Asymmetric Legislative Bargaining

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Abstract

In the Baron and Ferejohn legislative bargaining model, assuming unequal discount factors and recognition probabilities, under minor parameter restrictions, the unique stationary subgame perfect Nash equilibrium has a simple closed form solution. It is characterized by each approving member of the legislature obtaining the same discounted expected continuation payoff. The non-approving members in equilibrium obtain a zero payoff. The intuition extends to the general case and to the case of risk aversion.

1 Asymmetric legislative bargaining

A distinctive feature of political (or *non-market*) settings is the lack of enforcement possibilities for political entities to keep to their promises. It is natural to think of a political outcome as a result of bargaining between some number of legislators, here $2n+1$, $n \geq 1$. In their celebrated model, Baron and Ferejohn [5] consider a distributive setting where these legislators bargain over allocating a budget normalized to 1. There are possibly many time periods $t = 1, 2, ..., \infty$, wherein the legislators can propose various ways to allocate the budget; in order to pass, a proposal must be approved by a majority. In each period t, some legislator i is randomly *recognized* as a proposer (all legislators are recognized with equal probability, $p_i = \frac{1}{2n+1}$), whom then proposes a split of the budget. If the proposal is voted up by a majority the bargaining ends and the legislators obtain their shares, otherwise the bargaining proceeds to the next time period. Each legislator discounts the future by δ_i and [5] assume that $u_i(x,t) = \delta_i^t x$, where x is the amount of surplus that i receives in period $t \geq 0$; they further assume that the discount factors are equal, $\delta_i = \delta \in (0,1)$. In that setting [5] show that in a stationary subgame perfect Nash equilibrium (from hereon *equilibrium*) the first recognized proposer at $t = 1$ compensates precisely n legislators with their discounted expected future payoffs, $x^a = \frac{\delta}{2n+1}$, the proposer collects the rest, $x^p = 1 - \frac{n\delta}{2n+1}$, and all other legislators obtain shares equal to 0. The proposal then passes the majority vote.[1]

One shortcoming of [5] has been its reliance on the symmetry of the environment. In many political settings where the model has been applied such assumption is restrictive. For example, the model has been used as a testbed for laboratory experiments, where differences in discount factors (and recognition probabilities) have been been a key parameter of interest.[2] To address asymmetric environments, some authors have considered altering the assumptions of the bargaining protocol,[3] others have resorted to notions from cooperative game theory, and in related settings, notions of coalitional bargaining.[4] The advances of the ensuing literature studying the Baron and Ferejohn model in asymmetric environments

[1] Assume a symmetric equilibrium, so that each i in the role of proposer (which happens with probability $p_i = \frac{1}{2n+1}$) selects approvers with equal probabilities, so that each legislator is an approver with a probability $\frac{n}{2n+1}$. To vote for a proposal, an approver must be at least indifferent (in equilibrium, exactly indifferent) so that the two equilibrium equations obtain,

$$x^a = \delta\left(\frac{1}{2n+1}x^p + \frac{n\delta}{2n+1}\right) \text{ and } x^p = 1 - nx^a.$$

In follow-up work, [12] shows that the equilibrium in [5] is unique.

[2] See e.g., [16], [15], [6], and also the recent work by [23].

[3] See [1], who consider a process where proposers in different periods are pre-determined. [10] provide a static model for general multi-dimensional policy spaces to study the effect of a fixed proposer's power on political outcomes; [11] provide a restricted version of that model in a distributive setting to study the formation of political parties.

[4] See, e.g., [21], who extend the Nash Bargaining solution to the political setting. The connection between Nash bargaining and non-cooperative bargaining is known as the Nash program, see e.g., [7] and the references therein. See [25] for a model of coalitional bargaining in a demand game between political parties.

have been technically demanding and laborious.[5,6] The sources of these difficulties are the possibly infinite horizon of the bargaining process and the discontinuity in the payoffs to a legislator depending on whether she is chosen to be an approver or not.

We provide a simple and intuitive closed-form characterization of the unique equilibrium in the Baron and Ferejohn model for asymmetric environments. Our characterization is closely related to perfect competition: when the discount factors (or recognition probabilities) are not too dissimilar, the compensation received by the legislators in the approving majority must be the same.[7] This yields a closed-form solution for equilibrium strategies and payoffs: if a legislator is more patient, then in equilibrium, this is off-set by a lower (*ex ante*) probability of being selected into the approving majority. To deliver this result, we build on the original insight of Baron and Ferejohn, borrow an idea from the literature on discontinuous games, i.e., that the probability of choosing the approving legislators is endogenous,[8] and use the results of the existence and uniqueness of equilibrium for the general versions of the model. Our characterization is so simple that we state it with some diffidence, however, this simplicity will allow for applications, comparative statics, and estimation.

