents who choose to have high-skill children;

   (d) The future utility of each type of individual (or function value);

   (e) The tax rate if there are government policies in the economy.

3. From these initial values, calculate the supply of each type of labor.

4. With the price p_S guessed initially, find the wages and the manufacturing price p_M. From these results, get the education and raising costs of each child for high- and low-skill parents.

5. Now form a vector of possible numbers of children:

   (a) Skilled that skilled parents may have;

   (b) Low-skill that low-skill parents choose;

   (c) High-skill that low-skill parents may have.

6. Perform the iteration of the value function for the problem of each type of parent.

7. From the result of the optimal number of children, compute the measure of individuals of each type and the income tax rate.

8. Verify if the condition (19) is satisfied. If the ratio of the value function of the high-skill child with their cost (right side of equation (19)) is greater than the ratio of the low-skill child, update the fraction of low-skill parents who decide to have high-skill children, increasing it. Otherwise, this fraction should decrease.

9. Calculate the difference between the new and the former fraction.

10. Update the measure of individuals of each type, the number of children, the two value functions and the income tax rate, making a convex combination of the old value with the new value of each variable.

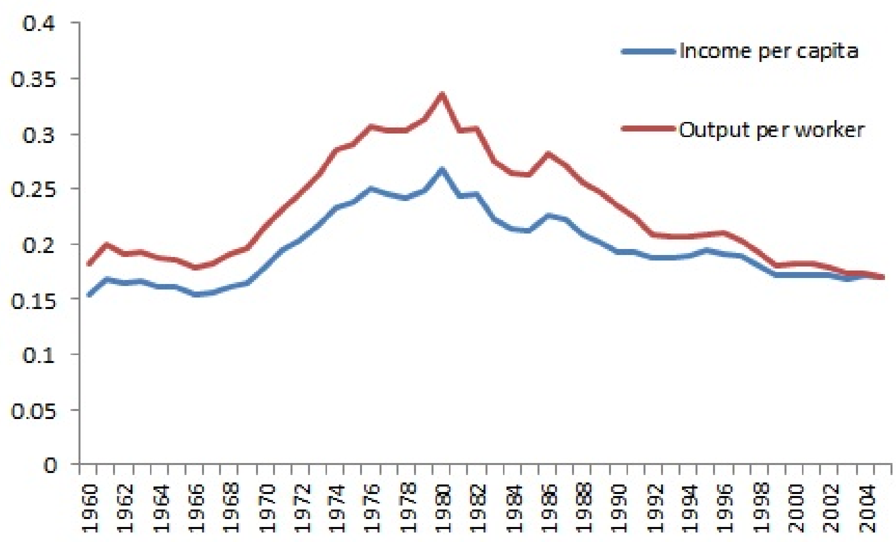
11. While the criterion, formed by the sum of the maximum difference between the old value and the new value of the function of each type with the difference between the old and new fractions of low-skill parents who decide to have high-skill children, is high enough or the number of iterations is less than a maximum established, steps 3. to 10. should be repeated.

12. After all these computation, for each possible equilibrium price p_S, there will be the values of the equilibrium variables defined in the subsection 4.1.

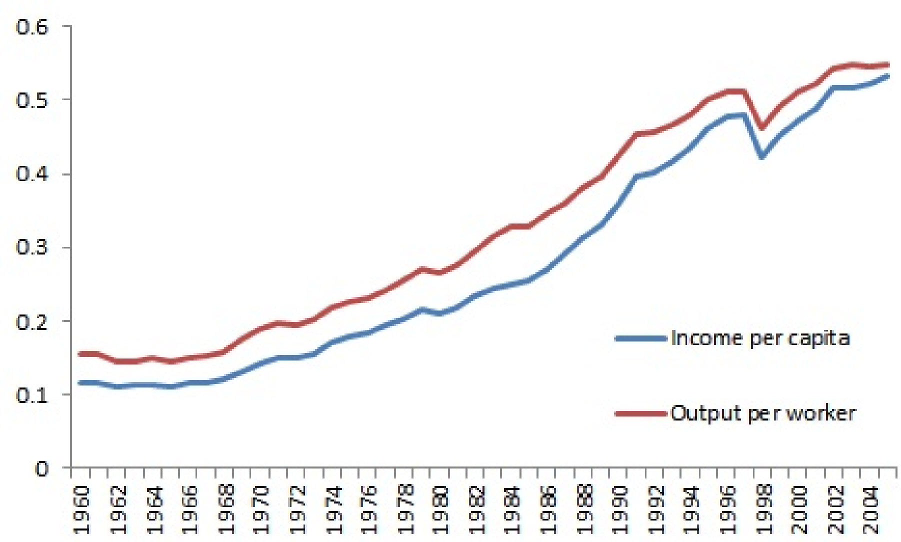
13. The equilibrium price p_S will be the one that the aggregate consumption is closest to the value of production of the goods.

