Industry Dynamics, Investment and Uncertainty*

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Abstract

When there are non-convexities in firm level technologies and firms operate under uncertainty, dispersion in marginal products can arise as the outcome of an efficient allocation. Reallocation of factors that equalize marginal products may not be efficiency improving. I analyze an economy where heterogeneous firms compete monopolistically, invest in technology and decide when to enter and exit the market. The competitive allocation is inefficient because i) firms compete monopolistically, and ii) inefficient entry, exit and investment patterns induced by imperfect competition feed back into equilibrium dispersion in marginal products. The latter generates endogenous TFP losses through misallocation of factors. I characterize and compute the efficient allocation for a calibrated economy to the US manufacturing sector. I find that productivity can be increased by 11 percent when implementing the efficient allocation via state contingent taxes and subsidies. Only a third of the gains are explained by reallocation of inputs across incumbents that reduces marginal product dispersion. Most of the gains are accounted by changes in equilibrium exit, entry and technology investment. Finally, I show that the equilibrium level of dispersion in marginal products depends on the degree of uncertainty that firms face. It is possible for low and high uncertainty economies to display similar measures of dispersion in marginal products. However, low uncertainty economies are more efficient as long as they display better selection of firms into the market and across technologies. [JEL Codes: E32,L11,E23].

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1 Introduction

Dispersion in marginal products within narrowly defined industries is a stylized fact of modern economies[1]. There are many reasons for which marginal productivity of inputs may differ across firms. Some of the most extensively analyzed mechanisms in the literature are size dependent policies [2], subsidies or taxes for particular firms[3] and market incompleteness (i.e. financial frictions [4] or more generally, contractual frictions[5]). These mechanisms can explain a large portion of the documented dispersion. In this paper, I argue that dispersion in marginal products may arise as the outcome of an efficient allocation. Hence, some of the observed pattern in the data need not be detrimental for productivity or welfare. In this paper, I characterize optimal policy in an economy in which dispersion in marginal product of capital arises endogenously through irreversibility in investment when firms operate under uncertainty. Inefficiency in the model stems from imperfect competition. Disparities between the market and efficient allocation are reflected in aggregate capital accumulation, the measure of firms operating in the market, and equilibrium dispersion in marginal products.

In the economy that I study, irreversibility in investment when firms operate under uncertainty, generates dispersion in marginal products. Consider the following example. There are two firms that have access to the same set of production technologies and different marginal products: an incumbent operating at high capacity and an entrant operating at a lower scale. If firm level technologies were convex, such disparity could be interpreted as an inefficiency. Suppose, however, that the incumbent firm had entered the market at an earlier time during a boom, made a (partially) irreversible investment, and current market conditions have worsened since. Given current conditions, the entrant finds it optimal to wait to scale up its capacity until market conditions improve, while the incumbent does not exit because its option value of remaining in the market is positive[6]. Hence, a gap between the marginal products of capital for these two otherwise identical firms is consistent with

[1]For cross country evidence refer to Asker et al. (2011). For evidence for Korea, refer to Midrigan and Xu (2009). Also, Hsieh and Klenow (2009) provide evidence for the US, India and China. For evidence in Latin America, see Buso et al. (2013).

[2]Barstelman et al. (2013) document and study the impact of distortions that are correlated with the size of firms.

[3]Restuccia and Rogerson (2008) analyze a broad range of policy distortions.

[4]See Buera and Shin (2011), Moll (2013), Midrigan and Xu (2009) and the extensive literature thereafter.

[5]As in (Eisfeldt and Rampini (2006)).

[6]The intuition is analogous to the impact of factor specificity as analyzed in Caballero and Hammour (1998).

optimal investment strategies.

In the model, decisions to enter and exit the market, as well as technology selection are costly and modelled as real options. A technology is a productivity level and an associated minimum capacity in terms of capital. More productive technologies have a higher minimum capacity associated to them. I assume away idiosyncratic shocks, so that at the moment of entry, each investor is assigned a blueprint (a technique to produce a good), the quality of which varies over a continuum of types and is constant in time[7]. I solve for the industry equilibrium by means of a centralized problem with transfers, whose allocation coincides with the market one.

I calibrate this stylized economy to the US manufacturing sector, and ask whether the allocation can be Pareto improved either by narrowing differences in marginal product or by changing the industry dynamic (entry, exit, adoption). I show that shifts in the patterns of entry, exit and investment have a larger quantitative contribution to productivity gains than those associated to drops in marginal product dispersion. This finding is consistent with micro empirical analysis that documents substantial productivity improvements associated to shifts in the patterns of firm churning (Haltinwanger, Davis et al. (2007) and Eslava et al. (2004)). It is also consistent with the literature that studies the impact of the slow down in firm churning in the US for overall productivity (Thomas et al. (2014), Pugsley and Sahin (2014)).

The main feature that generate equilibrium dispersion in marginal products is the combination of minimum capacity constraints that ocasionally bind with uncertainty. There are two pieces of empirical evidence, that put together, suggest that minimum capacity constraints can be in line with the data. First, measures of dispersion in the marginal product of capital fluctuate with the cycle (Eisfeldt and Rampini (2006)), they are countercyclical. In the model the aggregate state of the economy dictates fluctuations in the distribution of marginal products along this line. Second, there is also evidence that dispersion in revenue TFP at the plant level is countercyclical, and that the increase in dispersion is explained mostly by a larger right tail, i.e. more firms with lower revenue productivity (Kehrig (2011)). With constant return technologies, revenue productivity is proportional to marginal product

[7] Firm level uncertainty can be allowed, and may reinforce the channel studied in this paper. Higher uncertainty increases the value of operating a given technology, and may induce firms to stay at their minimum capacity (with low MPK) longer than without it. However, more uncertainty implies that the decision to invest in a more productive (higher minimum capacity) technology is delayed. Towards the end of paper I vary the level of uncertainty in the economy to illustrate these two counteracting effects.

of inputs. It is possible to argue that part of the increase in the right tail observed in the data is accounted for by an increase in dispersion in marginal product of capital, mostly due to lower marginal products. Our economy implies that recessions are periods where more firms operate with low marginal product of capital along the lines of the empirical evidence[8].

In this economy, the equilibrium allocation is inefficient. Monopolistic competition generates a gap between the social and private of the firm, equal to a constant markup charged by the firms in the decentralized market. Ceteris paribus, entry, exit and investment in technologies is inefficient in the market allocation. Additionally, the endogenous path of capital accumulation is also distorted, as its return departs from the efficient one. This is standard in models with imperfect competition. In this paper however, distorsions in the allocation of firms across technologies interacts with the endogenous distribution of marginal products inducing static allocative inefficiencies. In other words, which firms are minimum capacity constrained (and hence the distribution of marginal products) depends on both the aggregate supply of capital, and the number of operating firms in the market.

The study of optimality in models with aggregate uncertainty, heterogenous firms and irreversibility is challenging. I show that if costs of adjustment are sunk[9] and there is a continuum of firm types, i) the market allocation can be mimic as the solution to a centralized problem with transfers (pseudo-planner)[10]; ii) that the planner's allocation can be decentralized as a market allocation, i.e. the second welfare theorem holds. To bring decentralized and efficient allocations together, the optimal policy entails a transfer scheme similar to a state dependent Pigouvian tax/subsidy. Whether the efficient allocation is associated to more or less dispersion than the market outcome depends through general equilibrium, on the equilibrium cost of capital and allocation of firms. For a given cost of capital, the planner generates higher entry and investment and lower exit. When a negative shock hits the economy, there are more firms operating technologies with higher minimum capacities in the efficient allocation, than there are in the market one. Hence, it is possible for the efficient allocation to generate more equilibrium dispersion. Whether selection outweighs

[8]The financial frictions story predicts more firms with higher marginal product of capital. Hence, such theory predicts that during recession, dispersion in revenue TFP should increase because there are relatively more firms operating with high marginal products (the left tail of the distribution has more mass).

[9]These results hold whether sunk costs are denominated in final goods or input costs. This paper presents the former, for an analysis of the latter see follow up work on Caunedo, 2014.

[10]The equivalence result follows closely the result described in Jones and Manuelli (1990) to study policy questions in convex economies with growth.

this movement or not depends on equilibrium movement in profits.

As a by-product of the equivalence result between the pseudo-planner and the market allocation, I can sidestep the standard approximation methods of Krusell and Smith (1998) to solve economies with heterogeneity and aggregate shocks. I solve the planner's allocation and use the decentralization result to compute the equilibrium allocation of this economy.

I calibrate the economy to the US manufacturing sector. The planner's allocation dictates higher equilibrium investment, and a shift in output production towards larger, more productive firms. Improvements in aggregate productivity are 11% under the optimal policy. Suppose that instead of characterizing the efficient allocation, the economist assumes all dispersion in marginal products is associated to inefficiencies. He will compute gains from full elimination of dispersion in marginal products of 76%. The efficiency improvement would be widely overestimated. Efficiency gains from the implementation of the optimal policy are accounted mostly by a change in firms entry, exit and investment patterns. Only a third of the gains in productivity are explained by reallocation of labor and capital across incumbent firms. The employment distribution varies slightly between the decentralized and planner's allocation. The optimal policy implies subsidies to entry, and the size of the subsidy is predicted higher in good times. In equilibrium, there are more firms operating in the market under the efficient allocation. Upgrade costs are subsidized to induce better selection of firms in the market. The policy in terms of scrap values varies with the aggregate state and the technology operated by the firm. In good times, scrap values are lower for all capacities except for the bottom ones, to generate exit of the least productive units. In bad times, scrap values for the lowest capacities need to drop, and the scrap value of the firms at the top of the productivity/size distribution have to increase. The latter induces exit by large firms that are possibly capacity constrained and have high option value to wait and remain in the market.

Finally, I study the impact of the level of uncertainty that firms face (given calibrated costs, and technology ladders in the economy) over observed measures of marginal product dispersion and aggregate TFP. In our economy, it is possible for low and high uncerainty economies to have similar dispersion marginal products and substantial differences in aggregate productivity. At one extreme, when the volatility of the aggregate productivity process is low, the economy approximates a stationary one. There is exit and entry in equilibrium as well as upgrades in technology. However, because the size of the aggregate shock is small, the main determinant of investment decisions is the firm's idiosyncratic productivity (as it will be in an economy with no shocks). The mechanism discussed in the example at the beginning becomes irrelevant. At the other extreme, when the volatility of the process is very high, incumbent firms find it more valuable to wait and not upgrade. Hence, in equilibrium upgrades in technology are delayed. Exit rates increase so that firms holding capital away from the level that they would have chosen in the current period are selected out of the market whenever a bad shock hits the economy. The mechanism described above vanishes again. While both economies display low dispersion in marginal products, the one with higher volatility is on average less productive than the one with lower volatility. Hence, the link between aggregate productivity and dispersion in marginal products depends on features of the macroeconomy and the patterns of firms entry, exit and investment.

Next, I review the literature. The rest of the paper is organized as follows. Section 2 presents the model, Section 3 describes the equilibrium, Section 4 explores the quantitative implications of the model, and Section 5 concludes.

1.1 Literature Review

Models of industry equilibrium with complete markets (as Lucas (1978), Jovanovic (1982), Hopenhayn (1992)) share the feature that capital labor ratios are equalized across firms. Hence, heterogeneity can only affect equilibrium allocations through selection. As marginal product and capital labor ratios are equalized, the model boils down to one of a representative firm with average productivity (as in Melitz (2003)). Firm selection determines the equilibrium mean productivity in the market. When the relationship between productivity, size (employment or assets) and output is non-monotonic (as is the case with non-convexities in production) heterogeneity matters also for the equilibrium allocation of factors.

Restuccia and Rogerson (2008) study the impact of a wide variety of distorsions that may affect the allocation of resources across production units away from that dictated by differences in establishment level productivity. For a calibrated economy to the US, they find that TFP losses are between 30% to 50%. Distinctively, our model implies that when accounting for the endogenous characteristic of part of those wedges, TFP losses are lower (one third). In addition, that shifts to entry and investment patterns may account for a large share of those. In related work, Hsieh and Klenow (2009) and Alfaro et al. (2009) study the impact of allocation inefficiencies in explaining differences in TFP and income per capita across countries. In both cases, the frictions that induce wedges in marginal products are kept unspecified, and assumed constant in time.

There is an extensive literature that study economies where marginal products differ endogenously across production units (Lee and Mukoyama (2008), Clementi and Palazzo (2010), Veracierto (2002) with adjustment costs, Albuquerque and Hopenhayn (2004), Midrigan and Xu (2009) with contracting frictions, and Khan and Thomas (2008) with both). Lee and Mukoyama (2008) provide evidence of differential entry and exit behavior along the business cycle and propose a model to quantitatively explain those facts. They analyze the effect of fluctuations in fixed production costs and labor adjustment costs on the industry dynamic in a model with no capital. Clementi and Palazzo (2010) analyze the propagation of aggregate shocks due to entry and exit of firms when firms are allowed to accumulate capital. Khan and Thomas (2008) study the effect of irreversibility and collateral constraints in equilibrium allocations in an economy with idiosyncratic shocks and without exit and entry. Veracierto (2002) studies the implications of investment irreversibility and aggregate shocks for the volatility of aggregate investment. His set up abstracts from endogenous entry and exit, which is key in our paper to explain efficiency gains [11]. In these papers, capital labor ratios are endogenous and possibly disparate across firms. The focus of analysis however on the implications of inaction for the dynamics of GDP, investment and entry and exit. The main contribution of our paper is to characterize and study optimal industrial policy in an environment with endogenous entry, exit and investment and non-convex production technology.

There are two papers that are closely related to this one in that the focus of analysis are productivity gains from reallocation. Both, Asker et al. (2011) and Cooper and Schott (2013) study the impact of capital adjustment costs for factor allocations, and gains from reallocation. Asker et al. (2011) studies an stationary economy, with no endogenous entry and exit. This paper shows that such margin is key in assessing gains from reallocation. The authors emphasize the link between dispersion in marginal products and volatility. Here, I show that both of them interact with the selection process as in turn the trade off between efficiency and selection, which determines the efficiency with which an industry operates. Cooper and Schott (2013) study productivity gains in the US manufacturing

[11] As can be seen from table 4 in Veracierto (2002), when there is full irreversibility, the change in the exogenous death rate has considerable effect on investment dispersion across production units.

sector in response to cyclical factor reallocation. In their environment aggregate shocks do not generate cyclical losses in productivity, but shocks to the shadow value of capital or the dispersion in idiosyncratic shocks do. The endogenous entry and exit margin is abstracted away.

Finally, as mentioned in the introduction, one of the main contributions of the paper is the characterization of the optimal industrial policy (as in Lucas and Prescott (1971)). A recent paper that also studies the characteristics of the constrained optima allocation in models with industry dynamic and wedges in marginal products is Fattal Jaef and Hopenhayn (July 2012).As in this paper, they restrict the planner to face the same distortions that the firms in the economy face. They find that while the competitive allocation generates the efficient allocation of resources across a given set of technologies (and distortions), it fails to generate the efficient level of entry and exit, and hence the measure of active firms. In our paper, the allocation of technologies run by firms is endogenous, hence the allocation of factors across incumbents firms is not always constrained efficient. Furthermore, because wedges in marginal products are endogenous in our economy, changes in the extensive margin, imply changes in the equilibrium distribution in marginal product.

Industrial policy has been studied in models of international trade under oligopolistic competition in prices and quantities (Eaton and Grossman (1986)). For a model of industry dynamic without capital accumulation Lee and Mukoyama (2008) studied the impact of alternative policies on labor regulations (both i.i.d. taxes to output and inputs and policies correlated with firm productivity). However, their policies were ad hoc in the sense that no notion of efficiency was associated to them. Gourio and Roys (2014) and Guner et al. (2008) also studied policies correlated with the size of the establishment, which in turn is correlated with their idiosyncratic productivity, and find substantial role in shaping aggregate productivity. In these set up however, the efficient allocation is by construction one in which marginal products are equalized across production units. Distinctively, this paper characterizes the optimal policy in an environment in which the efficient allocation does not dictate equalization of marginal products across all firms in the economy.