To keep our exposition accessible and transparent, we present our analysis for the case where the legislators differ only in their discount factors, $0 \leq \delta_1 \leq \delta_2 \leq ... \leq \delta_{2n+1}$. We assume that the legislators' discount factors satisfy the following restrictions,

$$\frac{1}{\delta_1} - \frac{1}{2} \leq \frac{1}{2n}\left(\sum_{i=2}^{2n+1} \frac{1}{\delta_i}\right), \tag{1}$$

$$\frac{1}{\delta_{2n+1}} + \frac{1}{2} \geq \frac{1}{2n}\left(\sum_{i=1}^{2n} \frac{1}{\delta_i}\right). \tag{2}$$

These conditions assure that the most impatient and the most patient legislators are not dramatically more impatient (or patient) than the rest.[9]

Theorem 1. *If (and only if) the conditions (1) and (2) hold, then in the unique equilibrium,*

[5] In a technical note, [19] shows a proportionality property of players' payoffs when discount factors and recognition probabilities may differ. [14] show existence and uniqueness of the stationary subgame perfect Nash equilibrium for such environments. [13] extends this result to the case of risk aversion, and shows a general monotonicity properties of the legislators' payoffs. See e.g., [18] for a computational approach.

[6] [4] proposed a notion of simplicity as a reasonable criterion for equilibrium selection in such dynamic models and applied it to the Baron and Ferejohn legislative bargaining game.

[7] To achieve this, we slightly restrict the parameter space. In contrast, the existing theoretical literature, [14], [18] and especially [13], consider the unrestricted case. Then our intuition does not hold precisely and our characterization is hard to obtain. See also Section 3.

[8] In the literature on games with discontinuous payoffs, this idea is used to assure that the best-reply correspondence is convex, see e.g., [30], [31], and [28]

[9] To fall out of the scope of the present analysis, the differences in the legislators' discount factors would have to be beyond unrealistically large. As noted below, Theorem 1 extends easily to the case when the legislators differ also in their recognition probabilities. It is evident from the proof that the theorem also easily applies to other other voting rules, i.e., where the proposal must be approved by more than a simple majority. For the case of risk aversion, the same intuitions presented here apply, but the equilibrium might no longer be computed in closed form.

x^p *and* x^a *are given by,*

$$x^a = \frac{1}{\sum_{i=1}^{2n+1} \frac{1}{\delta_i}} \textit{ and } x^p = 1 - nx^a = \frac{1}{\sum_{i=1}^{2n+1} \frac{1}{\delta_i}} \left(\sum_{i=1}^{2n+1} \frac{1}{\delta_i} - n \right). \tag{3}$$

The widespread attention received by the Baron and Ferejohn model is not surprising. On the theoretical side, modeling difficulties of political processes are well known and Baron and Ferejohn provided a simple and appealing paradigm with sensible predictions, at least for the case of distributive politics.[10] Their model has consequently been applied to many political settings,[11] and it has been used as a testbed in laboratory environments, as well as a building block for more complex political models, see e.g., [6], [26], [2], etc. We hope that our result should facilitate further applications of the Baron and Ferejohn model.

As a final note, in recent work, [13] derives some (but not all) of the equilibrium conditions below, building also on the ideas in [19]. However, these authors miss the key idea of the present paper: under the conditions (1) and (2) the competition between the legislators forces the approvers' equilibrium shares to be equal. This is a crucial equilibrium condition yielding the characterization presented here. As noted above, without taking into account that all the approvers' shares will be equal *only* under these conditions, it does not seem obvious to us how to derive the present equilibrium characterization. Indeed, this then easily allows for the extensions to the case where (1) and (2) do not hold and to the general case of risk averse legislators, which we present in Section 3. The results in both, [13] and [19] are therefore immediate corrolaries of the results presented here.[12] In Section 2 we present the proof of our theorem. In Section 3 we discuss the extensions.

2 Proof of Theorem 1

We argue that the *ex ante* expected payoffs are equal for all legislators, construct an equilibrium under that assumption, and then verify that the assumption holds under the above conditions (1) and (2).