E Appendix: Figures

Figure 1: Evolution of per capita income and output per worker relative to United States

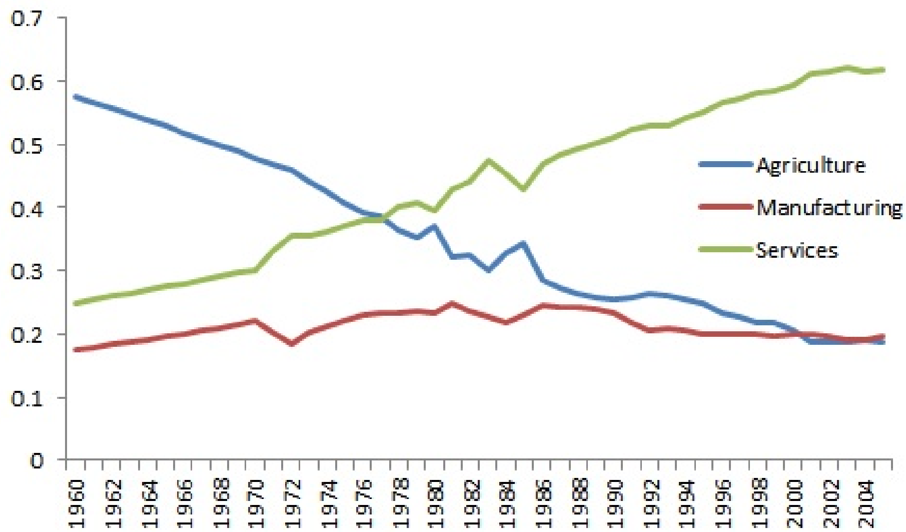


(a) Brazil

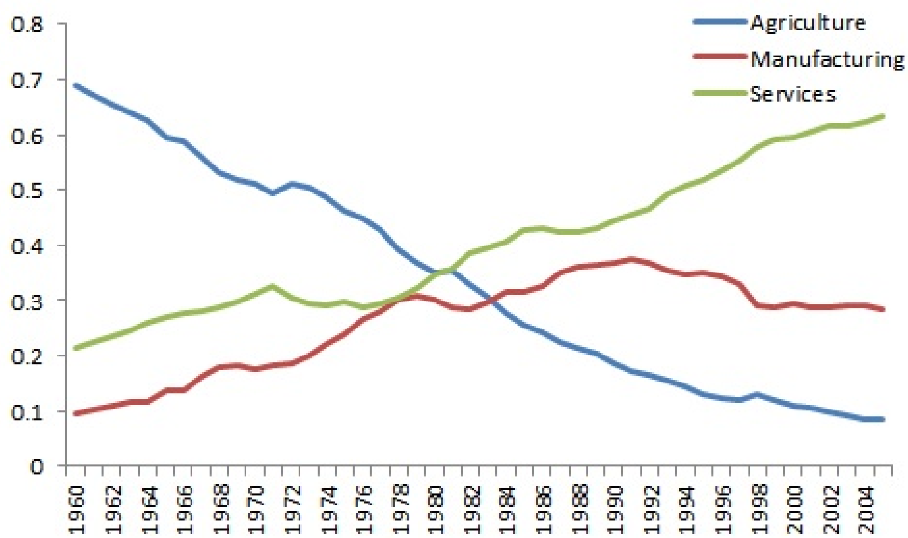


(b) South Korea