2 Model

This is an infinite horizon economy with time indexed by t. There is a final good which agents use for consumption and capital accumulation. It is produced by means of a continuum of intermediate goods. Intermediate goods are produced by combining capital and labor. Each intermediate good is perfectly differentiated and each firm producing it faces a constant elasticity demand. Final goods are traded competitively while there is monopolistic competition in intermediate goods. Technology for production of intermediate goods is endogenously chosen. There is a finite set of J technologies, each one characterized by a productivity shifter and a minimum running capacity in terms of capital (further details below). Profits of the firm vary with an exogenous aggregate shock, s_t with finite support, $\{s_1, ...s_S\}$, $s_i > s_{i-1}$ for any $i = 1 : S$. Transition probabilities are given by $\mathbf{P}$, which is assumed to satisfy the Feller property.

2.1 Households

The representative household derives utility from consumption of the final good C_t. Preferences are characterized by a concave, monotonic and differentiable function $U(C_t)$, with $U'(0) = +\infty$.The household is endowed with a unit of labor that for simplicity is supplied inelastically. She receives a wage w_t for these services. She can also accumulate capital K_t, priced in terms of the final good (the numeraire) and rent it at price r_t to the firms. The aggregate stock of capital depreciates at rate $\widehat{\delta}$ and time is discounted at rate $\beta \in (0, 1)$. Households can buy shares of mutual funds, n_t^j, that yield dividends of firms operating alternative technologies, d_t^j. Each mutual fund consist of all firms running technology $j \in \{1, ..., J\}$. After dividends are paid, assets can be traded at price P_t^j. Her problem reads

$$V(K_0, X_0, \mathbf{n}_0) = \max_{C_t, K_{t+1}, \{n_t^j\}_{j=1}^J} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{1}$$

subject to

$$C_t + K_{t+1} - (1 - \widehat{\delta})K_t + \sum_{j=L,H} P_t^j n_t^j = w_t + r_t K_t + \sum_{j=L,H} \left(d_t^j + P_t^j\right) n_{t-1}^j$$

$$X_{t+1} = \Phi_c(X_t)$$

where X_t corresponds to the aggregate state, i.e. $X_t = \left(s_t, \{v_t^j\}_{j=1}^J, K_t\right)$ which includes the exogenous shock, s_t ; the distribution of firms per technology, v_t^j and the available aggregate stock of capital. To save on notation I denote the measure of firms with productivity at most z and technology j, $v_t^j(z) \equiv v_t^j([0,z))$. In computing returns to share holdings, the agent needs to forecast the law of motion of the distribution of firms in the market for each possible realization for the exogenous aggregate shock, s_t. The representative consumer expects the law of motion of the aggregate state to be characterized by Φ_c.

2.2 Final Goods Sector

There is a a representative competitive firm with a constant elasticity of substitution (CES) technology that produces final goods Y_t out of intermediate inputs y_{it}. The firm maximizes profits.

$$\max_{y_{it}} \; Y_t - \int p_{it} y_{it} \; di \tag{2}$$

subject to

$$Y_t \leq \left(\int y_{it}^{\rho} di \right)^{\frac{1}{\rho}}$$

where p_{it} is the cost of good y_{it}. Intermediate goods are assumed substitutes in production, $\rho \in (0,1)$.

The corresponding input demand for each variety i emerges from the first order conditions of this problem,

$$Y_t^{1-\rho} y_{it}^{\rho-1} = p_{it}$$

2.3 Intermediate Goods Sector

There is a continuum of differentiated goods produced with Cobb-Douglas technologies in labor and capital.

$$y_t \leq s_t z \psi^j l_t^{1-\alpha} k_t^{\alpha}$$

Productivity is Hicks neutral. It has an aggregate component s_t and an idiosyncratic component, $z\psi^j$. The first element of idiosyncratic productivity is exogenously given at the moment of entry and constant in time [12]. The second one is chosen optimally by each firm. There are J alternative technologies associated to a minimum capacity and a productivity shifter, $\left\{\underline{k}^j, \psi^j\right\}$ for $j = \{1, .., J\}$. Assume, $\psi^j > \psi^{j-1}$ and $\underline{k}^{j-1} < \underline{k}^j$ for all j. Capital choice sets are $\left[\underline{k}^j, \infty\right)$ for each technology, respectively. This minimum capacity constraint can be interpreted as the construction of a plant, or the set up of machinery or processes which entails a particular capacity. Technology adoption is costly.

The product space in the economy is endogenous. There is a continuum of firms per technology j, indexed by z. Endogenous exit and technology upgrade determines the marginal firms operating for each technology (z^{ej}, z^{uj}), respectively.

Capital and labor allocation In this section, I study the allocation of capital and labor given a particular technology.

Define x_t as the vector of idiosyncratic state variables to the firm, i.e. $x_t^j = (z, \psi^j)$. Let X_t be defined as before and define Γ_f as the law of motion for the aggregate state as perceived by any intermediate good producing firm; i.e. $X_{t+1} = \Gamma_f(X_t)$. The problem of each firm is

$$\pi(x_t^j, X_t) = \max_{p_t, l_t, k_t} (p_t y_t - w_t l_t - r_t k_t)$$

subject to

$$y_t \leq s_t z \psi_t^j l_t^{1-\alpha} k_t^{\alpha}$$

$$\left(\frac{Y(X_t)}{y_t}\right)^{1-\rho} = p_t$$

$$k_t = \left[\underline{k}^j, \infty\right)$$

Firms are assumed to be entirely equity owned. Because the elasticity of demand is constant, the optimal price set by a firm is a constant markup over marginal cost,

$$p_t = \frac{(r_t - \lambda_t)^{\alpha} w_t^{1-\alpha}}{\rho \alpha^{\alpha} (1-\alpha)^{1-\alpha} s_t z}$$

where $\lambda_t \geq 0$ is the lagrange multiplier associated to the feasible set for capital. If the

[12] I assume that log productivity is drawn from an exponential distribution, so that the model can be interpreted as the limiting case of a model in which firms idiosyncratic productivity is stochastic and follows a Brownian Motion (See Luttmer (2010)). The mechanism studied in the paper may be reinforced when idiosyncratic productivity is allowed to vary stochastically.

minimum capacity requirement is not binding then, $\lambda_t = 0$, otherwise $\lambda_t > 0$ and the markup for this firm is lower than otherwise.

From the first order conditions we can compute labor and capital demands as follows

$$l_t = (s_t z \psi^j)^{\frac{\rho}{1-\rho}} \left[\left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\rho}-\alpha} \left(\frac{\alpha}{r_t - \lambda_t}\right)^{\alpha} \right]^{\frac{\rho}{1-\rho}} \rho^{\frac{1}{1-\rho}} Y(X_t)$$

$$k_t = \max\{\underline{k}^j, \left[s_t z \psi^j \left(\frac{1-\alpha}{w_t}\right)^{(1-\alpha)} \left(\frac{\alpha}{r_t}\right)^{\frac{1}{\rho}-(1-\alpha)} \right]^{\frac{\rho}{1-\rho}} \rho^{\frac{1}{1-\rho}} Y(X_t)\}$$

The higher the relative efficiency in production the higher the demand of labor when intermediate goods are substitutes in production. Labor and capital demands are non-increasing in their costs, and they are increasing in the demand level as summarized by $Y(X_t)$.

Importantly, capital labor ratios need not be equal across all firms in the economy, as the shadow value of capital depends on whether firms are constrained or not

$$\frac{k_t}{l_t} = \frac{w_t}{r_t - \lambda_t} \frac{\alpha}{1-\alpha}$$

If the minimum capacity requirement is binding, the firm adjusts its resource allocation through the flexible factor, in this case labor. However, the last condition indicates that constrained firms' labor demand does not increase enough to equalize capital labor ratios across all firms [13].

Define $Z^l(X_t) = \int (\frac{z\psi^j}{\left(r_t(X_t) - \lambda(x_t^j, X_t)\right)^{\alpha}})^{\frac{\rho}{1-\rho}} dx_t^j$ and $Z^k(X_t) = \int (\frac{z\psi^j}{\left(r_t(X_t) - \lambda(x_t^j, X_t)\right)^{\alpha}})^{\frac{\rho}{1-\rho}} \frac{1}{\left(r_t(X_t) - \lambda(x_t^j, X_t)\right)} dx_t^j$, both statistics of productivity adjusted by the shadow value of capital across firms in the economy. Labor and capital demand are proportional to these statistics

$$l(x_t^j, X_t) = \frac{1}{Z^l} (\frac{z\psi^j}{\left(r_t(X_t) - \lambda(x_t^j, X_t)\right)^{\alpha}})^{\frac{\rho}{1-\rho}} \tag{3}$$

$$k(x_t^j, X_t) = \frac{K_t}{Z^k} (\frac{z\psi^j}{\left(r_t(X_t) - \lambda(x_t^j, X_t)\right)^{\frac{1-(1-\alpha)\rho}{\rho}}})^{\frac{\rho}{1-\rho}} \tag{4}$$

If no firm is constrained, shadow values of capital equalize across firms and capital and

[13] In models where firms can be financially constrained, capital labor ratios of constrained firms is usually lower than that of unconstrained firms. Constrained firms hold lower capital than they would if unconstrained, while in our model, constrained firms hold more capital.

labor demand are only a function of the relative productivity of the firms versus the average in the economy. When some firms are capacity constrained ($\lambda_t(x_t^j, X_t) > 0$ for some x_t^j), the allocation of labor and capital is adjusted so that constrained firms can indeed retain more capital and labor inputs than if they were unconstrained.

The profits of any firm operating in the market can be described by

$$\pi(x_t^j, X_t) = ((1-\rho)Y_t^{1-\rho}[\frac{s_t K_t^\alpha}{(Z^k)^\alpha (Z^l)^{1-\alpha}}]^\rho - \frac{\lambda(x_t^j, X_t)}{r_t(X_t) - \lambda(x_t^j, X_t)} \frac{K_t}{Z^k})(\frac{z\psi^j}{(r_t(X_t) - \lambda_t(x_t^j, X_t))^\alpha})^{\frac{\rho}{1-\rho}} \tag{5}$$

Profits depend on aggregate demand, a measure of productivity in the economy summarized by $(Z^k)^\alpha (Z^l)^{1-\alpha}$ and the productivity of the firm, adjusted for the value of its marginal product of capital, $r_t(X_t) - \lambda_t(x_t^j, X_t)$. Whenever the minimum capacity constraint is binding the marginal product of capital of the firm is lower than the cost of capital in the market, and profits fall in proportion. If the firm is not constrained, $\lambda_t(x_t^j, X_t) = 0$ and

$$\pi(x_t^j, X_t) = ((1-\rho)Y_t^{1-\rho}[\frac{s_t K_t^\alpha}{(Z^k)^\alpha (Z^l)^{1-\alpha}}]^\rho)(\frac{z\psi^j}{(r_t(X_t))^\alpha})^{\frac{\rho}{1-\rho}}$$

Exit and upgrade Firms are exogenously liquidated with probability δ, getting a scrap value of $\Pi^{e,f}$. They can select out voluntarily, getting a scrap value of Π^e, net of exit costs. Assume $\Pi^{e,f} < \Pi^e$ so that the option to exit is meaningful and, without loss of generality, $\Pi^{e,f} = 0$. An incumbent firm in the market may choose to upgrade its technology at any point in time at cost I^u. For expositional purposes, I assume the cost structure is independent of the realization of the exogenous aggregate shock, s_t and equal across different technologies (scrap values and upgrade costs are the same for $j = 1, ..., J$). In the quantitative exercise I relax both these assumptions.

Timing for exit and upgrade decisions is as follows. At the beginning of each period the aggregate shock s_t realizes and every firm decides whether to operate or exit. If it decides to produce and the firm was operating technology j^* so far, it can choose any technology $j \geq j^*$ if paying the corresponding upgrade costs. After production takes place, dividends are paid to households and firms are exogenously liquidated at rate δ.

Let $W(x_t^j, X_t)$ be the value of a firm with type x_t^j when the aggregate state of the economy is X_t. For $j = J$ there is no further technology upgrade available, so the value is

$$W(x_t^J, X_t) = Max\left\{\Pi^e, \pi(x_t^J, X_t) + E_t\left(\widetilde{\beta}_{t+1} W_{t+1}(x_t^J, X_{t+1})\right)\right\} \tag{6}$$

subject to

$$X_{t+1} = \Phi_f(X_t)$$

where $\widetilde{\beta}_{t+1}(X_t, X_{t+1}) \equiv \beta\,(1-\delta)\,\frac{U'(C(X_{t+1}))}{U'(C(X_t))}$ is the stochastic discount factor of the household adjusted for the probability of survival of the firm, $\widetilde{\beta}_{t+1}$ to save notation.

For any firm operating technology $j < J$, the value of the firm includes also, the value of upgrade, i.e.

$$W(x_t^j, X_t) = Max\{\Pi^e, W(x_t^{j+1}, X_t) - I^H, \widetilde{W}(x_t^j, X_t)\} \tag{7}$$

subject to

$$X_{t+1} = \Phi_f(X_t)$$

where $\widetilde{W}$ is the value of the firm on the continuation region.

$$\widetilde{W}(x_t^j, X_t) = \pi(x_t^j, X_t) + E_t\left(\widetilde{\beta}_{t+1} W(x_t^j, X_{t+1})\right)$$

Entry There is a continuum of firms ready to enter the market at any period t. They pay a cost I^{ent} and draw their productivity from a distribution $G(z)$ with finite support $[\underline{z}, \overline{z}]$. For the problem to be well defined we need to assume $I^L \geq \Pi_e$. Otherwise, entrepreneurs could create resources by entering and exiting immediately from the market. After entry, they may choose to upgrade technology immediately at cost I^u.

The mass of entrants M_t^{ent} is determined by the free entry condition,

$$I^{ent} \geq \int\limits_{z^e(\psi^1, X_t)} W(z, \psi^1, X_t) dG(z/z \geq z^e(\psi^1, X_t)) \tag{8}$$

with equality if $M_t^{ent} > 0$.

It is worth noting that the equilibrium distribution of firms across productivity and technologies, which is used in the computation of the expected value of the firms (summarized in X_t), is indeed endogenously determined by the choice of exit and upgrade thresholds of firms in the market. Entrants correctly anticipate their future expected profits, so that pre-entry expected profit equalizes their post entry value.

2.4 Aggregates

Replacing capital and labor demands in the aggregate production function, we obtain

$$Y(X_t) = TFP_t K_t^{\alpha}$$

where

$$TFP_t = s_t M_t^{\frac{1-\rho}{\rho}} \left(Z^l\right)^{\frac{1-\rho}{\rho}} \left(\frac{Z^l}{Z^k}\right)^{\alpha}$$

In other words, aggregate efficiency is determined by the realization of the exogenous shock, the measure of firms operating in the market (as usual in models of monopolistic competition), and a moment of the productivity of the firms operating in the market.

If there are no firms capacity constrained, $\frac{Z^l}{Z^k} = r$, and the model boils down to the canonical firm dynamic one where

$$TFP_t = s_t M_t^{\frac{1-\rho}{\rho}} \left(\sum_j \int (\psi_i^j z_i)^{\frac{\rho}{1-\rho}} d\widehat{v}_t^j(z_i)\right)^{\frac{1-\rho}{\rho}}$$

where $\widehat{v}_t^j = \frac{v_t^j}{M_t}$ is a scaled measured of the firms operating in the market $M_t = \sum_{j=1}^{J} \int dv_t^j(z)$.

Also, as the share of capital in production, α, goes to zero, disparity in marginal products becomes irrelevant for aggregate productivity because the share of the factor for which the minimum constraint may bind becomes negligible. In general none of those is the case. Note also that it is possible for multiple allocations (distributions across technologies) to yield the same TFP_t conditional on the aggregate exogenous shock s_t and the measure of operating firms.