Consider the probability $\alpha_{i,j} \in [0, 1]$ with which legislator i is selected to be an approving member of the legislature when j is the proposer, and define,

$$\alpha_i = \sum_{j \neq i} p_j \alpha_{i,j},$$

[10] In settings with a more general (possibly multi-dimensional) policy space, modeling issues are well known: in general settings [3] proved impossibility of reasonable aggregation rules, [17] and [29] proved impossibility of strategy-proof aggregation rules (other than the dictatorial rule), and [22] proved non-transitivity properties of the majority-voting correspondence for multi-dimensional domains; while in single peaked domains the median voter theorem holds in a one-dimensional setting, [27] proved a generic impossibility of similar results for multi-dimensional settings. Allocating a budget between many legislators is a specific case of such a multi-dimensional single-peaked preference domain.

[11] For example, see [20] and [24] for general overviews of international relations and institutions, and, e.g., [32] for a specific model of insitutional bargaining. [8] constructs a related model to study legislated trade agreements.

[12] On a separate and less relevant note, the analysis here has been derived entirely independently, see [9] for an earlier version of the ideas presented here.

so that α_i denotes the *average probability* with which i is an approver, and in the present case, the recognition probability $p_j = \frac{1}{2n+1}$ for all j. In equilibrium each approver is offered her discounted expected continuation payoff, and she then votes in favor of the proposal. The probabilities $\alpha_{i,j}$ are determined in equilibrium such that all the non-proposing members of the legislature obtain the same discounted expected continuation payoff. The remainder of the surplus then goes to the proposer, in particular, in the role of the proposer, all legislators obtain the same amount of surplus. Denote by x^p the share received by the proposer, and let x^a denote the share that the proposer allocates to each legislator whom she entices to approve the proposal; call every such legislator *an approver*. Since each legislator needs at least n approvers to pass the proposal, we have $\sum_{i \neq j} \alpha_{i,j} = n$, so that,

$$\sum_{i=1}^{2n+1} \alpha_i = \sum_{i=1}^{2n+1} \sum_{j \neq i} p_j \alpha_{i,j} = \sum_{j=1}^{2n+1} \left(\sum_{i \neq j} p_j \alpha_{i,j} \right) = \sum_{j=1}^{2n+1} p_j \left(\sum_{i \neq j} \alpha_{i,j} \right) = n.$$

We therefore have the following equilibrium conditions,

$$x^p = 1 - n x^a, \tag{4}$$

$$x^a = \delta_i \left(p_i x^p + \alpha_i x^a \right), i \in \{1, ..., 2n+1\}, \tag{5}$$

$$\sum_{i=1}^{2n+1} \alpha_i = n. \tag{6}$$

Divide each of the $2n+1$ equalities in (5) by δ_i, and sum these across all i's to obtain,

$$\sum_{i=1}^{2n+1} \frac{x^a}{\delta_i} = x^p + x^a \left(\sum_{i=1}^{2n+1} \alpha_i \right) = x^p + n x^a,$$

so that by (4), we obtain,

$$x^a \sum_{i=1}^{2n+1} \frac{1}{\delta_i} = 1.$$

Therefore,

$$x^p = 1 - n x^a = \frac{1}{\sum_{i=1}^{2n+1} \frac{1}{\delta_i}} \left(\sum_{i=1}^{2n+1} \frac{1}{\delta_i} - n \right).$$

What remains to verify is that the above computations are valid, i.e., that these weights α_i are indeed probability weights. For that to be the case it must be that the least patient legislator, that is legislator 1, has a continuation payoff that is not lower than x^a. Therefore, when 1 is an approver in all cases when she is not recognized, i.e., $\alpha_1 = \frac{2n}{2n+1}$,

$$x^a \leq \delta_1 \left(\frac{x^p}{2n+1} + \frac{2n x^a}{2n+1} \right).$$

Substituting (3) and rearranging the expression yields (1). Similarly, the most patient legislator, that is legislator $2n+1$, does not have a continuation payoff that is higher than x^a.

That is, when $2n+1$ is an approver with probability $\alpha_1 = 0$, her expected continuation payoff is not greater than x^a, so that,

$$x^a \geq \frac{\delta_{2n+1} x^p}{2n+1}.$$

Substituting (3) and rearranging the expression yields (2).

The proof now follows by uniqueness of the stationary subgame perfect Nash equilibrium, see [14].