Figure 2: Evolution of labor force share by sector

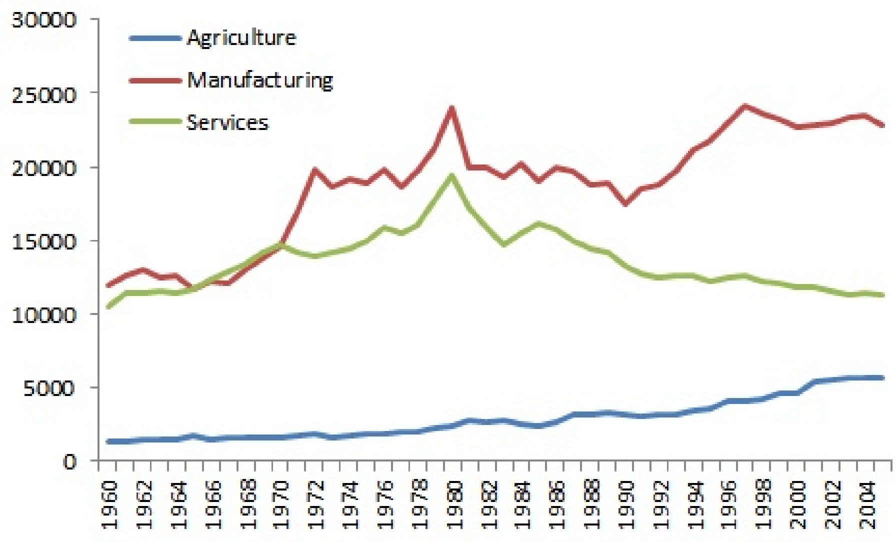


(a) Brazil

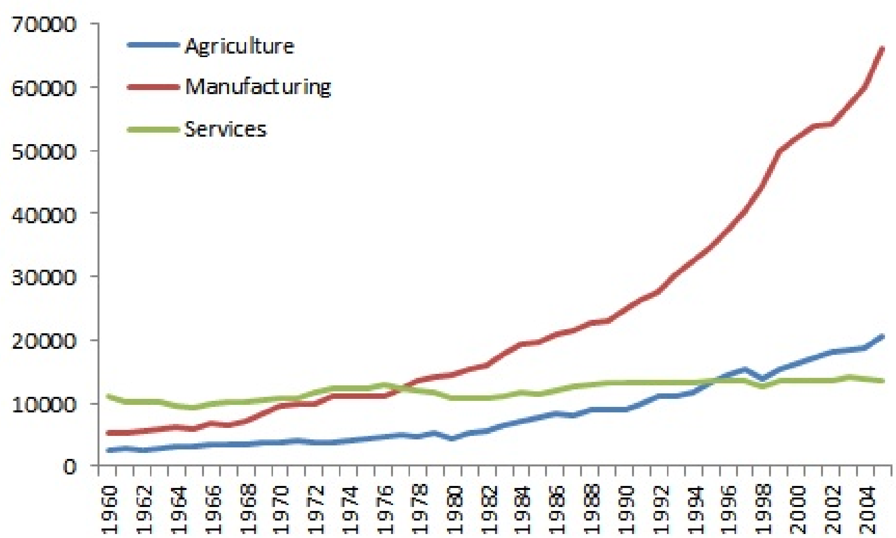


(b) South Korea

Figure 3: Evolution of output per worker by sector (US$ PPP)