3 Equilibrium

Let the measure of firms operating technology j exiting the market endogenously at the beginning of period t be M_t^{ej}. Let the measure of firms upgrading to technology $j+1$ in period t be $M_t^{u,j+1}$.

Definition 1 *A competitive equilibrium is a system of thresholds* $\{[z^e(\psi^1, X_t), ..., z^e(\psi^J, X_t)], z^u(X_t)\}_{t=0}^{\infty}$, *distribution of firms* $\left\{[v_t^1(z), ..., v_t^J(z)]\right\}_{t=0}^{\infty}$, *a law of motion for the dynamic of the distributions of firms* Λ[14], *a path for the measure of entrants* $\{M_t^{ent}\}_{t=0}^{\infty}$ *with productivity drawn from* $G(z)$, *and consumption, aggregate capital and share holdings functions,* $\left\{C(X_t), K_{t+1}(X_t), [n^1(X_t), ...n^J(X_t)]\right\}_{t=0}^{\infty}$ *such that given prices* $\{r(X_t), w(X_t), [P^1(X_t), ..., P^J(X_t)]\}_{t=0}^{\infty}$, *the cost structure* $\Upsilon_c = [\Pi^e, I^u, I^{ent}]$, *the initial stock of capital in the economy* K_0, *share holdings,* $n_0^j, .., n_0^J = 1$, *and the exogenous law of motion for aggregate shocks* s_t *as characterized by* $\mathbf{P}$,

i) the representative consumer maximizes utility (as in (1)*)*

ii) firms in the intermediate goods sector maximize their value as described by (6) and (7) given their residual demand and productivity z.

iii) firms in the final good sector maximize profits (as in (2)).

iv) the free entry condition (8) holds, (with equality if $M_t^{ent} > 0$*)*

v) the dynamic of the measure of firms in the market follows $M_t = M_t^{ent} + (1-\delta) M_{t-1} - \sum_{j=1}^{J} M_t^{ej}$

vi) Markets clear

(a) $\sum_{j=1}^{J} \int l(x_t^j, X_t) dv_t^j(z) = 1$

[14] The endogenous dynamic of the distribution of productivities across technologies described by Λ has three components. Let P_Λ be the operator described by

$$\begin{aligned} P_\Lambda(A) &= (1-\delta)\int_{z\in A} dz_i \qquad \text{for } z \geq z_t^{e,j} \\ &= 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{aligned}$$

Let $G_\Lambda(z)$ be the distribution of entrants across types z conditional on entry, i.e.

$$\begin{aligned} G_\Lambda(z) &= \frac{G(z) - G(z_t^{u,j})}{1 - G(z_t^{e,1})} \qquad \text{for } z \geq \max\{z_t^{e,1}, z^{u,j}\} \\ &= 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{aligned}$$

Let $A_t^u(k,j)$ the type of incumbent firms that upgrade to technology j when using technology k in the previous period. Then, Λ can be described as

$$v_t^j = P_\Lambda([z_t^{e,j}, z^{u,j}])v_{t-1}^j + G_\Lambda M_t^{ent} + \sum_{k\leq j} P_\Lambda(A_t^u(k,j))v_{t-1}^k$$

(b) $\sum_{j=1}^{J} \int k(x_t^j, X_t) dv_t^j(z) = K_t$

(c) $n_t^j = 1 \; \forall j$.

(d) $C_t + K_{t+1} - (1-\widehat{\delta})K_t + I^{ent} M_t^{ent} + I^u \sum_{j=1}^{J-1} M_t^{u,j+1} = Y_t + \Pi^e \sum_{j=1}^{J} M_t^{ej}$

vii) *The law of motion of the aggregate state satisfy:* $\Phi = \Phi_f = \Phi_c$ *(consistency)*

Existence of the equilibrium is proved after describing the planner's problem. I show existence of **a** centralized allocation, whose equilibrium allocation coincides with the market outcome, and hence existence of the market allocation. Detailed analysis can be found in section 3.2.

3.1 Properties of the allocation

Let the function $z^e(\psi^j, X_t)$ determine the threshold for exit of j technology firms when the aggregate state of the economy is X_t. Let the function $z^u(\psi^{j+1}, X_t)$ determine the threshold for upgrade from technology j to $j+1$.

Proposition 1 (Thresholds) $\widetilde{W}(x_t^j, X_t)$ *is monotonic increasing in idiosyncratic productivity,* z. *The optimal exit and upgrade strategy for the firm is such that if* $z < z^e(\psi^j, X_t)$*, the firm exits the market; if* $z \geq z^u(\psi^{j+1}, X_t)$ *the firm upgrades.*

Proposition 2 (Allocation) *The optimal allocation is such that*

1. *if costs and technologies satisfy* $\frac{\Pi^e(\psi^{j+1})}{(\psi^{j+1})^{\frac{1-\rho}{\rho}}} < \frac{\Pi^e(\psi^j)}{(\psi^j)^{\frac{1-\rho}{\rho}}}$*, then exit thresholds satisfy* $z^e(\psi^j, X_t) > z^e(\psi^{j+1}, X_t)$ *whenever neither firm is constrained by the minimum capacity contraint or both are.*

2. *exit thresholds are increasing in the cost of capital, i.e.* $\frac{\partial z^e(\psi^j, X_t)}{\partial r_t} \geq 0$.

3. *for each technology* j *the upgrade threshold is higher than the exit threshold for the next available technology, i.e.* $z^u(\psi^{j+1}, X_t) > z^e(\psi^{j+1}, X_t)$.

4. *the measure of entrants is procyclical.*

The first result indicates that firms running the lower minimum capacity technology find optimal to exit before firms of the same idiosyncratic productivity z running higher minimum capacity technologies. The second result shows that increases in the cost of capital, raise the likelihood of voluntary exit as equilibrium profits drop. The third result is important as it assures that costs are such that there is no upgrade in technology and immediate exit. Finally, the levels of entry are procyclical, as they are in the data.

3.2 Industry Equilibrium (PP)

To prove existence of the competitive equilibrium I define a centralized problem, whose solution coincides with the decentralized allocation under certain conditions.

Before describing the centralized problem, it is important to understand why the first welfare theorem does not hold. The market allocation is not surplus maximizing because competition is imperfect in the intermediate good sector. Imperfect competition not always yields inefficient allocations[15]. In our case, it does because the supply of factors (in particular, capital and the measure of firms) is elastic. Imperfect competition distorts returns to investment in technology upgrades and through it, the equilibrium measure (and type z) of firms that are at the minimum capacity constraint in any given period. Intuitively, upgrade to a new technology can be interpreted as an entry decision to a new sector. Imperfect competition distorts entry incentives for all these "sectors" and the productivity of firms operating in each of them. Equilibrium capital labor ratios are hence distorted away from the efficient ones. In addition, imperfect competition shifts the return to capital away from its efficient level. Incentives to aggregate capital accumulation are also distorted.

To ease the exposition, the efficient allocation is described in detail in the appendix. Here, we focus on describing a pseudo-planner's problem, whose allocation coincide with the market one. To mimic the market allocation, I modify the cost of entry, exit, and upgrade in technology. I also distort the feasibility condition in capital and labor, to mimic the return to factors in the decentralized allocation. This distortion accounts for a wedge in the rate of transformation of goods to capital/labor.

Before going into the core of this section, let's define some notation. The relevant aggregate state space from the point of view of the planner is $\Xi_t = \left\{s_t, [v_{t-1}^j, ...v_{t-1}^J], K_t\right\}$,i.e.

[15] When factor supply is inelastic and sunk costs are denominated in factor prices, as it is the case in Melitz (2003), the market allocation under monopolistic competition is efficient.

the realization of the shock, the distribution of firms across technologies carried over from the previous period, as well as the stock of capital. The dynamic for the measure of firms across technologies and idiosyncratic productivity (ψ^j, z) is characterized endogenously by Λ, as function of the exit and upgrade thresholds, and the measure of entrants in the market. The cost structure is given by tuples including entry and upgrades costs, scrap values and a transfer T necessary to mantain budget balance $\widehat{\Upsilon}(\Xi_t) = \left[\widehat{I}^{ent}, \widehat{\mathbf{I}}^u, \widehat{\Pi}^e, T\right]$[16], where $\widehat{\mathbf{I}}^u = [\widehat{I}^u(\psi^2),...,\widehat{I}^u(\psi^J)]$ and $\widehat{\Pi}^e = [\Pi^e(\psi^1), ...\widehat{\Pi}^e(\psi^J)]$. Each of the elements of $\widehat{\Upsilon}(\Xi_t)$ except T, equalizes the cost/scrap value from the decentralized allocation plus a wedge, i.e. $\widehat{\Upsilon}(\Xi_t) = \Upsilon(1 + \tau(\Xi_t))$ where $\tau(\Xi_t)$ is the vector of subsidies/taxes. To assure that those wedges are not lost, the total value of the wedges as well as any difference generated in total output, is transferred back lump sum to the pseudo-planner, $T(\Xi_t) = \Upsilon\tau(\Xi_t) + Y_t - \widehat{Y}_t$. Finally, we add a wedge in the rate of transformation between final goods and capital and labor allocated to firms. We define it as a function of the elasticity of substitution across intermediate goods in the market allocation, $\varepsilon = \frac{1}{1-\rho}$. In particular, $\tau^k = \tau^l = \frac{1}{\rho} = \frac{\varepsilon}{\varepsilon - 1}$.

The pseudo-planner's problem reads

$$V(\Xi_t) = \max_{C_t,\ K_{t+1},\ \{z_t^e(\psi^j)\}_{j=1}^J, \{z_t^u(\psi^{j+1})\}_{j=1}^{J-1}, M_t^{ent}, l_t^i, k_t^i} U(C_t) + \beta E V(\Xi_{t+1}) \tag{PP}$$

subject to

$$C_t + \widehat{I}_t^{ent} M_t^{ent} + \sum_{j=1}^{J-1} \widehat{I}_t^u(\psi^{j+1}) M_t^{u,j+1} + K_{t+1} - K_t(1-\widehat{\delta}) \leq \widehat{Y}_t + T_t + \sum_{j=1}^{J} \widehat{\Pi}_t^e(\psi^j) M_t^{ej} \quad (\zeta_t)$$

$$s_t(\sum_{j=1}^{J} \int \left(\psi^j z_i l_i^{1-\alpha} k_i^{\alpha}\right)^{\rho} dv_t^j(z_i))^{\frac{1}{\rho}} = \widehat{Y}_t$$

$$\sum_{j=1}^{J} \int \tau^l l_i dv_t^j(z_i) = 1 \quad (\lambda_t^l\ \zeta_t)$$

$$\sum_{j=1}^{J} \int \tau^k k_i dv_t^j(z_i) = K_t \quad (\lambda_t^k\ \zeta_t)$$

$$\tau^k k_i \geq \underline{k}^j \text{ if } \psi_i = \psi^j \quad \forall j = 1:J \quad (\widehat{\lambda}_i t^j\ \zeta_t)$$

[16] Each element of the vector depends on the aggregate state, but Ξ_t has been dropped for notational convenience.

$$v_t^j = \Lambda\left(v_{t-1}^j\right) \qquad \forall j = 1 : J \qquad\qquad (\mu_t^j\ \zeta_t)$$

Hence, given the distribution of firms in the market, the realization of the aggregate shock and the available stock of capital, the planner chooses how much capital to accumulate for next period, the allocation of firms across technologies, firm's labor and capital distribution. The lagrange multipliers associated to the feasibility constraint in goods, labor and capital, minimum capacity and dynamic of the measure of firms per technology are ζ_t, $\lambda_t^l \zeta_t$, $\lambda_t^k \zeta_t$, $\zeta_t \widehat{\lambda}_{it}^j$ and $\zeta_t \mu_t^j$, respectively[17].

Theorem 1 *a) For a given transfer scheme Υ^p, the solution to the centralized problem exists and it is unique.*

b) There exist a cost structure $\{\Upsilon^p(\Xi_t)\}_{t=0}^{\infty}$ such that the allocation of firms that solves the planner problem (PP) coincides with the competitive allocation.

For expositional purposes the full proof can be found in the appendix. Heuristically it goes as follows. For part a) note that the problem would be a standard concave problem if there were no sunk costs to technology adoption and no minimum capacity constraint that may bind in equilibrium. The presence of a continuum of heterogenous firms mitigates potential non-convexities as in Mas-Colell (1977). The continuum of firms works as the divisible commodity necessary to convexify the aggregate feasible set. For part b), the proof has two steps. Analogous to Jones and Manuelli (1990), first we define an operator on the transfers, $\Omega(T(\Xi_t))$ and prove that it has a fixed point. At the fixed point, the feasibility constraint of the planner and competitive equilibrium are the same. Second, we define a cost structure and characterize prices such that the optimality conditions hold for each allocation. I show that given τ^k the planner's optimal consumption and capital accumulation paths coincide with the market allocation, ceteris paribus. To assure that the allocation of firms across technologies and the measure of active firms coincide, I use the linearity of the optimality conditions of the (PP) for the allocation of firms, as well as in the indifference conditions for the firms in the decentralized problem (as described in (6), (7) and (8)). I show that one can define a unique set of subsidies/taxes, $\widehat{\tau}(\Xi_t)$ such that the thresholds of the decentralized problem satisfy the necessary conditions for optimality in (PP). I show that the transfer generated by $\widehat{\tau}(\Xi_t)$, $T(\widehat{\tau}(\Xi_t))$ is a fixed point of Ω. Hence, the equivalence

[17] I have redefined multipliers to ease notation when describing optimality conditions.

is proven. Note that if the equilibrium was pareto optimal, then $\widehat{\tau}(\Xi_t)$ should be equal to zero across all states.

Corollary 1 *The solution to the competitive equilibrium exists*[18]

3.2.1 Analysis

Before moving to the quantitative results, it is useful to illustrate how the optimality conditions for technology selection and input allocation may differ between the centralized and decentralized allocation. To do that I assume that firm types belong to a discrete set $Z = [z_1, ...z_N]$.Hence, the dynamic of the measure of firms of type z_i operating technology j, $\underline{v}_t^j(z_i)$, is decribed by

$$\underline{v}_t^j(z_i) = (1-\delta)\underline{v}_{t-1}^j(z_i) + M_t^{ent}\frac{g(z_i)}{1-G(z_t^{e,1})} + M_t^{u,j}(z_i) \qquad \text{for } i = 1, ..., N \qquad (\mu_t^j(z_i)\zeta_t)$$

Capital-labor ratios The allocation of employment and capital across firms in (PP) and in the decentralized allocation are dictated by

$$(Y_t^*)^{1-\rho}(1-\alpha)\frac{y_{it}^{\rho}}{l_{it}} = \tau^l\lambda_t^l \qquad\qquad (Y_t)^{1-\rho}(1-\alpha)\frac{y_{it}^{\rho}}{l_{it}} = \frac{w_t}{\rho}$$

$$Y_t^{*1-\rho}\alpha\frac{y_{it}^{\rho}}{k_{it}} = \tau^k(\lambda_t^k - \widehat{\lambda}_{it}) \qquad\qquad Y_t^{1-\rho}\alpha\frac{y_{it}^{\rho}}{k_{it}} = \frac{r_t - \lambda_{it}}{\rho}$$

Hence, the shadow value of labor (capital) in the centralized allocation equals the wage (interest) rate in the decentralized allocation adjusted by the elasticity of substitution in intermediate inputs. This is the well known gap introduced by the monopolistic competition assumption.