3 More general cases

The above analysis applies to the more general case, when the recognition probabilities p_i may also differ across the legislators, and in order to pass, the proposal needs $q+1$ votes in the legislature, where $q \geq n$. Without loss of generality assume that,

$$\frac{1}{p_1\delta_1} - \frac{1}{p_1} = \max_i \frac{1}{p_i\delta_i} - \frac{1}{p_i}, \text{ and } \frac{1}{p_{2n+1}\delta_{2n+1}} = \min_i \frac{1}{\delta_i p_i}.$$

Now the analogous restrictions to (1) and (2) are given by

$$\frac{1}{p_1\delta_1} - \frac{1}{p_1} + q - 1 \leq \sum_{i=1}^{2n+1} \frac{1}{\delta_i}, \tag{7}$$

$$\frac{1}{p_{2n+1}\delta_{2n+1}} + q \geq \sum_{i=1}^{2n+1} \frac{1}{\delta_i}. \tag{8}$$

These restrictions assure that the legislators' continuation payoffs (or their *bargaining powers*, in the sense of their recognition probabilities) do not differ too extravagantly. For example, in Section 1 where recognition probabilities were assumed to be equal, conditions (1) and (2) assured that the most impatient legislator was not dramatically more impatient than the other legislators, and that the most patient legislator was not dramatically more patient than the rest. Conditions (7) and (8) here play a similar role in that they guarantee that in the most extreme cases, the legislators' continuation payoffs can still be made equal through different probabilities with which they are selected into the approving (supra-)majority. In this more general case, Theorem 1 necessitates only a minor restatement and its proof is practically unaltered (we leave it to the reader to verify that).

Theorem 2. *Let $n \leq q < 2n$. If (and only if) conditions (7) and (8) are satisfied, then in the unique equilibrium, x^p and x^a are given by,*

$$x^a = \frac{1}{\sum_{i=1}^{2n+1} \frac{1}{\delta_i}} \text{ and } x^p = 1 - qx^a = \frac{1}{\sum_{i=1}^{2n+1} \frac{1}{\delta_i}} \left(\sum_{i=1}^{2n+1} \frac{1}{\delta_i} - q \right). \tag{9}$$

All the above intuitions apply to the case of risk aversion, i.e., under certain conditions, an interior solution obtains, in which case the *share* received by all members of the approving majority must be the same, and the *utility from her share* must for each legislator equal the discounted expected *utility* of the legislator should the game proceed to the next period. Except for very rare cases, it is impossible to compute the resulting equilibrium in closed form.[13] Finally, we note that the monotonicity results in [19] and [13] are a direct corollary of the present model.

Finally, one may ask whether some of the intuitions of the present approach apply when conditions (1) and (2) do not hold. Indeed, these intuitions continue to hold - Theorem 1 presents the case of interior solutions to the problem and when one or the other conditions isn't satisfied, the problem has a corner solution. For example, suppose all the recognition probabilities are equal but legislator 1 is too impatient relative to the rest, so that condition (1) fails. Then, even if 1 were always included in the approving majority, she would still receive a lower discounted expected payoff than the rest. In that case simply set $\alpha_1 = \frac{2n}{2n+1}$, consider legislator 1 separately from the rest, and note that every other proposer needs $n-1$ other approvers (apart from herself and legislator 1). Legislator 1 as an approver obtains a share equal to her discounted expected payoff (which is lower than the others') so that all other legislators as proposers obtain a higher share –when 1 is the proposer she obtains a slightly lower share since she must woo n relatively expensive approvers. No new difficulties arise if more than one legislator is quite impatient, except that one must keep careful book-keeping of what legislators different proposers select for approvers and with what shares.

If one or more, but fewer than a majority of the legislators are very patient relative to the rest, then condition (2) fails and these very patient legislators are never included among the approvers. Their expected discounted payoffs are higher than those of the rest of the legislature. These legislators are considered separately from the rest, and every proposer will now selects n approvers from amongst the other legislators. Again, this extends to the more general case of unequal recognition probabilities and risk aversion.