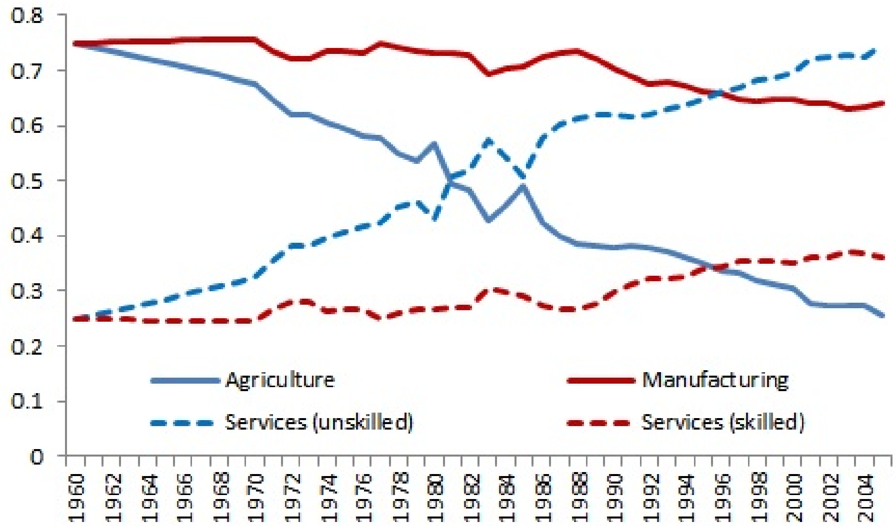


(a) Brazil

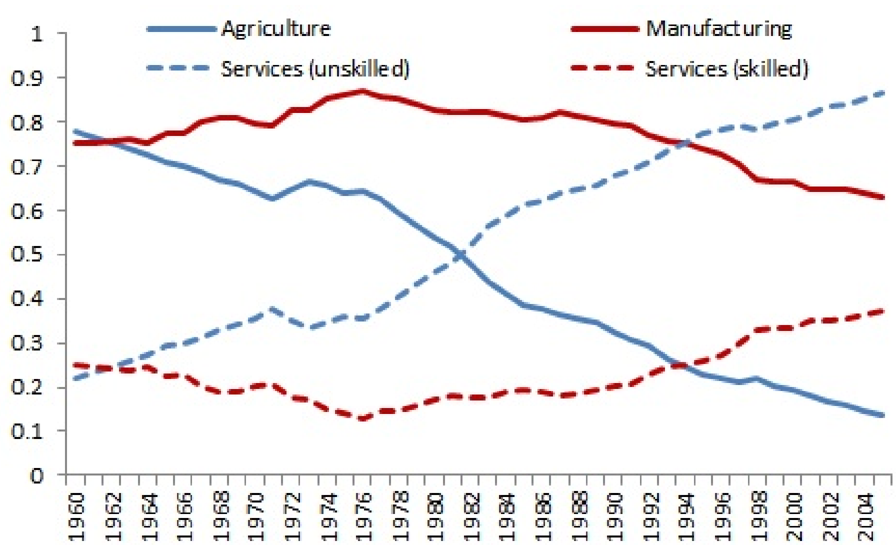


(b) South Korea

Figure 4: Evolution of labor share of each type (skilled or unskilled) by sector

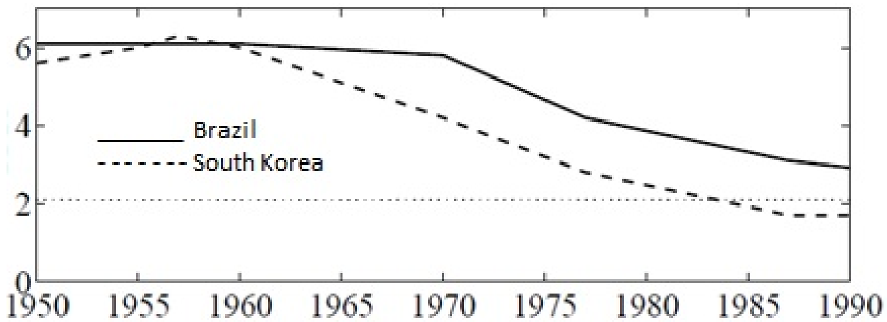


(a) Brazil



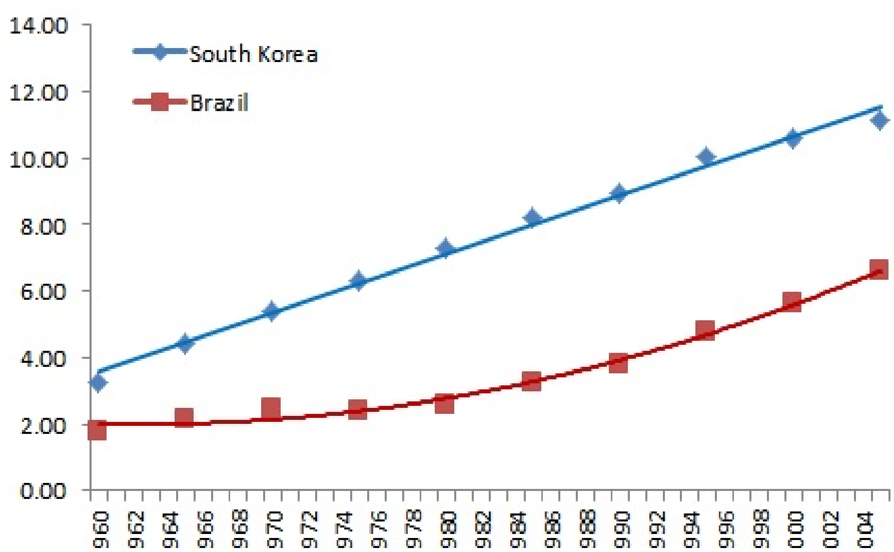
(b) South Korea

Figure 5: Evolution of total fertility



Source: [Doepke, 2004]