[18] We cannot say much about the determinacy of the competitive equilibrium. Let me illustrate an example with no aggregate uncertainty. Suppose that the household would like to consume more in the current period and less in the following one. It implies that the marginal utility of consumption today is higher and the marginal utility tomorrow lower than in the stationary equilibrium. If this is the case, the intertemporal Euler equation of the household would not hold. However, higher demand today for final goods implies higher demand for all intermediate goods, which triggers entry, raising labor demand and wages. The productivity cutoff for exit may raise, as profits are now lower than before on average. The shift in the cutoff implies that the average productivity in the market goes up, and the average markup drops. If the overall effect induces lower equilibrium profits, the price of shares P_t^k can drop bringing back the Euler equation to hold which yields the indeterminacy. If entry costs were denominated in terms of labor cost instead of the composite good, higher demand may not induce entry, as the costs of entry raises with the number of firms operating in the market.

Return and aggregate capital accumulation Let the set of firms that are constrained by the minimum capacity constraint be $\Delta_i = \{i : y_{it} > 0 \text{ and } \lambda_{it} > 0\}$ (name it $\widehat{\Delta}_i$ for the centralized allocation) , and let $K_t^{\Delta_i} = \sum_{\Delta_i} k_{it}$ be the demand of capital for constrained firms. The shadow value of capital in each allocation solves

$$\alpha \frac{\widehat{Y}_t}{\widehat{K}_t - K_t^{\widehat{\Delta}_i}} = \tau^k \lambda_t^k \qquad \text{and} \qquad \alpha\rho \frac{Y_t}{K_t - K_t^{\Delta_i}} = r_t$$

Suppose that $\widehat{\Delta}_i = \Delta_i$. The latter implies that the allocation of firms across technologies is constrained efficient. Capital labor ratios across production units can coincide in the decentralized and centralized allocation, iff $\frac{\tau^k \lambda_t^k - \widehat{\lambda}_{it}}{\lambda_t^l} = \frac{r_t - \lambda_{it}}{w_t}$ for all firms i. In other words, whenever $\widehat{\Delta}_i = \Delta_i$. Hence, in this case, the allocation of capital and labor across firms is also constrained efficient. Still the shadow value of capital may differ across allocations because of imperfect competition. This is important because incentives to capital accumulation are hence distorted. The optimal path for aggregate capital in (PP) and the decentralized allocation are dictated by

$$U'(C_t) = E_t\left[U'(C_{t+1})\,\beta\left(\lambda_{t+1}^k + 1 - \widehat{\delta}\right)\right]$$

$$U'(C_t) = E_t\left[U'(C_{t+1})\,\beta\left(r_{t+1} + 1 - \widehat{\delta}\right)\right],$$

respectively. To be able to replicate the decentralized allocation, it is necessary a tax the return on capital proportional to the elasticity of substitution across goods. The shadow value of capital λ_{t+1}^k that solves the problem (PP) is identical to the price of capital in the competitive allocation. To be able to do it, we introduced a wedge in the feasibility condition for capital, τ_k.

To make sure that allocations across technologies are constrained efficient, exit, entry and upgrade conditions need to coincide in the decentralized and centralized allocations.

Entry The optimality condition associated to the measure of entrants in (PP) is

$$\widehat{I}_t^{ent} = \mu_t^1 \frac{G(z_t^{u,2}) - G(z_t^{e1})}{1 - G(z_t^{e1})} + \sum_{j=1}^{J-1} (\mu_t^j - I_t^{u,j}) \frac{G(z_t^{u,j+1}) - G(z_t^{u,j})}{1 - G(z_t^{e1})} + (\mu_t^J - I_t^{u,J}) \frac{1 - G(z_t^{u,J})}{1 - G(z_t^{e1})} \quad (9)$$

where μ_t^j is the shadow value of a firm (of any type z) operating technology j. The value to the planner of such firm can be calculated from the optimality conditions with respect to the measure of firms of type j. Let the distribution of firms across productivity z, normalized by the measure of firms operating a particular technology j be $\widetilde{\underline{v}}_t^j = \frac{\underline{v}_t^j}{M_t^j}$.

$$\mu_t^j = \frac{1-\rho}{\rho} Y_t^{1-\rho} \sum_{z=z_t^{e,j}}^{z_t^u} (y_t(\psi_t^j, z, \Xi_t)^\rho - \widehat{\lambda}_t(\psi_t^j, z, \Xi_t)\tau^k \underline{k}^j)\widetilde{\underline{v}}_t^j(z) + \beta(1-\delta)E_t(\frac{\zeta_{t+1}}{\zeta_t} \max\{\mu_{t+1}^j, \widehat{\Pi}_{t+1}^{e,j+1}\})$$

Note that the continuation value of a firm operating technology j is the present discounted value of the contribution to output of the average productivity firm operating such technology. The analogous condition in the market allocation is the continuation value of the average firm using technology j,

$$\widetilde{W_t^j} = (1-\rho) Y_t^{1-\rho} \sum_{z=z_t^{e,j}}^{z_t^u} (y_t(\psi_t^j, z, \Xi_t)^\rho \widetilde{\underline{v}}_t^j(z) + \beta(1-\delta)E_t(\frac{U(C_{t+1})}{U(C_t)} W_{t+1}^j{}^{19})$$

In general, $\mu_t^j \neq \widetilde{W_t^j}$. For the entry measure to coincide in the centralized and decentralized allocation (in (9)) $\widehat{I}_t^{ent} = (1+\tau(\Xi_t))I_t^{ent}$ for $\tau(\Xi_t) \neq 0$.

Analogous disparities in the valuation of a production unit for the planner, and its market value (the discounted value of its profits) will show up in the upgrade and exit conditions. Those are presented next for completedness. Intuitively, scrap values and upgrade costs are adjusted as we did for the entry cost to assume that the marginal firm exiting and upgrading technology is the same.

Exit The exit condition for a firm operating technology j in the planners' problem reads

$$\begin{aligned} \widehat{\Pi}_t^{e,j} = & \frac{(1-\rho)}{\rho} \widehat{Y}_t^{1-\rho} y(\psi_t^j, z^{ej}, \Xi_t)^\rho - \widehat{\lambda}_{et} \tau^k \underline{k}^j + \\ & E_t[\beta \frac{\zeta_{t+1}}{\zeta_t}(1-\delta)(\max\{\mu_{t+1}^j(z^{ej}), \widehat{\Pi}_{t+1}^{e,j}, \mu_{t+1}^{j+1}(z^{ej}) - \widehat{I}_{t+1}^{u,j+1}\})] \end{aligned} \quad (10)$$

Hence, the planner equalizes the scrap value of the firm, to the marginal output generated by keeping that technology active plus the expected value of the technology. That includes

[19] Where for notational convenience I have drop the dependence of the value of firms with respect to the aggregate state of the economy, so $W_t^j \equiv W^j(X_t) = \int W^j(x_t^j, , X_t) d\widetilde{v}_t^j(z)$

the option to exit in the future and receive a scrap value of $\widehat{\Pi}^e_{t+1}$ or to keep operating. The value of an active firm with technology j is μ^j_t.

Upgrade The upgrade condition for a firm operating technology j in the planners' problem reads

$$\begin{aligned}\widehat{I}^{u,j+1}_t &= \frac{(1-\rho)}{\rho}\widehat{Y}^{1-\rho}_t(y(\psi^{j+1}_t, z^{u,j+1}, \Xi_t)^\rho - y(\psi^j_t, z^{u,j+1}, \Xi_t)^\rho) - \tau^k(\widehat{\lambda}^{j+1}_{ut}\underline{k}^{j+1} - \widehat{\lambda}^j_{ut}\underline{k}^j) \quad (11)\\ &+ E_t[\beta\frac{\zeta_{t+1}}{\zeta_t}(1-\delta)(\max\{\mu^{j+1}_{t+1}(z^{u,j+1}), \widehat{\Pi}^{e,j+1}_{t+1}, \mu^{j+2}_{t+1}(z^{u,j+1}) - \widehat{I}^{u,j+2}_{t+1}\}\\ &- \max\{\mu^j_{t+1}(z^{u,j+1}), \widehat{\Pi}^{e,j}_{t+1}, \mu^{j+1}_{t+1}(z^{u,j+1}) - \widehat{I}^{u,j+1}_{t+1}\}]\end{aligned}$$

Hence, the planner equalizes the scrap value of the firm, to the marginal output generated by keeping that technology active plus the expected value of the technology. The latter includes the option to exit in the future and receive a scrap value of $\widehat{\Pi}^{e,j}_{t+1}$ or to keep operating and possibly upgrade. If the firm is upgraded, its expected value correspond to the value of the new technology,$\mu^{j+1}_{t+1}(z^{u,j+1})$, minus upgrading costs.

4 Quantitative analysis

In this section I assume there is a finite level (N) of minimum capacities/technologies, and there is no further investment in capacity conditional on a particular technology. I assume that there is a stock of capital ready to be used in any particular company. The stock is large enough so that any firm that decides to invest in capacity or enter the market can be supplied with the corresponding stock. The dynamic of the aggregate stock of capital will be pinned down by the consumption decisions of the planner, which in turn will pin down the dynamic of the measure of firms in the economy.

Production under each alternative technology is given by

$$y_t = s_t z k^\alpha_j l(x_t, X_t)^{1-\alpha} \text{ for } j = 1, ..., N$$

where $k_j < k_{j+1}$ for any j. A detailed explanation of the algorithm for computing the equilibrium is provided in Appendix A.

4.1 Calibration

The model is calibrated to the USA economy[20]. Although business cycle statistics are typically presented at quarterly frequency, industry dynamics statistics are only available on a yearly basis. Hence, the time unit of the model is a year. Some of the calibrated parameters are standard in the RBC literature. The persistence of expansions and recession periods were set to match the average duration of the phase of the business cycle in the USA. In particular, $\gamma_s = 1 - 1/t_s$ where t_s is the average length of a particular phase of the business cycle $s = g, b$. The average duration of an expansion was set to 3.175 years (or 12.7 quarters), and that of a recession to 1.425 years (or 5.7 quarters). The discount factor was set to match a steady state interest rate of 2%, $1 + r = \beta^{-1}$. Log utility was assumed.

The substitutability across intermediate goods in the final good aggregator was set to match returns to entrepreneurship (ρ shapes the curvature of the profit function). Atkenson and Kehoe (2005) set a value of 15% to the returns to entrepreneurship, whose analogous in the model is $1-\rho$ ($\rho = 0.85$). The share of capital in value added is set to 1/3 as standard in the literature. The hazard rate for exogenous exit, δ was set to 5,5%. It corresponds to the mean exit rate reported in Lee and Mukoyama (2008) based on statistics from the Annual Survey of Manufactures. Finally, the number of technologies is set arbitrarily to 4 and the lower bound of possible productivity equal to 0.01[21].

The remaining parameters of the model were calibrated jointly to match moments of the firm size distribution, as well as features of the industry dynamic and the aggregate volatility of the economy. To calibrate them I simulate the model economy via Montecarlo: I run the optimal policies for 1000 periods (discard the first 200) over 100 alternative paths for a variety of parameter specifications. The list of parameters calibrated jointly is presented in Table 2

While some parameters have closer tights to certain moments, they are not independent of the remaining variables of the economy. Let me describe their roles briefly. First, the size of aggregate shocks measured by $s_g - s_b$ is closely related to the volatility of the cyclical component of log GDP. The target in the data corresponds to the standard deviation of

[20]There are substantial differences in the firm size distribution of the USA versus other OECD countries (see Barstelman et al. (2009)). In particular, the right tail of the distribution is "fatter" in the USA than in other developed economies. Alternative calibrations can be accommodated.

[21]The minimum effective productivity operating in the market is determined endogenously.

the hp-filtered series of log GDP from 1930 to 2011, equal to 2.1%. Positive shocks take a value of 1.027 and negative shocks of 0.97 (shocks are assumed symmetric around one). The observed variation in aggregate output is not independent however of the cost structure of the economy, as the latter determines how much investment or exit is observed in equilibrium, which in turn affects aggregate output.

The set of capacities as well as the range for idiosyncratic productivity, are related to the levels of log employment produced by the model[22]. The upper bound on capacities was set to 4 while the upper bound on productivity was set to 4.25. The firms at the top of the employment have a level of employment slightly above 10000 employees, consistent with the data. The distribution of sizes in the economy inherits also some of the properties of the exogenous distribution of idiosyncratic productivity, $G(z)$. The distribution of entrants is calibrated such that the $log(z)$ is exponential with parameter $\zeta_G = 1.9$. In other words, $G(z)$ is Pareto with parameter ζ_G.

The generated firm size distribution is also related to the entry and upgrade costs per capacity, through the equilibrium allocations. To calibrate the cost structure, I assumed state independent costs for the pseudo planner problem. Once the allocations generated by the economy matched the targets for the US, I backed out the cost structure in the decentralized allocation. In other words, I computed the costs that would make the exit and upgrade threshold of the decentralized allocation coincide with the ones in the calibrated economy.

The total number of parameters for calibration is thirteen. The complete list of moments that were targeted to calibrate them are found in Table 3.The identified costs indicate slightly higher entry costs during expansions, fairly constant scrap values across states, but increasing in the capacity of the firms as expected. Upgrade costs are identified higher during expansions. In the ergodic distribution of the model, upgrades in capacity for incumbent firms average 2.8% of the total population of active firms, costs of upgrade should raise when incentives to upgrade increase to avoid shifts in the firm size distribution that will make it inconsistent with its fairly constant shape in the data. The establishment and employment shares are as reported by Lee and Mukoyama (2008), as well as the average exit and entry

[22]The finite level of capacities model predicts that relative labor demands are described by $\frac{l_i}{l_j} = \frac{(z_i k_i^{\alpha})^{\frac{\rho}{1-\rho(1-\alpha)}}}{(z_i k_j^{\alpha})^{\frac{\rho}{1-\rho(1-\alpha)}}}$

rates. Overall, the model predicts well the behavior of the establishment and employment distribution. The share of employment for firms at the top of the log employment distribution is slightly under predicted. The model predicted share of establishments with less than 19 employees is below the observed number in the data. The firms at the top of the distribution reported by the BDS have 10.000 or more employees. They correspond to 6% of the total population of establishments in the economy. The model is conservative in this sense as the largest firm in the economy employs 10.829 employees.

In terms of firm entry and exit rates the model over predicts exit rates by 0.7%, and under predicts entry rates 0.6%. For the measure of firms to be stable in the ergodic distribution, these flows should be roughly the same, the model is calibrated to go half the way the difference in entry and exit rates reported in the data. I also targeted the percentage of firms with positive investment spikes as reported by Dums and Dunne (1998). A spike is defined as firm that reports an investment rate of 30% or higher in any given year. Given the capacity grid, any upgrade in capacity will be considered an investment spike, as well as any entry decision. The model produces a measure of spikes of about 1% higher than in the data once we account for investment of entrants. In the model, 40% of the measure of firms with investment spikes corresponds to incumbent firms. The contribution is rather small as for the calibrated aggregate shocks, investment thresholds move mildly. The introduction of firm specific shocks will increase fluctuations in the thresholds, potentially inducing more equilibrium investment for incumbents.

4.2 Results

4.2.1 Productivity

We first describe the predictions of the model for aggregate Total Factor Productivity (TFP). To express the results as close as those in the literature, note that when technology is Cobb[Douglas, total factor physical productivity ($TFPQ_i$) per firm is proportional to a geometric average of capital and labor productivity

$$TFPQ_i \triangleq MPK_i^{\alpha} MPL_i^{1-\alpha}$$

where the marginal product of capital and marginal product of labor are defined as $MPK_i = \rho\alpha\frac{y_i}{k_i}$ and $MPL_i = \rho(1-\alpha)\frac{y_i}{l_i}$respectively. Aggregating up, we obtain

$$TFP_t = \left(\sum_{j=L,H} \int (TFPQ_{z_i})^{\rho} \, dv_t^j(z_i) \right)^{\frac{1}{\rho}} \tag{12}$$

This expression is analogous to (9) presented in the aggregates section and is our baseline measure.