To put some flesh on the bones, consider an example with $n = 1$ so that there are a total of 3 legislators, and assume that two legislators have the same discount factor.[14] First consider the case where $\delta_1 = \delta_2 < \delta_3$, that is, legislator 3 is more patient than 1 and 2. Condition (2) is then,

$$\frac{1}{\delta_3} + \frac{1}{2} \geq \frac{1}{\delta_1}. \tag{10}$$

In particular, if $\frac{1}{\delta_3} + \frac{1}{2} < \frac{1}{\delta_1}$, then the condition is violated and no proposer selects legislator 3 as an approver because legislator 3 is too expensive so that $\alpha_1 = \alpha_2 = \frac{1}{2}$ and $\alpha_3 = 0$.[15] Therefore, in equilibrium,

$$x^a = \delta_1\left(\frac{x^p}{3} + \frac{1}{2}x^a\right),$$

and from $x^p = 1 - x^a$ we obtain, $x^a = \frac{2\delta_1}{6-\delta_1}$. Since $\frac{1}{\delta_3} + \frac{1}{2} < \frac{1}{\delta_1}$, we have $\frac{x^p\delta_3}{3} > x^a$, i.e., even if legislator 3 is offered the approver's share with probability 0, legislator 3's discounted expected payoff is still higher than the payoff she would obtain as an approver. Hence, whenever offered, legislator 3 would decline x^a while either 1 or 2 would accept. For an

[13]A notable exception is the case of quadratic utilities, but even there the solution is rather laborious.

[14]That is too keep the example parsimonious; the above logic, or the specific analysis below can easily be adapted to the case where the three discount factors are all different.

[15]That is, $\alpha_{1,2} = \alpha_{2,1} = 1$ and $\alpha_{1,3} = \alpha_{2,3} = \frac{1}{2}$, so that $\alpha_1 = \alpha_2 = \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{2} = \frac{1}{2}$.

illustration, if $\delta_1 = \delta_2 = \frac{1}{2}$, then condition (10) holds if and only if $\delta_3 \leq \frac{2}{3}$. If, for instance, $\delta_3 = \frac{3}{4}$, then $x^a = \frac{2}{11}$, $x^p = \frac{9}{11}$, so that indeed, $\frac{\delta_3 x^p}{3} = \frac{9}{4} \times \frac{1}{11} > \frac{2}{11} = x^a$.

Next, consider the case where $\delta_1 < \delta_2 = \delta_3$, that is, legislator 1 is more impatient than 2 and 3. Condition (1) is then,

$$\frac{1}{\delta_1} - \frac{1}{2} \leq \frac{1}{\delta_2}. \tag{11}$$

If $\frac{1}{\delta_1} - \frac{1}{2} > \frac{1}{\delta_2}$, then this condition is violated and even when every other legislator as a proposer selects legislator 1 to be an approver, 1's continuation payoff is still lower so that legislator 1's vote is the cheapest. Consequently, 1's share as a proposer, x_1^p, and her share as an approver, x_1^a, are both lower than the respective shares x_2^p and x_2^a of the other two legislators. In equilibrium, when selecting an approver, legislators 2 and 3 therefore choose legislator 1 for sure (since she is cheaper than the other possibility), while legislator 1 randomizes equally between 2 and 3 (who are both equally expensive). That is, $\alpha_1 = \frac{2}{3}$, $\alpha_2 = \alpha_3 = \frac{1}{6}$ and we have the following equilibrium conditions,

$$\begin{aligned}
x_1^a &= \delta_1 \left(\frac{1}{3} x_1^p + \frac{2}{3} x_1^a \right) \\
x_1^p &= 1 - x_2^a \\
x_2^a &= \delta_2 \left(\frac{1}{3} x_2^p + \frac{1}{6} x_2^a \right) \\
x_2^p &= 1 - x_1^a.
\end{aligned}$$

From here, we obtain, after some manipulation,

$$\begin{aligned}
x_1^a &= \frac{2\delta_1 - \delta_1\delta_2}{6 - 4\delta_1 - \delta_2}, \\
x_2^a &= \frac{2\delta_2 - 2\delta_1\delta_2}{6 - 4\delta_1 - \delta_2}.
\end{aligned}$$

For example, when $\delta_1 = \frac{1}{2}$ and $\delta_2 = \delta_3 = \frac{3}{4}$, then $x_1^a = \frac{5}{26}$ and $x_2^a = \frac{6}{26}$. Indeed, it can be easily shown that the ratio of the above $\frac{x_1^a}{x_2^a} < 1$ if and only if, $\frac{1}{\delta_1} > \frac{1}{\delta_2} + \frac{1}{2}$, that is, when condition (11) is violated.

References

[1] S. N. Ali, B. D. Bernheim, and X. Fan. Predictability and power in legislative bargaining. *Mimeo*, 2014.

[2] V. Anesi and D. J. Seidmann. Bargaining in standing committees. *Mimeo*, 2012.