Figure 6: Average years of schooling of economically active population



F Appendix: Tables

Table 1: Calibration

Parameter	Value
α	0.6
σ	0.5
β	0.132
ϵ	0.5
b	0.15
$\bar{c}^{Kor}_{A,60-82}$	0.55
$\bar{c}^{Kor}_{A,83-05}$	0.35
$\bar{c}^{Br}_{A,60-82}$	0.7
$\bar{c}^{Br}_{A,83-05}$	0.55
$\bar{c}^{Kor}_{S,60-05}$	1.3
$\bar{c}^{Br}_{S,60-05}$	1.6
ϕ	0.155
ϕ_H	0.04
ϕ_L	0.07
δ	0.5
$\phi^g_{L,60-82}$	0.069
$\phi^g_{L,83-05}$	0

Table 2: Evolution of public expenditure on education in Brazil

Year	Public expenditure on education (%GDP)	Cost per pupil (%GDP per capita) Primary	Secondary	Tertiary
1950	1.4	10	133	750
1955	1.6	10	95	950
1960	1.7	8	78	939
1965	2.4	11	42	873
1970	2.9	11	32	384
1975	2.6	11	27	167
1980	2.4	10	16	157
1985	2.9	12	18	161
1990	3.8	15	18	233
1995	3.9	14	16	201
2000	4	13	14	210

Source: [Maduro Junior, 2007]

Table 3: Adjusted enrollment ratios

	Primary		Secondary	
Year	Brazil	Korea	Brazil	Korea
1960	0.95	0.94	0.11	0.27
1965	1.0	1.0	0.16	0.35
1970	0.72	1.0	0.26	0.42
1975	0.88	1.0	0.26	0.56
1980	0.99	1.0	0.34	0.76
1985	1.0	0.96	0.35	0.95
1990	1.0	1.0	0.39	1.0

Source: [Doepke, 2004]

Table 4: Brazil: Percentage of population with highest level attained

		Primary		Secondary		Tertiary	
Year	No schooling	Total	Completed	Total	Completed	Total	Completed
1960	56.00	36.75	15.71	5.97	3.62	1.21	1.10
1965	49.37	41.42	17.56	7.54	3.84	1.57	1.40
1970	42.50	46.00	19.40	9.60	3.70	2.00	1.70
1975	32.70	57.30	4.30	5.70	2.28	4.30	3.47
1980	32.90	55.30	4.90	6.90	2.90	5.00	3.69
1985	28.87	55.44	19.53	10.24	4.54	5.45	4.05
1990	24.99	54.88	27.04	14.18	6.66	5.94	4.35
1995	20.69	53.79	30.52	18.96	9.52	6.56	4.76
2000	17.75	49.54	30.44	25.35	13.85	7.30	5.25
2005	15.66	41.79	26.51	34.21	21.25	8.09	5.77

Table 5: South Korea: Percentage of population with highest level attained

		Primary		Secondary		Tertiary	
Year	No schooling	Total	Completed	Total	Completed	Total	Completed
1960	56.90	29.60	26.20	10.90	5.80	2.60	1.88
1965	43.60	35.20	33.50	17.50	7.55	3.60	2.67
1970	34.30	38.10	26.50	21.80	9.90	5.70	4.29
1975	25.20	39.20	37.40	28.70	13.04	6.90	5.29
1980	19.70	34.50	33.04	36.90	18.70	8.90	6.58
1985	15.41	27.62	26.55	45.26	24.42	11.71	8.19
1990	14.22	24.35	23.43	41.86	25.14	19.57	12.67
1995	8.70	18.20	17.30	51.90	36.20	21.10	12.62
2000	7.45	14.79	14.21	50.90	35.98	26.82	14.75
2005	5.62	12.70	12.27	49.69	37.12	31.99	16.26

Table 6: Percentage of high-skill individuals in population

Year	Brazil	Korea
1960	7.18	13.50
1965	9.11	21.10
1970	11.60	27.50
1975	10.00	35.60
1980	11.90	45.80
1985	15.69	56.97
1990	20.12	61.43
1995	25.52	73.00
2000	32.65	77.72
2005	42.30	81.68

Table 7: Distribution of the variables of interest in the initial period (1960-1982)

	Brazil		Korea	
	Data	Model	Data	Model
N_H	0.10	0.31	0.29	0.49
$L_{A,L}/L_L$	0.64	0.63	0.65	0.65
$L_{M,H}/L_H$	0.74	0.77	0.81	0.82
$L_{S,L}/L_L$	0.36	0.37	0.35	0.35
$L_{S,H}/L_H$	0.26	0.23	0.19	0.18

Table 8: Distribution of the variables of interest in the final period (1983-2005)