If there is no dispersion in marginal product across firms, aggregate total factor productivity simplifies to

$$TFP_t^{MC} = s_t \left[\sum_{j=L,H} \int (z_i)^{\frac{\rho}{1-\rho}} \, dv_t^j(z_i) \right]^{\frac{1-\rho}{\rho}} \tag{13}$$

Although in this case there are no losses in efficiency stemming from the technological friction, the presence of monopolistic competition might still affect productivity through the equilibrium number of operating firms in the market. We use this measure to test the properties of the baseline model against.

Table 4 shows the effect of irreversibility and indivisibilities in production on computed aggregate TFP. All values are reported in log points. The first column reports the statistic described in (12) .The second column reports the same statistic for the optimal allocation of firms which is computed imposing the decentralized cost structure into the pseudo-planner problem absent of transfers. The first row reports aggregate productivity and the second row the standard deviation of the time series. The third row reports a measure of dispersion in computed TFPQ across firms. I report the coefficient of variation across economies.

Aggregate productivity under the optimal allocation is 11% higher than in the Baseline economy. While the optimal policy induces a drop in the coefficient of variation of TFPQ across firms, it induces higher volatility of productivity in the time series. From the definition of TFP one can see that the gains in efficiency in the constrained optima may stem from disparities in the allocation of firms across technologies and productivity, or from differences in the equilibrium measure of firms operating in the market. Further analysis on the sources of gains is included when describing the optimal policy.

To isolate the effect of irreversibility and indivisibility from the changes in the equilibrium measure of firms due to the monopolistic competition, I normalize the measure of active firms to one. Table 5 reports the statistics described in the previous table for the baseline economy, the optimal policy, and an economy in which marginal product of inputs in production equalizes across firms, i.e.(13). The allocation in which marginal products are equalized across firms yields the highest aggregate productivity and the lowest coefficient of variation for TFPQ. This is not surprising since the constrained optima cannot completely undo the impact of indivisibility and irreversibility on marginal product dispersion. The differences between them are large, while aggregate productivity almost double, the cross sectional dispersion drops to a third. Also time series productivity volatility raises even more when marginal products are equalized. Fluctuations in productivity in such economy stem from changes in the productivity of the marginal firm operating in the market. The irreversibility and indivisibility in the model induce lower adjustment, and less volatile aggregate productivity.

The measure of dispersion in TFPQ potentially hides distributional issues, i.e. the distortion generated by the irreversibility and the indivisibility is disparate across capacities/technologies. I compute the ratio of mean productivity per capacity in the model and under the assumption that firms equalize marginal products. An entry equal to 1 in Table 6 indicates the same mean productivity. The results suggest that the friction in the model generates firms with low capacity to held few resources (hence high marginal products), and productive firms running high capacity technologies, with too many resources compared to what they would held if marginal products were equalized. The friction in the model generates selection towards bigger more productivity firms. In the economy with equalization of marginal products, labor is shifted from the high capacity, low marginal productivity firms to low capacity higher marginal productivity ones. It is worth noting that the improvement in aggregate productivity induced for the optimal policy, is attained for a distribution of employment that resembles largely the one in the baseline economy.

4.2.2 Optimal Policy

As mentioned in the previous section efficiency gains may stem from improvements in the allocation of firms across technologies and productivity, or from differences in the equilibrium measure of firms operating in the market. For the calibrated economy, while total efficiency gains associated to the optimal policy are 11%, only a third of them stem from pure reallocation of resources. The rest, is induced by a larger measure of firms operating in the market in equilibrium: 17% more firms than in the baseline economy.

Accordingly, the industry dynamic is different. While entry and exit rates are lower under the optimal policy, the upgrade rate increases. Both combined indicate that there is a shift toward more productive larger firms. Upgrade rates of incumbent firms raises by 1% if compared to the baseline economy. Table 10 reports the firm dynamic. These patterns are consistent with the planner assigning a higher value to holding an additional large capacity firm than the private value of the firm in the decentralized equilibrium. The thresholds for upgrade and exit move accordingly. While in the baseline economy the exit thresholds are lower, the upgrade threshold are above the optimal levels as dictated by the efficient allocation. Average output per firm increases in the optimal allocation by 24.7% and average consumption increases 27%. The consumption equivalent compensation that would make an agent indifferent between living in the efficient or in the baseline economy should be 44% of the consumption in the baseline economy. Note that in this economy consumption equals output minus the good cost of entries and upgrades, plus the scrap value of the firms in the economy. Differences in the firm dynamic across allocations will be reflected in differences consumption equivalent measures even if the yield the same levels of output.

The optimal policy induces shifts in the contribution to output across firm sizes. It predicts a slightly larger share of output to be accounted for firms with more than 500 employees, as well as a larger contribution in employment. Capital however is allocated in the opposite direction, with a slightly higher share of the total used by the firms at the bottom of the distribution. This is not surprising since the marginal products at the bottom tend to be higher than those predicted by an economy with equalization of marginal products. Table 9 compares the predictions of the model and the optimal policy for the distribution of output, capital and employment.

One of the advantages of having the second welfare theorem to hold, is that we can study the characteristics of the optimal industrial policy, i.e. the cost structure that would induce a decentralized allocation that is efficient. Table 11 reports such cost structure and the one from the calibrated economy. The optimal policy dictates subsidies to the cost of entry in recessions and higher entry costs during booms. Both policies combined induce less fluctuations in the measure of entrants to the market. Upgrade costs are subsidized across all aggregate states. Less costly upgrades induce shifts in the productivity distribution of the firms operating in the market to the right. Scrap values are identified lower than in the calibrated economy for all capacities except at the very bottom. Lower scrap values are consistent with lower exit rates predicted in by the optimal policy.

Note that I only describe differences across stationary equilibria. The exercises are silent as of the gains/losses that the economy may incur along the transition. Studying the path across equilibria is particularly challenging in economies like this one, where not only a statistic of the distribution needs to be carried along in the state space, but potentially full histories of a continuum of firms need to be considered. In the case where only two capacities are operated and there is no aggregate uncertainty the transition can be computed. In that case, the gains across stationary equilibrium are a lower bound to total gains whenever the transition occurs from an economy with a relatively low measure of active firms, to one with higher level of operating firms. For an increase in the measure of firms comparable to the one observed across steady states in the full model (17%), predicted transition gains are 60% larger than the steady state gains. Steady state gains in the simplified economy are 1%. This number is not readily comparable to the ones in the full economy because the cost structure and investment strategies do not map to each other. However, the exercise is useful to gain intuition. Gains are larger accounting for the transition because consumption convergence occurs from "above". By doing so, the planner avoids entering firms in the transition that will later on find themselves holding more capital that what they would need at the new steady state. In the transition the upgrade threshold jumps an overshoots the new steady state upgrade threshold. Any entrant that finds optimal to upgrade in the beginning of the transition will find optimal to do so all along it. Also, induced entry decreases the relative measure of firms that are holding more capacity that what they would have chosen if entering the market this period. Hence, if the measure of firms is increasing in the market, the effect of the irreversibility on firms holding high capacity in the initial steady state vanishes in the aggregate.

4.2.3 Volatility and Aggregate TFP

In this section I investigate how features of the business cycle impact the entry and exit behavior of firms as well as our measures of aggregate productivity. The spirit of the exercise is to understand how the level of uncertainty that firms face affects aggregate productivity and equilibrium dispersion in marginal products.

In particular, I focus on changes in the unconditional variance of the shock. Suppose the aggregate shock s_t follows an AR(1)

$$s_t = \phi s_{t-1} + e_s$$

where ϕ is the persistence of the shock and e_s an i.i.d. shock with mean zero and standard deviation σ_e. The unconditional volatility of the aggregate shock is

$$\sigma_s^2 = \frac{\sigma_e^2}{1-\phi^2}$$

Hence, changes in unconditional volatility can be brought about by changes in the persistence or in the variance of the e_s shock. If the AR(1) process is approximated by a two state Markov chain, a la Tauchen (1986), then

$$\left(\frac{s_g - s_b}{2}\right)^2 = \sigma_e^2$$

and

$$\gamma_g + \gamma_b - 1 = \phi$$

I first study whether changes in the persistence and the variance of e_s (for a given unconditional volatility) have different impact in entry and exit patterns as well as in aggregate efficiency. Second, I vary the unconditional variance by changing the variance of e_s only, and assess the implications for aggregate efficiency.

I assume that expansions are shorter than in the calibrated economy ($\gamma_g = .237$), about 1.1 years on average. I will call this Case G, for change in gamma. Alternatively, I set γ_g back to its calibration value, and increase $s_g - s_b$ to generate the same unconditional volatility. I will call this Case S, for change in the size of the shock.

Table 7 reports the results. The first row reports aggregate TFP, the second its volatility. The third row reports the coefficient of variation of TFPQ across firms. The fourth, the ratio of aggregate TFP defined as (12)/(13) when the measure of firms is normalized to 1. The fourth row reports the implied volatility of output. The fifth and sixth columns report the cross sectional dispersion in productivity. As expected the predicted volatility of output is larger in the cases under study than under the calibrated model. In this particular example, the volatility of output is substantially higher when the size of shocks changes rather than when the persistence of the process does. On the one hand, lower persistence of the shock affects the discounting of future profits and hence the trade off between current and future consumption. While shocks are more frequent, firms are also less willing to respond to the aggregate fluctuations by investing or disinvesting On the other hand, the size of the shocks affects the actual payoffs of investment. Because the firms have an outside option given by their scrap value when exiting, increases in the size of the shock improve the payoffs of investment, inducing larger responses in output.

A feature to highlight is that the impact on aggregate TFP is not monotonous. While in Case G productivity raises about 10%, it drops one third in Case S. The cross sectional dispersion of TFPQ drops by similar magnitudes in both cases, yet aggregate efficiency is very different. The volatility of aggregate output raises substantially. In terms of allocations, the relative efficiency of these economies against their equal marginal products counterparts are fairly constant. Hence, much of the differences across economies stem from the equilibrium measure of firms in the market. The economy of Case G has 4 times more firms than the economy of Case S.

The underlying industry dynamic, i.e. patterns of entry, exit and investment, also differ. Table 8 depicts mean exit, entry and upgrade rates from montecarlo simulations. In both cases the increase in volatility induces higher upgrade rates. Although in Case S, upgrade rates augments almost 5 times with respect to the baseline, selection does not induce higher average productivity (in part because exit rates are also larger). In Case G instead, entry and exit rates drop with respect to the baseline, while upgrade increase and average productivity raises.

This example points out that different features of the underlying process of exogenous shocks, can produce substantially different responses of the economy even when the underlying measure of uncertainty (unconditional volatility) is the same. This is embedded in the non-convexities of the model. The disparity in the behavior of exit and entry rates as well as investment rates, may be a promising tool in identifying characteristics of the productivity process. A limitation however, is that the relationship between the industry dynamic and the nature of shock depends on the underlying friction in the economy.

Finally, I assess the impact of changes in the unconditional volatility of the shock from changes in the size of the shocks only. I simulate the economy for a grid of $s_g - s_b$ between 0.04 to 0.15 (equivalent to positive and negative shocks of sizes 0.02 and 0.07, respectively). The predicted relationship between the volatility of output (and hence the unconditional volatility of the aggregate shock) and the cross sectional dispersion in productivity is non-monotonic. Also, the relationship between dispersion in computed productivity at the firm level and aggregate productivity is not independent of aggregate uncertainty. Figure 3 displays a scatter plot of measures of dispersion and aggregate TFP under alternative shocks.

4.3 Sensitivity Analysis

I perform robustness check with respect to some of the parameters that characterize the size distribution of firms. In particular, the parameter ς_G that parameterizes the exponential distribution from which productivity draws for entrants are obtained. Second I compare the predictions of the calibrated Model to one in which the exogenous rate of exit is substantially lower.

I first set the parameter that characterizes the exponential distribution to 1.01. This number is not arbitrary as it correspond to the estimated parameter for the Pareto distribution that characterizes the firm size distribution in the data (See Axtell (2001)). The predicted distribution of establishment across log employment lies to the right of the calibrated one. Note that a lower parameter for an exponential distribution indicates a "fatter' tail. In other words, entrants in this alternative economy start too productive inducing selection at the bottom and a shift in the allocation towards larger firms.

As the parameter increases the average productivity of entrants gets lower. Entrants with lower productivity affect the average productivity in the market and the allocation of employment and capacity across productivity. Matching accurately the firm distribution by employment and establishment is important. The economy with $\varsigma_G = 1.2$ cannot match the employment distribution in the data. It generates a distribution highly skewed to the right.

I also test the predictions of the model when the exogenous exit rate drops to 1.1% per year. The equilibrium industry dynamic changes by construction generating lower entry and exit rates in equilibrium. The size distribution of firms gets skewed to the right, indicating reallocation towards high capacity more productive firms. The equilibrium number of firms operating in the market drops. Finally, the time of the transition to the stationary distribution of firms doubles. Although transitional dynamics is not the objective of this paper, this result indicates that the study of the impact of policies that changes the incentives to firm liquidation should account for longer or shorter transition paths.

5 Conclusion

This paper explores the implications of investment irreversibility and technology indivisibility for aggregate efficiency in production.The paper contributes to the study of non-convex economies with heterogeneous agents by providing an equivalence result. The equivalence result paves the way for the study of optimal policy in richer environments.

In the paper, I show that observed dispersion in marginal products is not independent of other features of the economy, such as the business cycle or more broadly the degree of demand uncertainty that firms face. The paper highlights that dispersion in marginal products is an imperfect measure of the associated efficiency losses.

When the industry dynamic is incorporated in a general equilibrium framework, high aggregate productivity allocations are associated with relatively low dispersion in marginal products. But low aggregate productivity allocations can also be associated to low dispersion in marginal products and hence in measured productivity. For a calibrated economy to the US manufacturing sector, I show that most of the gains in productivity from shifting to the efficient allocation of resources stem from changes in the industry dynamic rather than static reallocation of resources.

Partial irreversibility and higher divisibility in capital allocations will lessen the model generated excess dispersion in marginal products, for a given volatility of the aggregate process. However, as long as the movements in investment thresholds are such that the measure of incumbents firms holding capital away from the one chosen by entrants with the same blueprint does not vanish, non-convexities at the micro level will induce dispersion in marginal products and computed productivity.

I have abstracted from idiosyncratic risk. If incorporated in the model, I expect higher induced dispersion in marginal products. Higher uncertainty at the firm level will move optimal investment thresholds at the firm level even more than in the economy with aggregate shocks only. Large regions of inaction for alternative realizations of the idiosyncratic productivity shock or demand shock, are consistent with sustained disparities in marginal products.

While this paper focuses on the US manufacturing sector, the relationship between uncertainty, investment, industry structure and disparities in marginal products across production units might be a promising line of research in the context of the study of cross country differences in aggregate TFP. In other words, are economies characterized by more instability (i.e. political instability that leads to uncertainty on tax schemes, or fluctuations in the terms of trade in economies with a highly concentrated production base) prone to higher and persistent disparities in marginal products? How does the industry structure and firm dynamics vary across these economies? Can those patterns help us identify features of the aggregate productivity process?

Suppose that one would like to compare alternative economies for which we observe some statistic of marginal product dispersion. Suppose that these economies differ in the process characterizing the aggregate shock. In the model, it is possible for these economies to have similar dispersion in marginal products and substantial differences in aggregate productivity. At one extreme, when the volatility of the aggregate productivity process is low, the economy approximates a stationary one. There is exit and entry in equilibrium as well as upgrades in technology. However, because the size of the aggregate shock is small, the main determinant of investment decisions is the firm's idiosyncratic productivity (as it will be in an economy with no shocks). The mechanism discussed in the example at the beginning becomes irrelevant. At the other extreme, when the volatility of the process is very high, incumbent firms find it more valuable to wait and not upgrade. Hence, in equilibrium upgrades in technology are delayed. Exit rates increase so that firms holding capital away from the level that they would have chosen in the current period are selected out of the market whenever a bad shock hits the economy. The mechanism described above vanishes again. While both economies display low dispersion in marginal products, the one with higher volatility is on average less productive than the one with lower volatility. Hence, the link between aggregate productivity and dispersion in marginal products depends on features of the macroeconomy and the patterns of firms entry, exit and investment.