[3] K. J. Arrow. *Social Choice and Individual Values*. Wiley, 1963.

[4] D. Baron and E. Kalai. The simplest equilibrium of a majority-rule division game. *Journal of Economic Theory*, 61(2):290–301, 1993.

[5] D. P. Baron and J. A. Ferejohn. Bargaining in legislatures. *American Political Science Review*, 83(04):1181–1206, 1989.

[6] M. Battaglini, S. Nunnari, and T. R. Palfrey. Legislative bargaining and the dynamics of public investment. *American Political Science Review*, 106(02):407–429, 2012.

[7] K. Binmore, A. Rubinstein, and A. Wolinsky. The nash bargaining solution in economic modeling. *The RAND Journal of Economics*, 2(176–188), 17.

[8] T. R. Bowen. Legislated protection and the world trade organization. *International Economic Review*, 56(4):1349–1384, 2015.

[9] J. Copic. Legislative bargaining with unequal discount rates. *Mimeo*, 2013.

[10] J. Copic and J. N. Katz. Auctioning off the agenda: Bargaining in legislatures with endogenous scheduling. *Mimeo*, 2012.

[11] J. Copic and J. N. Katz. Scheduling auctions and proto-parties in legislatures. *Mimeo*, 2014.

[12] H. Eraslan. Uniqueness of stationary equilibrium payoffs in the baron–ferejohn model. *Journal of Economic Theory*, 103(1):11–30, 2002.

[13] H. Eraslan. Uniqueness of stationary equilibrium payoffs in the baron-ferejohn model with risk averse players. *Mimeo*, 2015.

[14] H. Eraslan and A. McLennan. Uniqueness of stationary equilibrium payoffs in coalitional bargaining. *Journal of Economic Theory*, 148(6):2195–2222, 2013.

[15] G. Fréchette, J. Kagel, and M. Morelli. Nominal bargaining power, selection protocol and discounting in legislative bargaining. *Journal of Public Economics*, 89(8):1497–1517, 2005.

[16] G. Fréchette, J. H. Kagel, and M. Morelli. Behavioral identification in coalitional bargaining: an experimental analysis of demand bargaining and alternating offers. *Econometrica*, 73(6):1893–1937, 2005.

[17] A. Gibbard. Manipulation of voting schemes: A general result. *Econometrica*, 41(4):587–601, 1973.

[18] T. Kalandrakis. Computation of equilibrium values in the baron and ferejohn bargaining model. 2014.

[19] T. Kawamori. Player's patience and equilibrium payoffs in the baron-ferejohn model. *Economics Bulletin*, 3(43):1–5, 2005.

[20] D. A. Lake and R. Powell, editors. *Strategic Choice and International Relations*. Princeton University Press, 1999.

[21] A. laruelle and F. Valenciano. Bargaining in committees as an extension of nash's bargaining theory. *Journal of Economic Theory*, 132(1):291–305, 2007.

[22] R. D. McKelvey and R. E. Wendell. Voting equilibria in multidimensional choice spaces. *Mathematics of Operations Research*, 1(2):144–158, 1976.

[23] L. Miller and C. Vanberg. Group size and decision rules in legislative bargaining. *European Journal of Political Economy*, 37:288–302, 2015.

[24] H. V. Milner. *Interests, Institutions, and Information: Domestic Politics and International Relations*. Princeton University Press, 1997.

[25] M. Morelli. Demand competition and policy compromise in legislative bargaining. *The American Political Science Review*, 93(4):809–820, 1999.

[26] S. Nunnari. Dynamic legislative bargaining with veto power. *Unpublished manuscript*, 2012.

[27] C. R. Plott. A notion of equilibrium and its possibility under majority rule. *The American Economic Review*, 57(4):787–806, 1967.

[28] P. J. Reny. On the existence of pure and mixed strategy nash equilibria in discontinuous games. *Econometrica*, 67(5):1029–1056, 1999.

[29] M. A. Satterthwaite. Strategy-proofness and arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of Economic Theory*, 10(2):187–217, 1975.

[30] L. K. Simon. Games with discontinuous payoffs. *The Review of Economic Studies*, 54(4):569–597, 1987.

[31] L. K. Simon and W. R. Zame. Discontinuous games and endogenous sharing rules. *Econometrica: Journal of the Econometric Society*, pages 861–872, 1990.

[32] A. Tarar. Constituencies and preferences in international bargaining. *Journal of Conflict Resolution*, 49(3):383–407, 2005.