	Brazil		Korea	
	Data	Model	Data	Model
N_H	0.27	0.35	0.70	0.74
$L_{A,L}/L_L$	0.35	0.36	0.27	0.28
$L_{M,H}/L_H$	0.67	0.67	0.74	0.69
$L_{S,L}/L_L$	0.65	0.64	0.26	0.72
$L_{S,H}/L_H$	0.33	0.34	0.73	0.31

Table 9: Counterfactual: Brazil with Korean public policies

	1960 - 1982	1983 - 2005
N_H	0.38	0.60
$L_{A,L}/L_L$	0.73	0.57
$L_{M,H}/L_H$	0.80	0.65
$L_{S,L}/L_L$	0.27	0.43
$L_{S,H}/L_H$	0.20	0.35

Table 10: Counterfactual: Korea with Brazilian public policies

	1960 - 1982	1983 - 2005
N_H	0.37	0.42
$L_{A,L}/L_L$	0.62	0.13
$L_{M,H}/L_H$	0.81	0.66
$L_{S,L}/L_L$	0.38	0.87
$L_{S,H}/L_H$	0.19	0.34

Table 11: Counterfactual: Brazil with Korean sectoral productivity

	1960 - 1982	1983 - 2005
N_H	0.42	0.44
$L_{A,L}/L_L$	0.74	0.23
$L_{M,H}/L_H$	0.90	0.72
$L_{S,L}/L_L$	0.26	0.77
$L_{S,H}/L_H$	0.10	0.28

Table 12: Counterfactual: Korea with Brazilian sectoral productivity

	1960 - 1982	1983 - 2005
N_H	0.37	0.67
$L_{A,L}/L_L$	0.58	0.34
$L_{M,H}/L_H$	0.66	0.59
$L_{S,L}/L_L$	0.42	0.66
$L_{S,H}/L_H$	0.34	0.41

Table 13: Counterfactual: Brazil with a total restriction on child labor

	1960 - 1982	1983 - 2005
N_H	0.48	0.61
$L_{A,L}/L_L$	0.79	0.45
$L_{M,H}/L_H$	0.77	0.64
$L_{S,L}/L_L$	0.21	0.55
$L_{S,H}/L_H$	0.23	0.36

Table 14: Counterfactual: Korea without restriction on child labor

	1960 - 1982	1983 - 2005
N_H	0.48	0.54
$L_{A,L}/L_L$	0.66	0.17
$L_{M,H}/L_H$	0.84	0.66
$L_{S,L}/L_L$	0.34	0.83
$L_{S,H}/L_H$	0.16	0.34

Table 15: Counterfactual: Brazil with policy of education subsidy

	1960 - 1982	1983 - 2005
N_H	0.38	0.45
$L_{A,L}/L_L$	0.73	0.44
$L_{M,H}/L_H$	0.79	0.67
$L_{S,L}/L_L$	0.27	0.56
$L_{S,H}/L_H$	0.21	0.33

Table 16: Counterfactual: Korea without policy of education subsidy

	1960 - 1982	1983 - 2005
N_H	0.41	0.69
$L_{A,L}/L_L$	0.54	0.23
$L_{M,H}/L_H$	0.80	0.66
$L_{S,L}/L_L$	0.46	0.77
$L_{S,H}/L_H$	0.20	0.34

Table 17: Classification of economic activities in sectors

	1976 - 1979	**1981 - 1990**	1992 - 2001	2002 - 2005
Agriculture	011-036,581	011-042,581	011-042,581	01101-05002
Mining	051-054	050-059	050-059	10000-14004
Manufacturing	100-300	100-300	100-300	15010-37000
Public Utilities	351-354	351-354	351-354	40010-41000
Construction	340	340	340	45005-45999
Trade, Hotels, Restaurants	420-434,511-512,521-528	410-424,511-512,521-525	410-424,511-512,521-525	50010-55030
Transport, Communication	471-483,582	471-482,583,586-588	471-482,583,586-588	60010-64020
Finance, Insurance, Real Estate	451-454,461-466,539,546,571-579,583-584,589,614	451-453,461-464,533,543,571-578,582,584-585,589,612,614	451-453,461-464,533,543,571-578,582,584-585,589,612,614	65000-74090
Community, Social, Personal, Government Services	531-532,541-542,547-549,551-552,611-613, 615-619,621-632,711-729,811-814	531-532,541-542,544-545,551-552,610,611,613, 615-619,621-632,711-727,801-902	531-532,541-542,544-545,551-552,610,611,613, 615-619,621-632,711-727,801-903	75011-99999