Finally, it is worth mentioning that the results presented in the paper correspond to the behavior of firm distributions in the long run. The properties of the transitions to the ergodic set remain to be studied. The presence of indivisibility in technologies may slow down the transition, affecting not only the equilibrium technologies adopted but also the return to capital and the path of output and capital accumulation, as well as the implications for the design of optimal policy.

References

R. Albuquerque and H. Hopenhayn. Optimal lending contracts and firm dynamics. *Review of Economic Studies*, 71:285–315, 2004.

Laura Alfaro, Andrew Charlton, and Fabio Kanczuk. Plant-Size Distribution and Cross-Country Income Differences. In *NBER International Seminar on Macroeconomics 2008*, NBER Chapters, pages 243–272. National Bureau of Economic Research, Inc, December 2009.

Y. Algan, O. Allais, and W. Den Haan. Solving the incomplete markets model with aggregate uncertainty using parameterized cross-sectional distributions. *Journal of Economic Dynamics and Control*, 34(1):59–68, January 2010.

J. Asker, A. Collard-Wexler, and J. De Loecker. Dynamic inputs and resource (mis)allocation. *CEPR Discussion Papers*, 8469, 2011.

R Axtell. Zipf distribution of us firm sizes. *Science*, 293:1818–1820, 2001.

E. Barstelman, J. Haltinwanger, and S. Scarpetta. *Price Dynamics*, chapter Measuring and Analizing Cross Country Differences in Firm Dynamics. University of Chicago Press, 2009.

E. Barstelman, J. Haltinwanger, and S. Scarpetta. Cross-country differences in productivity: The role of allocation and selection. *American Economic Review*, 103(1):305–334, 2013.

F. Buera and Y. Shin. Self-insurance vs. self-financing: A welfare analysis of the persistence of shoc. *Journal of Economic Theory*, 146(3):845–862, 2011.

M. Buso, L. Madrigal, and C. Pages. Productivity and resource misallocation in latin america. *The B.E. Journal of Macroeconomics*, 13(1):30, 2013.

R. Caballero and M. Hammour. Jobless growth: appropriability, factor substitution, and unemployment. *Carnegie-Rochester Conference Series, Elsevier*, 48(1):51–94, 1998.

G. Clementi and D. Palazzo. Entry, exit, firm dynamics, and aggregate fluctuations. *Unpublished*, 2010. Working Paper Series, Rimini Center for Economic Analysis.

R. Cooper and I. Schott. Capital reallocation and aggregate productivity. *NBER WP19715*, 2013.

S. Davis, J. Haltiwanger, R. Jarmin, C.J. Krizan, J. Miranda, and K. Sandusky. *Producer dynamics: new evidence from micro data*, chapter Measuring the Dynamics of Young and Small Businesses: Integrating the Employer and Nonemployer Universes. 2007.

M. Dums and T. Dunne. Capital adjustment patterns in manufacturing plants. *Review of*

Economic Dynamics, 1(2):409–429, 1998.

J. Eaton and G. Grossman. Optimal trade and industrial policy under oligopoly. *The Quarterly Journal of Economics*, 101(2):383–406, 1986.

A. Eisfeldt and A. Rampini. Capital reallocation and liquidity. *Journal of Monetary Economics*, 53:369–399, 2006.

M. Eslava, J. Haltiwanger, A. Kugler, and M. Kugler. The effects of structural reforms on productivity and profitability enhancing reallocation. *Journal of Development Ec.*, 2004.

R. Fattal Jaef and H. Hopenhayn. Constrained optimality and the welfare effects of misallocation. *Working Paper*, July 2012.

F. Gourio and N. Roys. Size-dependent regulations, firm size distribution, and reallocation. *Quantitative Economics*, 5:377–416, 2014.

N. Guner, G. Ventura, and Y. Xu. Macroeconomic implications of size-dependent policies. *Review of Economic Dynamics*, 11(4):721–744, 2008.

J. Haltinwanger. Job creation and firm dynamics in the united states. *Innovation Policy and the Economy*, 12.

H. Hopenhayn. Entry, exit and firm dynamics in long run equilibrium. *Econometrica*, 60: 1127–50, 1992.

C. Hsieh and Peter Klenow. Misallocation and manufacturing tfp in china adn india. *Quarterly Journal of Economics*, CXXIV, 2009.

L. Jones and R. Manuelli. A convex model of equilibrium growth: Theory and policy implications. *Journal of Political Economy*, 98:1008–1038, 1990. doi: dd.

B. Jovanovic. Selection and the evolution of industry. *Econometrica*, 50:649–70, 1982.

K. Judd, L. Maliar, S. Maliar, and R. Valero. Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain. *Journal of Economic Dynamics and Control*, 44(C):92–123, 2014.

M. Kehrig. The cycliclality of productivity dispersion. Working Paper, Northwestern University, 2011.

A. Khan and J. Thomas. Idyosincratic shocks and the role of nonconvexities in plant and aggregate investment. *Econometrica*, 76:385–436, 2008.

P. Krusell and T. Smith. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106, 1998. doi: pid.

Y. Lee and T. Mukoyama. Entry, exit, and plant-level dynamics over the business cycle.

Center for Economic Studies, US Census Bureau, 08-17, 2008.

R. Lucas. On the size distribution of business firms. *The Bell Journal of Economics*, 9: 508–523, 1978.

R. Lucas and E. Prescott. Investment under uncertainty. *Econometrica*, 39, 1971.

E. Luttmer. Models of growth and firm heterogeneity. *Annual Review of Economics*, 2: 547–576, 2010.

A. Mas-Colell. Indivisibile commodities and general equilibrium theories. *Journal of Economic Theory*, 16:443–456, 1977. doi: f.

M. Melitz. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725, 2003.

V. Midrigan and D. Xu. Accounting for plant-level misallocation. *Society of Economic Dynamics, Meeting Papers*, 2009.

B. Moll. Productivity losses from financial frictions: Can self-financing undo capital misallocation? *forthcoming American Economic Review*, 2013.

B. Pugsley and A. Sahin. Grown-Up Business Cycles. Staff Reports 707, Federal Reserve Bank of New York, 2014.

D. Restuccia and R. Rogerson. Policy distortions and aggregate productivity with heterogeneous plants. *Review of Economic Dynamics*, 11:707–720, 2008.

J. Thomas, B. Palazzo, A. Khan, and G. Clementi. Entry, Exit and the Shape of Aggregate Fluctuations in a General Equilibrium Model with Capital Heterogeneity. Technical report, 2014.

M. Veracierto. Plant level irreversible investment and equilibrium business cycles. *American Economic Review*, 92:181–197, 2002.

Appendix (A)

5.1 Numerical Solution

Given a cost structure, Υ,the solution to the pseudo-planner problem is a set of functions $z^{e*}(k_j, \Xi_t; \Upsilon)$, $z^{u*}(k_j, \Xi_t; \Upsilon)$ and a measure of entrants M^{ent*} that solves the corresponding optimality conditions. The algorithm to solve the equilibrium allocations is

1. Assume an arbitrary cost structure for the planner $\Upsilon = \left[\Pi^e, \Pi^e, I^H, I^L, 0\right]$ (with no transfers, T).

2. Compute the dynamic of the joint distribution of capital and productivity for an arbitrary initial distribution v_0.

   I follow the ideas set up by Algan et al. (2010) for the parameterization of cross-sectional distributions. In this model productivity distributions for each technology inherit properties of the distribution of entrants, and are fully characterized once we solve for upgrade and exit thresholds.

   To compute equilibrium thresholds I use anisotropic grids as introduced by Judd et al. (2014), and approximate the value function of the planner using a Smolyak-based projection algorithm.

3. Find the value function of the planner, and optimal policy (threshold functions).

4. For a given optimal policy for the planner, run Montecarlo simulations over the predicted distribution of $\{v_t\}_{t=1}^{T_M}$.

5. Calibration: The moments of $v = v_{T_M}$ for T_M large enough, are used to matched moments of firms dynamic in the data.

6. Use the calibrated cost structure of the planner Υ, and the optimality conditions delivered from the decentralized problem to compute the cost structure of the decentralized allocation $\Upsilon^c = \left[\Pi^c_e(k_j, \Xi_t), I^c_H(k_j, \Xi_t), I^c_L, 0\right]$.

7. Use the decentralized cost structure to solve for the optimal policy (planner's allocation).

5.1.1 Dynamic of the Distribution

We need first to construct the grid of capacity levels in the economy, Ψ^k and that of idiosyncratic productivity Ψ^z. The grid for capacities is equally spaced, and the grid of idiosyncratic productivity is log spaced. Points in the Ψ^z will be concentrated in the left tail.

Let J be the number of capacity levels. Define the grid if exit thresholds Ψ_j^e for $j = 1, ..., J$; and three grids for upgrade threshold grids Ψ_j^u for $j = 1, ..., J-1$ where Ψ_j^u indexes the grid of upgrade thresholds from capacity j to $j+1$. Finally, we need a grid for entry levels, Ψ^{ent}.

To generate the grids we do it jointly via the Smolyak algorithm. The algorithm constructs a sparse multidimensional grid.

The grid and transition matrix for the aggregate exogenous state s is constructed following Tauchen (1986).

For given Λ_0, I compute Λ_1 using the law of motion described in the body of the paper, for each of the points in the sparse grid.

5.1.2 Approximation of the Value Function

I implement standard value function iteration over the centralized problem.

To interpolate the value function, I use tensor products using the sparse grid as interpolation points.

I solve for the coefficients of the interpolating function given an initial guess of the value function, θ_0 and the cost structure of the model, Υ.

Then update the guess by optimizing numerically

$$\begin{aligned}
V_1(v, s, M_t) = & \max_{\left\{z_{jt}^x\right\}_{j=1}^J, \left\{z_{jt}^u\right\}_{j=1}^J, M_t^e} U\left(C_t(\left\{z_{jt}^e\right\}_{j=1}^J, \left\{z_{jt}^u\right\}_{j=1}^J, M_t^e)\right) \\
& + \beta[\Pr(s' = s_1/s)V_0\left(v', s_1, M_t(1-\delta) - M_t^{eL} - M_t^{eH} + M_t^{ent}\right) \\
& + \Pr(s' = s_2/s)V_0\left(v', s_2, M_t(1-\delta) - M_t^{eL} - M_t^{eH} + M_t^{ent}\right)]
\end{aligned}$$

subject to

$$C_t + I^{L*}M_t^{ent} + I^{H*}M_t^{up} \leq Y_t + T_t + \Pi_j^{e*}M_j^e$$

$$v' = \Lambda(v, \left\{z_{jt}^x\right\}_{j=1}^J, \left\{z_{jt}^u\right\}_{j=1}^J, M_t^{ent}, M_t)$$

$$\left(\sum_{\Psi^k}\sum_{\Psi^z}\left(z_i l_i^{1-\alpha} k_j^{\alpha}\right)^{\rho}\eta^j(z_i)\right)^{\frac{1}{\rho}} = Y_t$$

$$\frac{v_t^j(z_i) - v_t^j(z_{i-1})}{z_i - z_{i-1}} = \eta^j(z_i)$$

$$\int l_i di = 1$$

Using the updated value function V_1 recompute θ.Iterate until convergence.

5.1.3 Montecarlo Simulations

From the calibrated transition probabilities of the aggregate shock, generate 100 paths of 1000 periods each and simulate the path of allocations given the optimal policy of the planner.

Compute statistics of interest characterizing the firm dynamics of the economy, i.e. entry rates, exit rates and investment rates per capacity, dispersion in productivity, etc.

5.1.4 Cost Structure in the Decentralized Allocation

The optimality conditions for the firms, as well as those of the centralized problem are linear in the adjustment costs. Hence, if we replace the allocation that solves the pseudo planner problem into the system of equations that solves the decentralized allocation, we can infer the cost structure that decentralizes the allocation.

At the centralized allocation, the optimality conditions from the decentralized problem would typically not hold. To bring the equilibrium about, we redefine the adjustment costs faced by firms as

$$\Pi_j^c(k_j, s_t, v_t) = \Pi_e(1 + \tau^e(k_j, s_t, v_t))$$

$$I_j^{Hc}(k_j, s_t, v_t) = I_H(1 + \tau^u(k_j, s_t, v_t))$$

$$I^{Lc}(s_t, v_t) = I_L(1 + \tau^{ent}(s_t, v_t))$$

and solve a system of nonlinear equations for the tax/subsidy scheme. The cost structure of the decentralized allocation is $\Upsilon^c = \left[\left\{\Pi_j^c(k_j, s_t, v_t)\right\}_{j=1}^{J}, \left\{I_j^{Hc}(k_j, s_t, v_t)\right\}_{j=1}^{J-1}, I^{Lc}(s_t, v_t), 0\right]$.

Appendix (B)

5.2 Results

Figure 1: Establishment Distribution

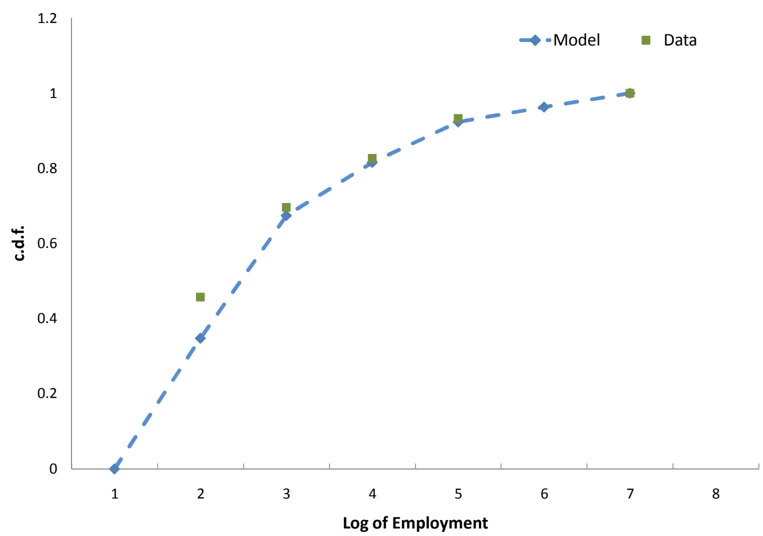
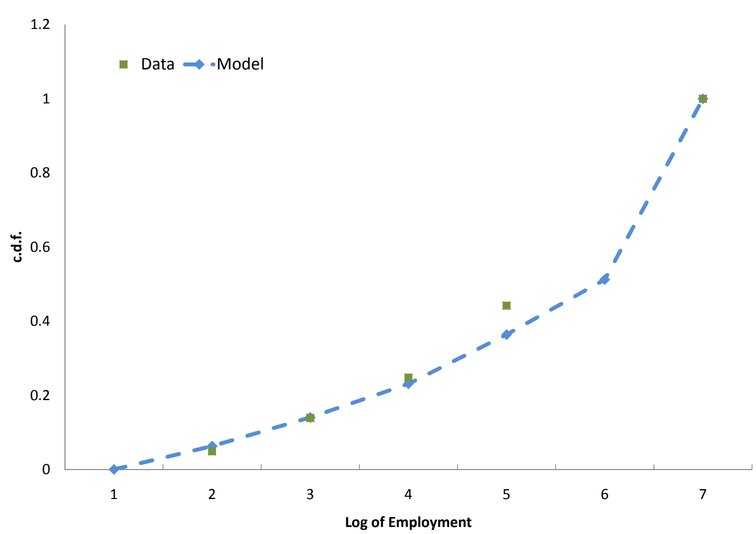


Figure 2: Employment Distribution



Parameter	Target	Value
γ_g	Persistence of Expansions	.685
γ_b	Persistence of Recessions	.298
β	Average Annual Interest Rate	.98
α	Share of Capital	33%
$\sigma(\rho)$	Returns to entrepreneurship	6.66 (0.85)
δ	Mean Exit Rate	0.055
θ	Intertemporal Elasticity of Substitution	1 (log utility)
$\underline{z}$	Lower Bound of Idyosincratic Productivity	0.01
N	Number of Technologies/Capacities	4

Table 1: Parametrization

Parameter	Definition	Value
$s_g - s_b$	Size of the Shocks (Symmetric)	$exp(0.0267) - exp(-0.0267)$
$[\underline{k}, \overline{k}]$	Range of Capacities	$[1, 4]$
$[\underline{z}, \overline{z}]$	Range for Idiosyncratic Productivity (Upper Bound)	$[0.01, 4.25]$
I^{L23}	Entry Costs	$\begin{bmatrix} 1.09 \\ 0.93 \end{bmatrix}$
I^H	Upgrade Costs	$\begin{bmatrix} 4.55 & 11.37 & 37.1 \\ 4.28 & 4.26 & 1.98 \end{bmatrix}$
Π^e	Scrap Values	$\begin{bmatrix} 0.85 & 2.47 & 9.27 & 9.1 \\ 0.86 & 2.46 & 9.2 & 9.13 \end{bmatrix}$
ς_G	Pareto Tail of the productivity distirbution at entry	1.9

Table 2: Jointly Calibrated Parameters

Moment	Data	Model	Moment	Data	Model
Emp. Share, 1-19	0.05	0.04	Estab. Share, 1-19	0.46	0.35
Emp. Share, 20-49	0.14	0.13	Estab. Share, 20-49	0.69	0.67
Emp. Share, 50-99	0.25	0.21	Estab., 50-99	0.83	0.82
Emp. Share, 100-249	0.44	0.36	Estab., 100-249	0.93	0.91
Entry Rate	6.9%	6.24%	Exit Rate	5.5%	6.23%
Investment Spikes[24]	8%	9.1%	Log Emp. (upper bound)	10000+	10829
Output Volatility	2.09%	2.1%			

Table 3: Targeted Moments

	Baseline	**Optimal Allocation**
Aggregate TFP	3.36	3.73
Standard Deviation TFP	7.9%	8.4%
Coefficient of Variation, TFPQ	3.01	2.66

Table 4: Productivity Statistics

	Baseline	**Optimal Allocation**	TFP^{mc}
Aggregate TFP	1.31	1.36	2.33
Standard Deviation TFP	2.6%	2.6%	3.3%
Coefficient of Variation, TFPQ	3.01	2.66	1.05

Table 5: Productivity Statistics: Normalized Measure

k	**Ratio mean** $TFPQ$
1	1.02
2	0.99
3	0.99
4	0.98

Table 6: Efficiency across capacities

	Baseline	**Case G**	**Case S**
	γ_g= **.685**	γ_g= **.237**	γ_g= **.685**
	$\mathbf{s}_g - \mathbf{s}_b$= **0.053**	$\mathbf{s}_g - \mathbf{s}_b$= **0.053**	$\mathbf{s}_g - \mathbf{s}_b$= **0.064**
TFP	3.36	3.72	2.19
Standard Deviation TFP	7.9%	9.1%	30.4%
Coefficient of Variation TFPQ	3.01	2.7	2.64
$\mathbf{TFP}_{M=1}/\mathbf{TFP}^{mc}$	0.56	0.58	0.56
Volatility of Output	2.1%	2.5%	8.6%

Table 7: Features of Aggregate Uncertainty

	Model	**Case G**	**Case S**
Entry Rate	6.24%	5.95%	20.4%
Exit Rate	6.23%	5.94%	12.6%
Upgrade Rate	9.1%	9.7%	45.1%

Table 8: Firm Dynamics

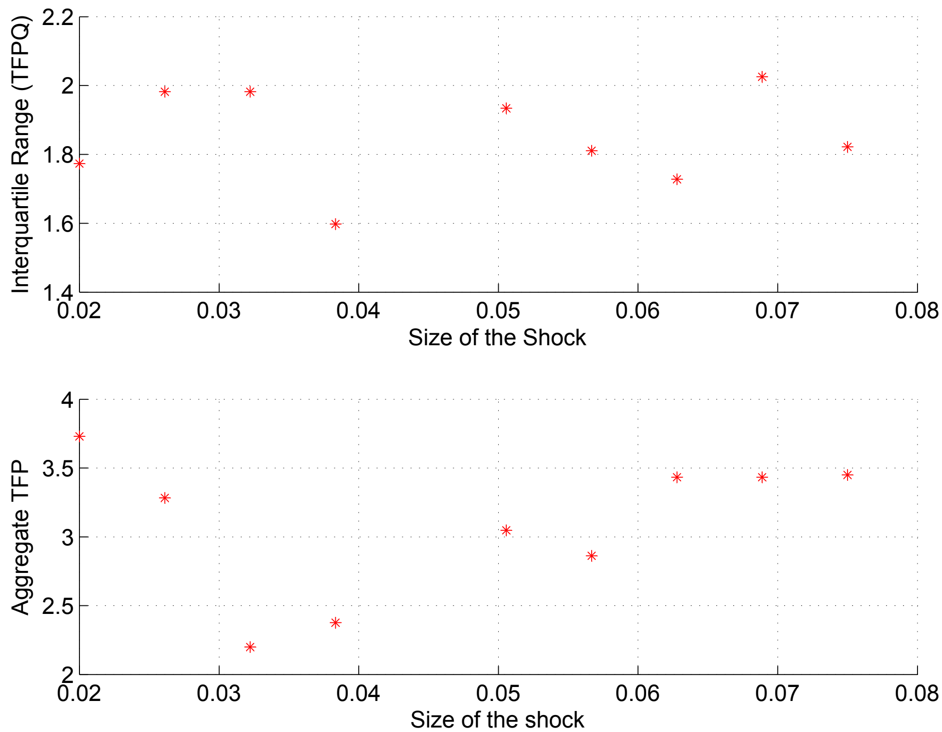


Figure 3: Dispersion in TFP, Aggregate TFP and the Cyclical Component of GDP

Employment	0-49	50-149	150-499	500+
Output Share	0.14	0.06	0.05	0.75
Opt. Policy	0.13	0.05	0.04	0.78
Capital Share	0.69	0.16	0.08	0.07
Opt. Policy	0.71	0.15	0.08	0.06
Employment Share	0.16	0.11	0.15	0.58
Opt. Policy	0.16	0.11	0.14	0.59

Table 9: Optimal Policy: Distributional Implications

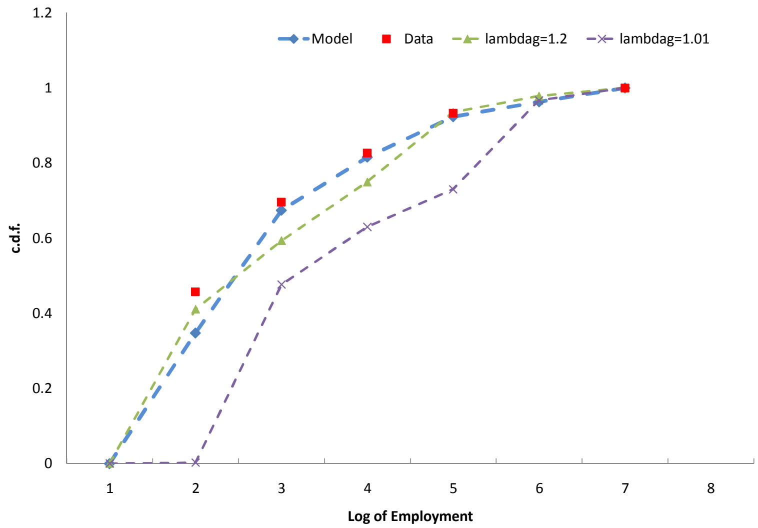
	Model	Optimal Policy
Entry Rate	6.24%	5.85%
Exit Rate	6.23%	5.84%
Upgrade Rate	9.1%	9.8%

Table 10: Optimal Policy: Firm Dynamics

	Good Times		Bad Times	
	Baseline	**Optimal Policy**	**Baseline**	**Optimal Policy**
I^L/Y	0.30	0.28	0.30	0.29
I^H/Y	$\left[\begin{array}{ccc} 0.87 & 3.28 & 3.22 \end{array}\right]$	$\left[\begin{array}{ccc} 0.45 & 0.81 & 2.51 \end{array}\right]$	$\left[\begin{array}{ccc} 0.87 & 3.25 & 3.23 \end{array}\right]$	$\left[\begin{array}{ccc} 0.43 & 0.83 & 2.57 \end{array}\right]$
Π^e/Y	$\left[\begin{array}{cccc} 0.39 & 1.61 & 4.02 & 13.11 \end{array}\right]$	$\left[\begin{array}{cccc} 0.50 & 0.49 & 2.27 & 8.33 \end{array}\right]$	$\left[\begin{array}{cccc} 0.33 & 1.51 & 1.51 & 0.70 \end{array}\right]$	$\left[\begin{array}{cccc} -0.02 & 0.29 & 0.79 & 1.84 \end{array}\right]$

Table 11: Tax/subsidy Structure in terms of output per worker

Figure 4: Establishment Distribution, Sensitivity Analysis



6 Appendix (C)

6.1 Features of the Solution

Proof (thresholds). First notice that $\pi(x_t, X_t)$ is bounded and continuous in z. (Replace the optimal factor demands in the profit function).

Second, let $W^*(\ x, X)$ be the unique fixed point to the operator T,

$$T(W(x, X_t)) = \max\left\{\Pi_e, \pi(x, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\}$$

We first show that $W^*(\ x, X)$ is non-decreasing in z.

Let $C(Z)$ be the set of continuous bounded functions in z, and let $C'(Z)$ a closed subspace of non-decreasing functions. Take $W \in C(Z)$ and $z_1 < z_2$. then

$$\begin{aligned}
T(W(z_1, \psi^j, X_t)) &= \max\left\{\Pi_e, \pi(z_1, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\} \\
&\leq \max\left\{\Pi_e, \pi(z_2, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W(x, X_{t+1})\right)\right\} \\
&= T(W(z_2, \psi^j, X_t))
\end{aligned}$$

so that $T(C'(Z)) \subseteq C'(Z)$. Hence by the Contraction Mapping Theorem $W^* \in C'(Z)$.

Now, we want to prove that for each (ψ^j, X) the function $\widetilde{W}\ (z,\psi^j, X_t)$ is strictly increasing in z. Note that the expectation operator in the last term of the previous equation defined over the aggregate of the economy and independent of the productivity of the firm except through the function W^*. Take $z_1 < z_2$

$$\begin{aligned}
\widetilde{W}(z_1, \psi^j, X_t) &= \pi(z_1, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W^*(z_1, \psi^j, X_{t+1})\right) \\
&< \pi(z_2, \psi^j, X_t) + E_t\left(\widetilde{\beta}_{t+1}(X_t, X_{t+1})\,(1-\delta)\,W^*(z_2, \psi^j, X_{t+1})\right) \\
&= \widetilde{W}(z_2, \psi^j, X_t)
\end{aligned}$$

which proves the claim.

Given the monotonicity of the continuation values, it is optimal to set thresholds such that, if productivity is relatively low firms exit, and if high firms upgrade. Suppose this strategy is not optimal. Hence, there is a firm with productivity $z < z^e(\psi^J, X_t)$ who does not exit the market. There is also a firm with productivity $z + \Delta < z^e(\psi^j, X_t)$ who does exot, so $\max\left\{\Pi_e, \widetilde{W}(z+\Delta, \psi^j, X_t)\right\} = \Pi_e$. From the monotonicity of $\widetilde{W}$ we know that $\widetilde{W}(z+\Delta, \psi^j, X_t) > \widetilde{W}(z, \psi^j, X_t)$ so that $\Pi_e > \widetilde{W}(z, \psi^j, X_t)$ and hence remaining in the market cannot be optimal, since $\max\left\{\Pi_e, \widetilde{W}(z, \psi^j, X_t)\right\} = \Pi_e$. Analogous arguments hold for upgrade thresholds. ■

Proof.

1. Note first that the profit function $\pi(x_t, X_t)$ described in (5) is monotonic in the firm idiosyncratic productivity and the technology shifter. We have shown that firms' continuation values are also monotonic. Hence $W(z, \psi^{j+1}, X_t) > W(z, \psi^j, X_t)$ for all z whenever the minimum capacity constraint is not binding, $\lambda(x_t, X_t) = 0$. The value of the firm is homogenous in the productivity of the process (follows from the form of the profit function) and optimality condition for the exit equalizes the value of the firm to its scrap value. Hence, if $\frac{\Pi^e(\psi^{j+1})}{(\psi^{j+1})^{\frac{\rho}{1-\rho}}} < \frac{\Pi^e(\psi^j)}{(\psi^j)^{\frac{\rho}{1-\rho}}}$ then $z^e(\psi^j, X_t) > z^e(\psi^{j+1}, X_t)$.When scrap values are the same across technologies, this condition hold trivially.

2. The profit function is such that $\frac{\partial \pi(x_t, X_t)}{\partial r_t} < 0$. Following the same strategy than for the monotonicity in idiosyncratic productivity one can show that $W(z, \psi^j, X_t)$ is non increasing in the cost of capital and the continuation value $\widetilde{W}(z, \psi^j, X_t)$ is decreasing in r_t. The exit condition of firms equalizes its scrap value (which in this case is independent of the cost of capital) to the value of the firm. The higher the cost of capital, the lower the value of firm, the higher the exit threshold.

3. Given that upgrades in technology are costly and the scrap value at exit is independent of the technology operated by the firm., it cannot be optimal to upgrade and exit immediately. Suppose a firm pays an upgrade cost and exits inmediatly after. Without upgrade, it receives the same scrap value and avoids paying I^u.

   In the general case where scrap values are allowed to change across technology, the condition on costs that generates the same result, i.e. no upgrade and inmediate exit, is $I^u > \Pi^e(\psi^{j+1}) - \Pi^e(\psi^j)$. The upgrade cost needs to exceed the scrap value differential.

4. $\widetilde{W}_t(z, \psi^1, X_t)$ is increasing in the aggregate state of technology s_t and decreasing in the measure of firms in the market. From the free entry condition the result follows.

■

6.2 Existence and uniqueness of the centralized allocation

Lemma 1 (AC) *The measure associated to the distribution of types is absolutely continuous(AC) with respect to the lebesque measure on the real line*

Proof. The claim follows from the absolute continuity of the exogenous distribution of types. We prove by induction.

By definition of v_t^j

$$v_1^1(z) = \left[\frac{G(z) - G(z^{e*}(\psi^1, X))}{1 - G(z^{e*}(\psi^1, X))}\right] \quad \text{for } z \leq z^{u*}(\psi^2, X)$$

$$v_1^j(z) = \left[\frac{G(z) - G(z^{u*}(\psi^{j+1}, X))}{1 - G(z^{e*}(\psi^1, X))}\right] \quad \forall j = 2 : J \quad \text{and} \quad z \leq z^{u*}(\psi^{j+1}, X)$$

Take a sequence of intervals $(a_n, b_n)_{n=1}^N$ such that

$$\sum_{n=1}^{N} \left| v_1^j(b_n) - v_1^j(a_n) \right| \leq \varepsilon$$

Replacing by the definition

$$\sum_{n=1}^{N} \left| \frac{1}{1 - G(z^{e*}(\psi^1, X))} \left(G(b_n) - G(a_n)\right) \right| \leq \varepsilon$$

Let $\overline{\varepsilon} = \varepsilon \left[1 - G(z^{e*}(\psi^1, X))\right]$ .By AC of G, there exist $\overline{\delta}$ such that

$$\sum_{n=1}^{N} |b_n - a_n| \leq \overline{\delta}$$

Because ε was arbitrary, and $(a_n, b_n)_{n=1}^N$ too, v_1^j is absolutely continuous.

Suppose v_T^j is absolutely continuous and let $m = M_T^{ent} \frac{G(z) - G(z^{u*}(\psi^{j+1}, X))}{1 - G(z^{e*}(\psi^1, X))}$. By definition,

$$v_{T+1}^j(z) = \frac{(1 - \delta)\, v_N^j(z)}{1 - v_T^j(z^{e*}(\psi^1, X))} + m \frac{G(z) - G(z^{e*}(\psi^1, X))}{G(z) - G(z^{u*}(\psi^{j+1}, X))} \quad \text{for} \quad z^{u*}(\psi^2, X) \geq z > z^{e*}(\psi^1, X)$$

If $z_T^{u*} \leq z_{T+1}^{u*}$

$$
\begin{aligned}
v_{T+1}^{j+1}(z) &= (1-\delta)\ v_T^{j+1}(z) & & z^{u*}(\psi^{j+1},X) \geq z > \ z^{e*}(\psi^{j},X) \\
&= (1-\delta)\, v_T^{j+1}(z) + m & 1 > z > \ z^{u*}(\psi^{j+1},X) &
\end{aligned}
$$

If $z_T^{u*} > z_{T+1}^{u*}$

$$
\begin{aligned}
v_{T+1}^{j+1}(z) &= (1-\delta)\ v_T^{j+1}(z) & z_{T+1}^{u*}(\psi^{j+1},X) \geq z > \ z^{e*}(\psi^{j},X) \\
&= (1-\delta)\left[v_T^{j+1}(z) + v_T^{j}(z) - v_T^{j}(\ z_{T+1}^{uj+1*})\right]\ + m & z_T^{u*}(\psi^{j+1},X) > z > \ z_{T+1}^{u*}(\psi^{j+1},X) \\
&= (1-\delta)\, v_T^{j+1}(z) + v_T^{j}(z_T^{uj+1*}) - v_T^{j}(\ z_{T+1}^{uj+1*}) + m & 1 > z > \ z_T^{u*}(\psi^{j+1},X)
\end{aligned}
$$

Therefore, each of the measures is the sum of absolutely continuous functions. Hence, v_{T+1}^{j} is absolutely continuous.

If v_t^j, is absolutely continuous then it is continuous. ■

Lemma 2 (M) *The feasible measure of firms in the market is bounded*

Proof. Using the aggregation results, one could right the feasibility constraint of the economy as

$$
M_t^{\frac{1}{\rho}}\widetilde{Y}_t + T_t + \Pi_t^e M_t^e - I_t^L M_t^{ent} - I_t^H M_t^u = C_t
$$
$$
M_t - (1-\delta)\, M_{t-1} - M_t^{ent} + M_t^e = 0
$$

where $\widetilde{Y}_t = Y_t M_t^{-\frac{1}{\rho}}$

A strategy to make the measure of firms grow without bound would be to never exit firms and enter as much as possible. Now, because entry is costly, optimality dictates that the marginal cost of an entrant equalizes the marginal return,

$$
\frac{1}{\rho}\left((1-\delta)\, M_{t-1} + M_t^{ent}\right)^{\frac{1}{\rho}-1} y_t = I_t^L
$$

which pins down a finite level of entry at each t. Replacing the entry level into the dynamic equation for the measure of firms we obtain

$$
M_t = \left(\rho \frac{I_t^L}{\widetilde{Y}_t}\right)^{\frac{\rho}{1-\rho}}
$$

which is bounded.

Alternatively, a strategy to make the measure of firms shrink without bound would be to never enter firms and exit as many as possible. Now, because exit is costly (in terms of foregone output), optimality dictates

$$\frac{1}{\rho}\left(\left(1-\delta\right)M_{t-1}-M_t^e\right)^{\frac{1}{\rho}-1}y_t=\Pi_t^e$$

which pins down a finite level of entry at each t. Replacing the entry level into the dynamic equation for the measure of firms we obtain

$$M_t=\left(\rho\frac{\Pi_t^e}{\widetilde{Y}_t}\right)^{\frac{\rho}{1-\rho}-1}$$

which is bounded at a positive number. ■

Before moving to the next result define Θ as the set of bounded absolutely continuous functions from $[\underline{z},\overline{z}]\rightarrow R^{+}$. Hence, $\left\{v_{t-1}^{j}\right\}_{j=1}^{J}\in\Theta^{J}$. Let, $\overline{K}\subset R$ the feasible set for capital. Because there are decreasing returns to capital in the aggregate it is without loss of generality to assume $\overline{K}$ is compact. Lemma (M) shows that the measure of firms in the market is bounded. Given the feasibility constraint in final goods, feasible levels of consumption are bounded also. Let $\Gamma_C:\Theta^J\times\overline{K}\rightarrow\mathbb{R}^{+}$ describe feasible levels of consumption.

Lemma 3 (U) *$U:\mathbb{R}^{+}\rightarrow\mathbb{R}$ is bounded and continuous in Γ_C*

Proof. As defined in the body of the paper U is CES with parameter θ. If $\theta<1$ then $U(C)$ is bounded below as $U(0)=0$. Now, potentially U is unbounded above. However, because the feasible measure of firms in the market is always bounded above (Lemma (M)), consumption is bounded and $U(.)$ too, along the relevant state space. $U(.)$ is continuous by assumption so the claim is proved. If instead $\theta\geq 1$, the return U is discontinuous at zero and potentially unbounded below. Because the feasible measure of firms is bounded away from zero, unboundedness below of U is also ruled out.

If $\theta\geq 1, U$ can be unbounded below, but the feasible measure of firms and consumption are bounded below hence U is bounded in the feasible set. ■

Theorem 2 *a) The solution to the Planner problem exist and it is unique*

Define $u : \Gamma_C \rightarrow R$, the utility associated to each feasible level of consumption. Let $\Gamma : S\times\ \Theta^J \times \overline{K} \rightarrow \Theta^J \times \overline{K}$ be the set of feasible firm distributions and aggregate capital stock. Let the set $\overline{\Xi} \equiv S\times\ \Theta^J \times \overline{K}$ such that $\Xi_t \in \overline{\Xi}$ for each t.

Proof. Let E_t be the expected value under the transition probabilities for the exogenous shock $\mathbf{P}$. We can write the planner's problem in terms of the operator $\digamma$ as

$$\digamma V\left(\Xi_t\right) = \max_{\left(\left\{v_t^j\right\}_{j=1}^J, K_{t+1}\right) \in \Gamma\left(s_t, \left\{v_{t-1}^j\right\}_{j=1}^J, K_t\right)} u(\left\{v_t^j\right\}_{j=1}^J, K_{t+1}) + \beta E_t\left[V\left(\Xi_{t+1}\right)\right]$$

Let $H(\overline{\Xi}, \theta)$ be the set of functions $f : \overline{\Xi} \rightarrow \mathbb{R}$ that are homogenous of degree $(1-\theta)$, continuous except potentially at the origin if $\theta > 1$ and bounded in the norm

$$\|f\| = \sup_{\|\Xi_t\|=1, \Xi_t \in \Theta^J \times \overline{K}} \|f(\Xi_t)\|$$

From Lemma (U) we have that $u : \Gamma_C \rightarrow R$ maps a convex compact set[25] into a closed subset. Also, given the structure of the stochastic process for s_t, $\mathbf{P}$ has the Feller property. Hence, $\digamma$ maps from the set of continuous and bounded functions into itself, $\digamma : H(\overline{\Xi}, \theta) \rightarrow H(\overline{\Xi}, \theta)$.

If $\digamma$ is a contraction, $\|\digamma f - \digamma g\| \leq \gamma \|f - g\|$ for $\gamma \in (0,1)$ and all $f, g \in H(\overline{\Xi}, \theta)$.

For $f \leq g$ (which is, the inequality is satisfied for every $\Xi_t \in \overline{\Xi}$) it is true that $f \leq g + \|f - g\|$. Using the definition of $\digamma$, we know that $\digamma f \leq \digamma g + \beta\|f - g\|$ and that if $f \geq g$,then $\digamma g \leq \digamma f + \beta\|f - g\|$. Or in other words, that $\|\digamma f - \digamma g\| \leq \beta\|f - g\|$ .Hence, $\digamma$ is a contraction.

Because $H(\overline{\Xi}, \theta)$ is a Banach space, the contraction mapping theorem implies there exist a unique fixed point, V^*. ■

6.3 Equivalence with the decentralized solution

To prove the equivalence between the centralized and decentralized solution define

$$\Omega(z^e, z^u, M^{ent}; \Xi_t) \equiv \tau\Pi^e M^e + \tau I^H M^u + \tau I^L M^{ent} + Y - Y^*$$

thresholds depend of the transfers, T_t

[25] Θ is a convex set as each convex combination of two AC functions is AC.

Lemma 4 *$\Omega(z^e, z^u, M^{ent}; \Xi_t)$ is continuous in the exit and upgrade thresholds as well as in the measure of entrants.*

Proof. Continuity in the measure of entrants is straightforward from the definition. Continuity in the thresholds follows from the definition of the measure of upgrades and exits in terms of the distribution of firms in the market and the absolute continuity of v_t^j that we proved in Lemma (AC), i.e. $M^{ej} = v^j(z^e(\psi^j, \Xi_t))$ for $j = 1,..J$ and $M^u = \sum_{j=1}^{J-1} \max\left\{v^j(z_t^u(\psi^{j+1}, \Xi_t)) - v^j(z_{t-1}^u(\psi^{j+1}, \Xi_{t-1})), 0\right\}$ ■

Lemma 5 *There exist a transfer scheme $T^*(\Xi_t)$ such that*

$$\Omega(\left\{z^e(\psi^j, T^*)\right\}_{j=1}^{J}, \left\{z^u(\psi^{j+1}, T^*)\right\}_{j=1}^{J-1}, M^{ent}(T^*); \Xi_t) = T^*$$

Proof. Lemma (M) shows that the measure of firms operating in the market is bounded. Hence, there exist B such that $\Omega(z^e, z^u, M^{ent}; \Xi_t) < B$[26]. The feasible measure of entrants is also bounded by Lemma (M). Let $\Phi \equiv [0, B]$, which is convex and compact by construction. The optimal thresholds are the maximizers of (PP). By the theorem of the maximum they are u.h.c. in $T^*(\Xi_t)$. Hence, Ω is an upper hemicontinuous convex valued correspondence and $\Omega \neq \varnothing$ for any $T \in \Phi$. Thus, Ω has a fixed point (Kakutani). ■

Note that there might be different combination of thresholds that generate the same transfer

Lemma 6 *If the allocation of firms in the decentralized and centralized problem are the same, there exist prices such that the dynamic of aggregate capital is the same across economies.*

Proof. The equivalence comes from setting $\tau^k r_t = \lambda_t^k$, i.e. the marginal product of capital in the economy, where $\tau^k = \rho$ the elasticity of substitution of goods in the intermediate sector. For $\tau^k = \rho$ the euler equation of the household in the decentralized allocation, and the planner coincide. If the allocation of firms across technologies is the same (because entry, exit and upgrade margins are not distorted), then endogenous TFP is the same. If we replace the equilibrium prices of firm shares in the budget constraint of the representative household, we obtain the feasibility condition in terms of goods for the aggregate economy.

[26] Output is bounded because the measure of firms is bounded and there are decreasing returns to capital in the economy.

The latter equals the budget constraint of the pseudo planner for $T = T^*$ the fixed point. Hence the allocations of capital are the same. ■

Theorem 3 *b) There exist a cost structure* $\{\Upsilon^p(\Xi_t)\}_{t=0}^{\infty}$ *such that the allocation of firms that solves the planner problem (PP) coincides with the competitive allocation.*

Proof. Define, $\Upsilon^p(\Xi_t) = \Upsilon^c\tau^*(\Xi_t)$ where $\tau^*(\Xi_t)$ generates $T^*(\Xi_t)$ (the fixed point of Ω)

When the PP is solved at $T = T^*(\Xi_t)$ the budget constraint reads

$$C_t + I_t^L M_t^{ent} + I_t^H \sum_{j=1}^{J-1} M_t^{u,j+1} + K_{t+1} - K_t(1-\widehat{\delta}) \leq Y_t + \Pi_t^e \sum_{j=1}^{J} M_t^{ej}$$

which is the market clearing condition in the decentralized allocation. Hence, for this cost structure the feasibility constraint of the planner coincides with that of the competitive equilibrium.

I argue that there exist an industrial policy $\widehat{\tau}$ such that at the thresholds of the competitive equilibrium, the generated transfer T is a fixed point of $\Omega; T(\widehat{\tau}(\Xi_t)) = \Omega(T)$. Note that the pseudo-planner's optimality conditions in terms of the allocation of firms across technologies and entry levels are linear in the cost of entry, upgrade and the scrap value (as described in (9), (11) and (10)). The indifference conditions for the firms in the decentralized problem are linear in the costs too (as described in (8), (6) and (7)). Define $\widehat{\tau}$ to satisfy the optimality conditions of the pseudo planner, at the thresholds solved using the market allocation. The industrial policy is well defined because it solves a system of linear equations perfectly identified.

Suppose that $T(\widehat{\tau}(\Xi_t))$ is not a fixed point of Ω. The level of output generated in by the centralized allocation is the same as in the decentralized allocation because the thresholds and measure of entries are the same. The budget constraint would read

$$C_t+I_t^L M_t^{ent}+I_t^H \sum_{j=1}^{J-1} M_t^{u,j+1}+K_{t+1}-K_t(1-\widehat{\delta}) \leq Y_t+\Pi_t^e \sum_{j=1}^{J} M_t^{ej}+\widehat{\tau}\Pi^e M^e+\widehat{\tau} I^H M^u+\widehat{\tau} I^L M^{ent}$$

which implies that the set of thresholds z^e, z^u, M^{ent} of the decentralized allocation violate the market clearing condition in the goods market, which yields a contradiction.

Finally, at the prices of capital and labor that we have chosen, the optimal investment and consumption of the representative consumer coincides with the allocation of the pseudo planner. ■

6.4 Efficient Allocation

In this section I describe the problem of the planner. Differently with the pseudo-planner's problem described in the body of the paper, there are no equilibrium transfers, no wedges in the rate of transformation between final goods and capital or labor, and no wedges in the cost of entry, upgrade and scrap values.

$$V\left(\Xi_t\right)=\max_{C_t,\ K_{t+1},\ Y_t,\ \{z_t^e(\psi^j)\}_{j=1}^{J},\{z_t^u(\psi^{j+1})\}_{j=1}^{J-1},M_t^{ent},l_t^i,k_t^i} U\left(C_t\right)+\beta EV\left(\Xi_{t+1}\right)$$

subject to

$$C_t+I_t^{ent}M_t^{ent}+\sum_{j=1}^{J-1}I_t^u(\psi^{j+1})M_t^{u,j+1}+K_{t+1}-K_t(1-\widehat{\delta})\leq Y_t+\sum_{j=1}^{J}\Pi_t^e(\psi^j)M_t^{ej} \qquad (\zeta_t)$$

$$s_t(\sum_{j=1}^{J}\int\left(\psi^j z_i l_i^{1-\alpha}k_i^{\alpha}\right)^{\rho}dv_t^j(z_i))^{\frac{1}{\rho}}=Y_t$$

$$\sum_{j=1}^{J}\int l_i dv_t^j(z_i)=1 \qquad (\lambda_t^l\ \zeta_t)$$

$$\sum_{j=1}^{J}\int k_i dv_t^j(z_i)=K_t \qquad (\lambda_t^k\ \zeta_t)$$

$$\tau^k k_i\geq\underline{k}^j \text{ if } \psi_i=\psi^j \quad \forall j=1:J \qquad (\widehat{\lambda}_i t^j\ \zeta_t)$$

$$\left(v_t^j\right)=\Lambda\left(v_{t-1}^j\right) \quad \forall j=1:J \qquad (\mu_t^j\ \zeta_t)$$

The optimality conditions of the planner are analogous to those described in the body of the paper for (PP) with no additional distorsions. Disparities between the market and efficient allocation show up in the equilibrium return to capital (and hence the path of accumulation K_t for $t>0$), the distribution of firms across technologies v_t^j and entry levels, M_t^{ent}.